Dispersive optical model (DOM)

- Some reminders about Green's functions
- Second order and physical interpretation of (e,e'p) data
- Relevant physics considerations
- Dyson equation —> Schrödinger-like equation

- Use Green's function framework combined with data to extract the nucleon self-energy in finite nuclei
  - idea launched by Claude Mahaux end of 1980s
  - recent developments and motivation
  - later most recent work

Link of G with two-particle propagator

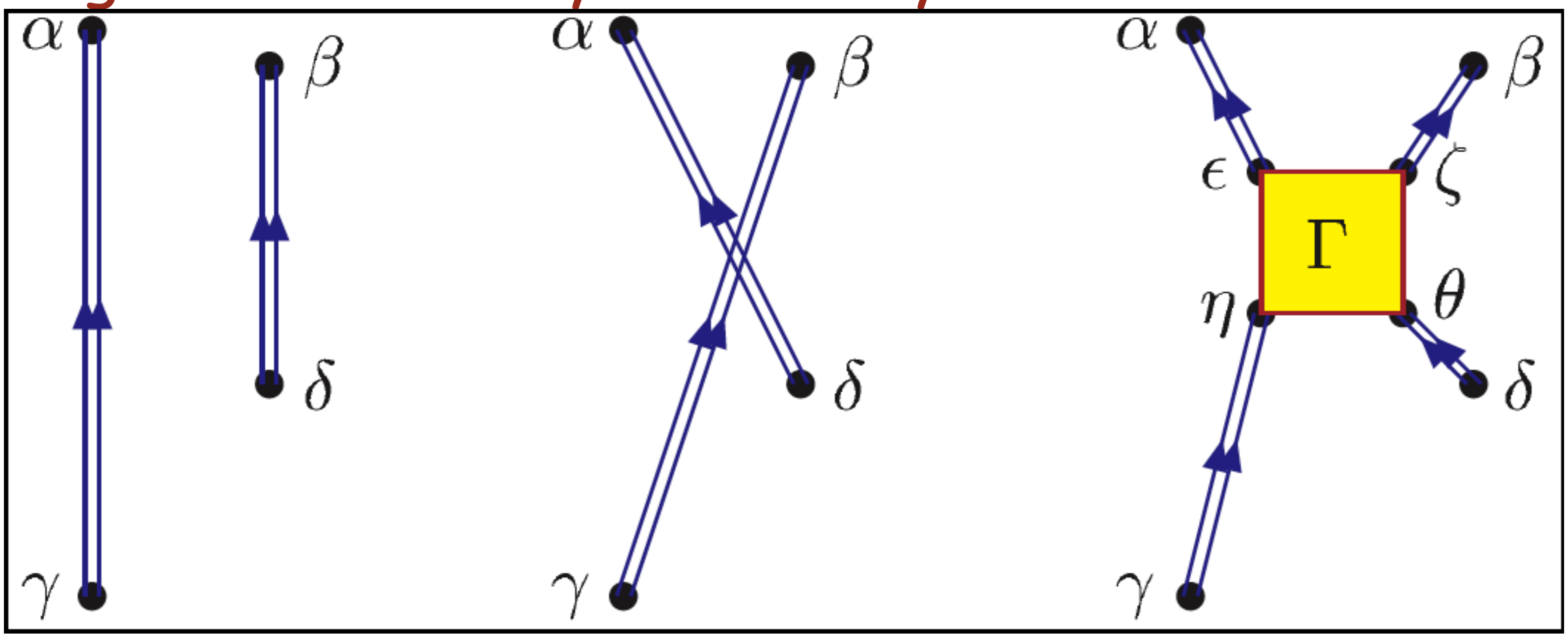
Equation of motion for G

$$i\hbar\frac{\partial}{\partial t}G(\alpha,\beta;t-t') = \delta(t-t')\delta_{\alpha,\beta} + \langle\Psi_0^N|\,\mathcal{T}[\frac{\partial a_{\alpha_H}(t)}{\partial t}a^\dagger_{\beta_H}(t')]\,|\Psi_0^N\rangle$$

$$= \delta(t-t')\delta_{\alpha,\beta} + \varepsilon_\alpha G(\alpha,\beta;t-t') - \sum_\delta \langle\alpha|\,U\,|\delta\rangle\, G(\delta,\beta;t-t')$$

$$+\frac{-i}{2\hbar}\sum_{\delta\zeta\theta}\langle\alpha\delta|\,V\,|\theta\zeta\rangle\,\langle\Psi_0^N|\,\mathcal{T}[a^\dagger_{\delta_H}(t)a_{\zeta_H}(t)a_{\theta_H}(t)a^\dagger_{\beta_H}(t')]\,|\Psi_0^N\rangle$$

Diagrammatic analysis of G^{II} yields



Γ is the effective interaction (vertex function) between correlated particles in the medium.

Rework

- Rearrange and do some relabeling: inverse FT
- Magic: again DE!!

$$G(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma,\delta} G^{(0)}(\alpha,\gamma;E)\Sigma^*(\gamma,\delta;E)G(\delta,\beta;E)$$

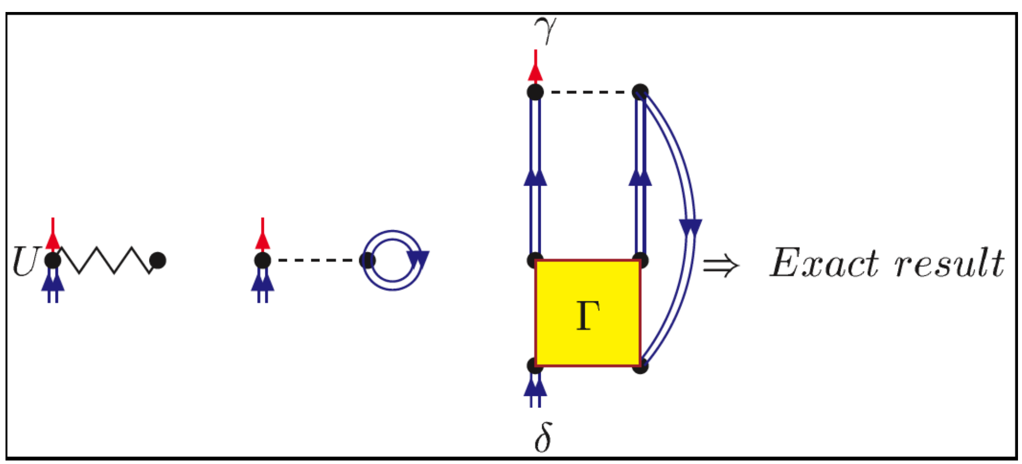
- with

$$\Sigma^*(\gamma,\delta;E) = -\langle\gamma|\,U\,|\delta\rangle - i\int_{C\uparrow}\frac{dE'}{2\pi}\sum_{\mu,\nu}\langle\gamma\mu|\,V\,|\delta\nu\rangle\,G(\nu,\mu;E')$$

$$+\frac{1}{2}\int\frac{dE_1}{2\pi}\int\frac{dE_2}{2\pi}\sum_{\epsilon,\mu,\nu,\zeta,\rho,\sigma}\langle\gamma\mu|\,V\,|\epsilon\nu\rangle\,G(\epsilon,\zeta;E_1)G(\nu,\rho;E_2)$$

$$\times G(\sigma,\mu;E_1+E_2-E)\,\langle\zeta\rho|\,\Gamma(E_1,E_2;E,E_1+E_2-E)\,|\delta\sigma\rangle$$

- Diagrammatically

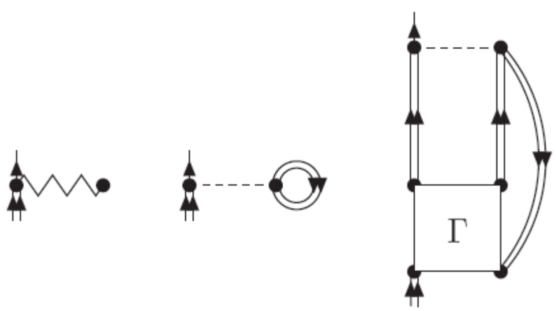


Beyond the mean-field approximation

- Consider again

$$\Sigma^*(\gamma,\delta;E) = -\langle\gamma|\,U\,|\delta\rangle - i\int_{C\uparrow}\frac{dE'}{2\pi}\sum_{\mu,\nu}\langle\gamma\mu|\,V\,|\delta\nu\rangle\, G(\nu,\mu;E')$$
$$+\frac{1}{2}\int\frac{dE_1}{2\pi}\int\frac{dE_2}{2\pi}\sum_{\epsilon,\mu,\nu,\zeta,\rho,\sigma}\langle\gamma\mu|\,V\,|\epsilon\nu\rangle\, G(\epsilon,\zeta;E_1)G(\nu,\rho;E_2)$$
$$\times G(\sigma,\mu;E_1+E_2-E)\,\langle\zeta\rho|\,\Gamma(E_1,E_2;E,E_1+E_2-E)\,|\delta\sigma\rangle$$

- self-energy



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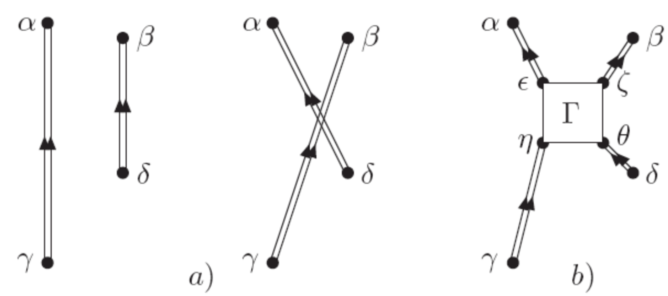


- When the two-body interaction is weak but not negligible, one can make the "Born" approximation for the two-body propagator
- The self-energy term then contains a dynamic second-order term

Beyond the mean-field approximation

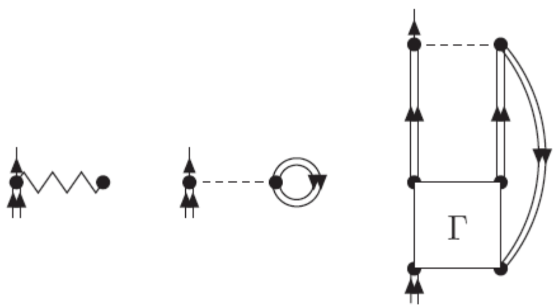
- Consider again

$$\Sigma^*(\gamma,\delta;E) = -\langle\gamma|\,U\,|\delta\rangle - i\int_{C\uparrow}\frac{dE'}{2\pi}\sum_{\mu,\nu}\langle\gamma\mu|\,V\,|\delta\nu\rangle\,G(\nu,\mu;E')$$

$$+\frac{1}{2}\int\frac{dE_1}{2\pi}\int\frac{dE_2}{2\pi}\sum_{\epsilon,\mu,\nu,\zeta,\rho,\sigma}\langle\gamma\mu|\,V\,|\epsilon\nu\rangle\,G(\epsilon,\zeta;E_1)G(\nu,\rho;E_2)$$

$$\times G(\sigma,\mu;E_1+E_2-E)\,\langle\zeta\rho|\,\Gamma(E_1,E_2;E,E_1+E_2-E)\,|\delta\sigma\rangle$$

- self-energy



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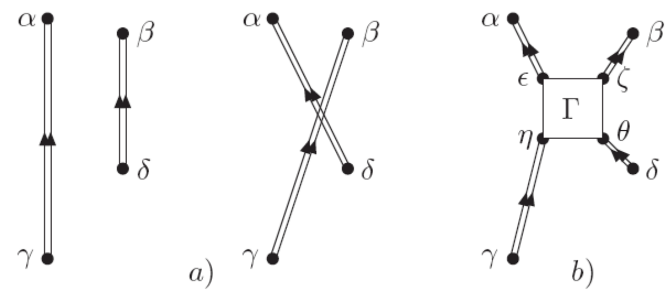


- When the two-body interaction is weak but not negligible, one can make the “Born” approximation for the two-body propagator
- The self-energy term then contains a dynamic second-order term

Second-order self-energy

- Expression with noninteracting propagators in Ch.9
- With self-consistent sp propagators

$$\Sigma^{(2)}(\gamma,\delta;E) = -\frac{1}{2}\int\frac{dE_1}{2\pi i}\int\frac{dE_2}{2\pi i}\sum_{\lambda,\epsilon,\nu}\sum_{\zeta,\xi,\mu}\langle\gamma\lambda|V|\epsilon\nu\rangle\langle\zeta\xi|V|\delta\mu\rangle \times G(\epsilon,\zeta;E_1)G(\nu,\xi;E_2)G(\mu,\lambda;E_1+E_2-E)$$

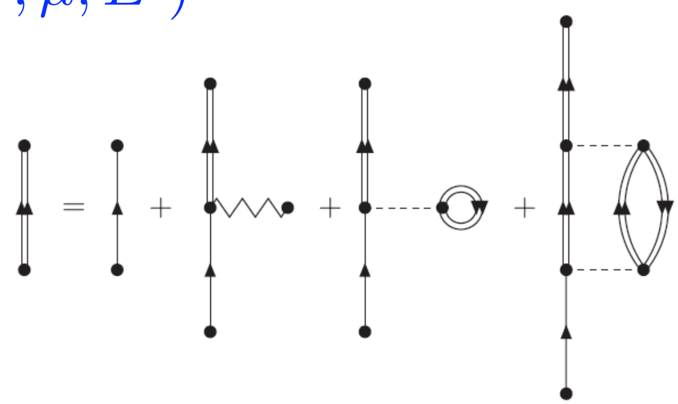
- Propagator therefore solves

$$G(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma\delta} G(\alpha,\gamma;E)\Sigma(\gamma,\delta;E)G^{(0)}(\delta,\beta;E)$$

$$\Sigma(\gamma,\delta;E) = -\langle\gamma|U|\delta\rangle + \Sigma^{(1)}(\gamma,\delta) + \Sigma^{(2)}(\gamma,\delta;E)$$

$$\Sigma^{(1)}(\gamma,\delta) = -i\int_{C\uparrow}\frac{dE'}{2\pi}\sum_{\mu\nu}\langle\gamma\mu|V|\delta\nu\rangle G(\nu,\mu;E')$$

- Diagrammatically



- Obtained from

$$\langle\zeta\rho|\Gamma(E_1,E_2;E_3,E_4)|\delta\sigma\rangle \equiv \langle\zeta\rho|V|\delta\sigma\rangle$$

Procedure

- Note first-order not equal HF!
- U term cancels as always
- Similar procedure as in HF
- Assume

$$G(\alpha,\beta;E) = \sum_m \frac{z_\alpha^{m+} z_\beta^{m+^*}}{E - \varepsilon_m^+ + i\eta} + \sum_n \frac{z_\alpha^{n-} z_\beta^{n-^*}}{E - \varepsilon_n^- - i\eta}$$

- Second-order self-energy by appropriate contour integration
- Integrals are of the form

$$I(E) = \int_{-\infty}^{+\infty} \frac{dE'}{2\pi i} \left(\frac{F_1}{E' - f_1 + i\eta} + \frac{B_1}{E' - b_1 - i\eta} \right) \times \left(\frac{F_2}{E' - E - f_2 + i\eta} + \frac{B_2}{E' - E - b_2 - i\eta} \right)$$

- Close contour in upper or lower half
- Four terms: two vanish with both poles on the same side
- Residue theorem: $I(E) = \dfrac{F_1 B_2}{E - (f_1 - b_2) + i\eta} - \dfrac{B_1 F_2}{E + (f_2 - b_1) - i\eta}$

Self-energy

- Apply to second-order self-energy

$$\Sigma^{(2)}(\gamma,\delta;E) = \frac{1}{2}\sum_{\lambda,\epsilon,\nu}\sum_{\zeta,\xi,\mu}\langle\gamma\lambda|V|\epsilon\nu\rangle\langle\zeta\xi|V|\delta\mu\rangle$$
$$\times\left(\sum_{m_1m_2n_3}\frac{z_\epsilon^{m_1+}z_\zeta^{m_1+^*}z_\nu^{m_2+}z_\xi^{m_2+^*}z_\mu^{n_3-}z_\lambda^{n_3^*}}{E-(\varepsilon_{m_1}^{+}+\varepsilon_{m_2}^{+}-\varepsilon_{n_3}^{-})+i\eta}\right.$$
$$\left.+\sum_{n_1n_2m_3}\frac{z_\epsilon^{n_1-}z_\zeta^{n_1-^*}z_\nu^{n_2-}z_\xi^{n_2-^*}z_\mu^{m_3+}z_\lambda^{m_3+^*}}{E+(\varepsilon_{m_3}^{+}-\varepsilon_{n_1}^{-}-\varepsilon_{n_2}^{-})-i\eta}\right)$$

- Remember: poles of propagator $\forall m,n:\varepsilon_n^- \leq \varepsilon_F^- < \varepsilon_F < \varepsilon_F^+ \leq \varepsilon_m^+$
- with $\varepsilon_F = \frac{1}{2}[\varepsilon_F^- + \varepsilon_F^+]$
- Therefore poles in self-energy obey

$$\forall m_i,n_i:\varepsilon_{n_1}^- + \varepsilon_{n_2}^- - \varepsilon_{m_3}^+ < \varepsilon_F < \varepsilon_{m_1}^+ + \varepsilon_{m_2}^+ - \varepsilon_{n_3}^-$$

- and have cuts when the spectra of N±1 have continuous parts

Solution of Dyson equation

- Fully self-consistent solution is possible (see later)
- First study how the presence of the energy dependence in the self-energy modifies the Dyson equation
- Start by solving HF first
- Then choose auxiliary potential to be HF potential so $G^{(0)} \equiv G^{HF}$
- Choose HF sp basis so $G^{HF}(\alpha,\beta;E) = \delta_{\alpha,\beta}\left[\frac{\theta(\alpha-F)}{E-\varepsilon_\alpha+i\eta} + \frac{\theta(F-\alpha)}{E-\varepsilon_\alpha-i\eta}\right]$
- is diagonal and to obtain 2nd order self-energy replace

$$z_\alpha^{m+} = \delta_{m,\alpha}\theta(\alpha-F); \quad z_\alpha^{n-} = \delta_{n,\alpha}\theta(F-\alpha)$$

- as a first iteration step in full solution

$$\Sigma^{(2)}(\gamma,\delta;E) = \frac{1}{2}\sum_{\lambda,\epsilon,\nu} \langle\gamma\lambda| V |\epsilon\nu\rangle \langle\epsilon\nu| V |\delta\lambda\rangle$$

$$\times \left(\frac{\theta(\epsilon-F)\theta(\nu-F)\theta(F-\lambda)}{E-(\varepsilon_\epsilon+\varepsilon_\nu-\varepsilon_\lambda)+i\eta} + \frac{\theta(F-\epsilon)\theta(F-\nu)\theta(\lambda-F)}{E+(\varepsilon_\lambda-\varepsilon_\epsilon-\varepsilon_\nu)-i\eta}\right)$$

Solution strategy

- Compact notation

$$\Sigma^{(2)}(\gamma,\delta;E)=\frac{1}{2}\left(\sum_{p_1p_2h_3}\frac{\langle\gamma h_3|V|p_1p_2\rangle\langle p_1p_2|V|\delta h_3\rangle}{E-(\varepsilon_{p_1}+\varepsilon_{p_2}-\varepsilon_{h_3})+i\eta}+\sum_{h_1h_2p_3}\frac{\langle\gamma p_3|V|h_1h_2\rangle\langle h_1h_2|V|\delta p_3\rangle}{E+(\varepsilon_{p_3}-\varepsilon_{h_1}-\varepsilon_{h_2})-i\eta}\right)$$

- identifies particles and holes
- Next solve

$$G(\alpha,\beta;E)=G^{HF}(\alpha,\beta;E)+\sum_{\gamma\delta}G(\alpha,\gamma;E)\Sigma^{(2)}(\gamma,\delta;E)G^{HF}(\delta,\beta;E)$$

- In principle, the solutions will contain nondiagonal contributions
- Sometimes (closed-shell atoms or nuclei) these can be neglected
- Corresponding self-energy

$$\Sigma^{(2)}(\alpha;E)=\frac{1}{2}\left(\sum_{p_1p_2h_3}\frac{|\langle\alpha h_3|V|p_1p_2\rangle|^2}{E-(\varepsilon_{p_1}+\varepsilon_{p_2}-\varepsilon_{h_3})+i\eta}+\sum_{h_1h_2p_3}\frac{|\langle\alpha p_3|V|h_1h_2\rangle|^2}{E+(\varepsilon_{p_3}-\varepsilon_{h_1}-\varepsilon_{h_2})-i\eta}\right)$$

Diagonal Dyson equation

- Corresponding DE

$$G(\alpha;E) = G^{HF}(\alpha;E) + G(\alpha;E)\Sigma^{(2)}(\alpha;E)G^{HF}(\alpha;E)$$

- Solution (like in the infinite HF case) algebraic

$$G(\alpha;E) = \frac{1}{\frac{1}{G^{HF}(\alpha;E)} - \Sigma^{(2)}(\alpha;E)} = \frac{1}{E - \varepsilon_\alpha - \Sigma^{(2)}(\alpha;E)}$$

- noting that

$$\frac{1}{G^{HF}(\alpha;E)} = E - \varepsilon_\alpha$$

- Physical information related to poles and residues
- Assume (for the sake of pedagogy) that the self-energy has poles at a set of discrete energies (isolated simple poles)
- Poles of propagator solutions of $E_{n\alpha} = \varepsilon_\alpha + \Sigma^{(2)}(\alpha;E_{n\alpha})$

More

- Solutions $$E_{n\alpha} = \varepsilon_\alpha + \Sigma^{(2)}(\alpha; E_{n\alpha})$$
- Residues from

$$\begin{aligned} R_{n\alpha} &= \lim_{E \to E_{n\alpha}} (E - E_{n\alpha}) G(\alpha; E) = \lim_{E \to E_{n\alpha}} \frac{E - E_{n\alpha}}{E - \varepsilon_\alpha - \Sigma^{(2)}(\alpha; E)} \\ &= \left(1 - \left. \frac{d\Sigma^{(2)}(\alpha; E)}{dE} \right|_{E = E_{n\alpha}} \right)^{-1} \end{aligned}$$

- noting that

$$\Sigma^{(2)}(\alpha; E) = \Sigma^{(2)}(\alpha; E_{n\alpha}) + (E - E_{n\alpha}) \left. \frac{d\Sigma^{(2)}(\alpha; E)}{dE} \right|_{E = E_{n\alpha}}$$

- Infinitesimal imaginary parts are irrelevant when dealing with discrete poles (not with continuum), since poles of self-energy are different from those of propagator
- Solution: plot $E - \varepsilon_\alpha$ and $\Sigma^{(2)}(\alpha; E)$
- Find intersections!

Graphical solution

Plot:

self-energy

ph gap

$\Delta = \varepsilon_p^{min} - \varepsilon_h^{max}$

centered on

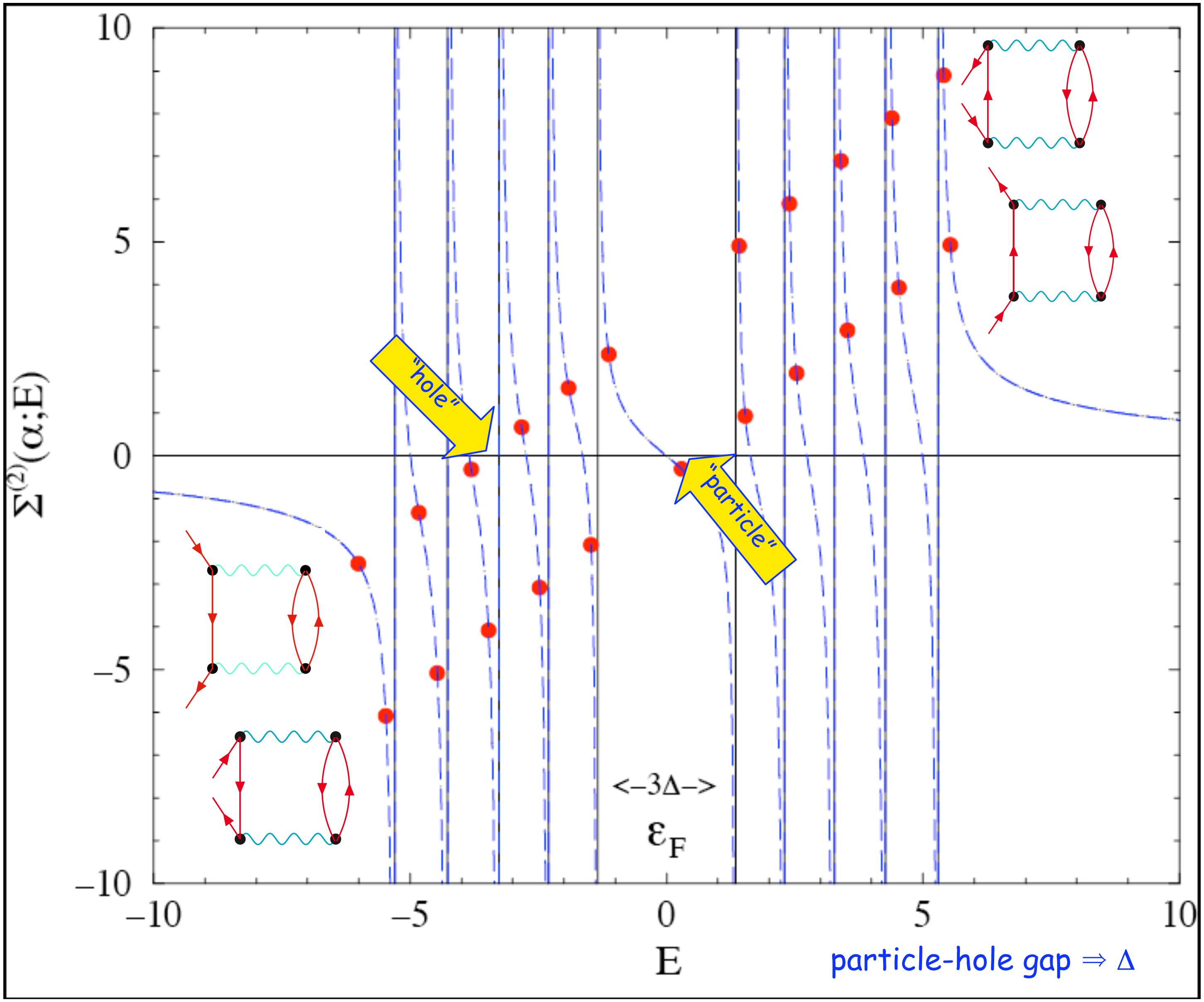
$\varepsilon_F = \frac{1}{2}(\varepsilon_p^{min} + \varepsilon_h^{max})$

gap 3Δ for

self-energy

Solutions:

intersect



with $E - \varepsilon_\alpha$ so D poles in self-energy yields D+1 solutions

Explains all qualitative features of sp strength distribution in nuclei!

Interpretation

- Poles in the removal domain: approximate energies of N-1 eigenstates $E_{n\alpha} \approx E_0^N - E_n^{N-1}$
- Corresponding residue: squared removal amplitude
$$R_{n\alpha} \approx |\langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle|^2$$
- Similarly in the addition domain: approximate energies of N+1 eigenstates $E_{n\alpha} \approx E_n^{N+1} - E_0^N$
- Addition probability:
$$R_{n\alpha} \approx |\langle \Psi_n^{N+1} | a_\alpha^\dagger | \Psi_0^N \rangle|^2$$
- Derivative of self-energy always negative so $0 \leq R_{n\alpha} \leq 1$
- Plot illustrates various possibilities and the relation with time-ordered diagrams further explored next...
- Note: no longer purely particle or hole interpretation possible

Mixing in nuclear physics I

Example: p and 2p1h

$$\begin{pmatrix} \varepsilon_p + \langle p|\, V\, |p\rangle & \langle p|\, V\, |2p1h\rangle \\ \langle 2p1h|\, V\, |p\rangle & \varepsilon_{2p1h} + \langle 2p1h|\, V\, |2p1h\rangle \end{pmatrix} \begin{pmatrix} \psi_p \\ \psi_{2p1h} \end{pmatrix} = E \begin{pmatrix} \psi_p \\ \psi_{2p1h} \end{pmatrix}$$

Assume little effect from $\langle 2p1h|\, V\, |2p1h\rangle \Rightarrow 0$

Equivalent to

$$\left(\varepsilon_p + \langle p|\, V\, |p\rangle + \underbrace{\langle p|\, V\, |2p1h\rangle \frac{1}{E - \varepsilon_{2p1h}} \langle 2p1h|\, V\, |p\rangle} \right) (\psi_p) = E\, (\psi_p)$$

In the continuum $\Rightarrow$ complex "optical" potential

Mixing in nuclear physics II

Yet another example: h and 1p2h

$$\begin{pmatrix} \varepsilon_h + \langle h|V|h\rangle & \langle h|V|1p2h\rangle \\ \langle 1p2h|V|h\rangle & \varepsilon_{1p2h} + \langle 1p2h|V|1p2h\rangle \end{pmatrix} \begin{pmatrix} \psi_h \\ \psi_{1p2h} \end{pmatrix} = E \begin{pmatrix} \psi_h \\ \psi_{1p2h} \end{pmatrix}$$

Assume little effect from $\langle 1p2h|V|1p2h\rangle \Rightarrow 0$

Equivalent to

$$\left(\varepsilon_h + \langle h|V|h\rangle + \underbrace{\langle h|V|1p2h\rangle \frac{1}{E - \varepsilon_{1p2h}} \langle 1p2h|V|h\rangle} \right) (\psi_h) = E\,(\psi_h)$$

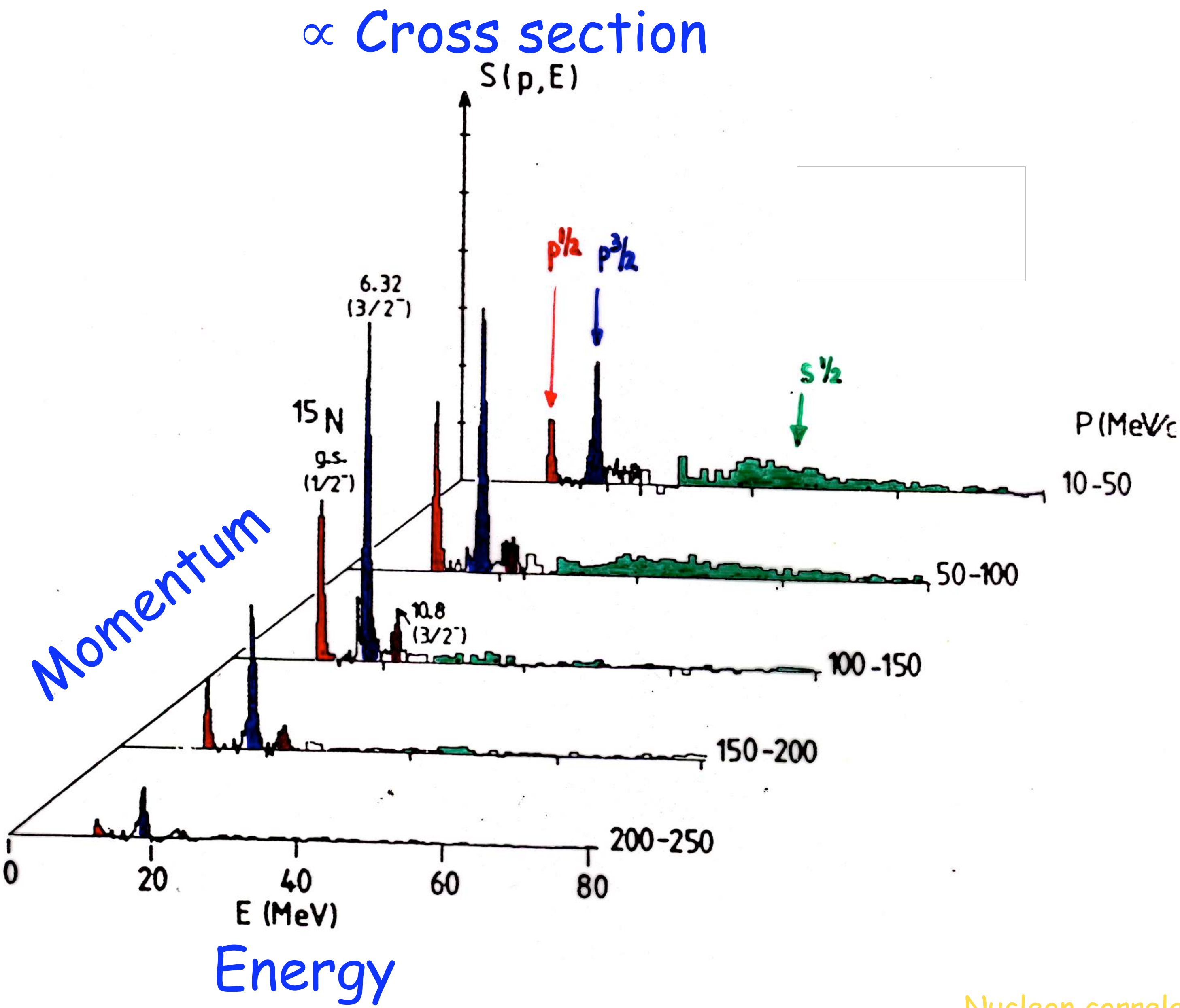
Energy-dependent self-energy below ε_F (and poles)

Explains fragmentation of single-particle strength ⇒ (e,e'p)

Note: so far only mixing on one side of ε_F

Mougey et al., Nucl. Phys. A335, 35 (1980)

$^{16}O(e,e'p)$



Mixing across the Fermi energy ⇒ inclusion of ground-state correlations

Example: α=p/h and 1p2h and 2p1h

$$\begin{pmatrix} \varepsilon_\alpha + \langle\alpha| V |\alpha\rangle & \langle\alpha| V |1p2h\rangle & \langle\alpha| V |2p1h\rangle \\ \langle 1p2h| V |\alpha\rangle & \varepsilon_{1p2h} + \langle 1p2h| V |1p2h\rangle & 0 \\ \langle 2p1h| V |\alpha\rangle & 0 & \varepsilon_{2p1h} + \langle 2p1h| V |2p1h\rangle \end{pmatrix} \begin{pmatrix} \psi_\alpha \\ \psi_{1p2h} \\ \psi_{2p1h} \end{pmatrix} = E \begin{pmatrix} \psi_\alpha \\ \psi_{1p2h} \\ \psi_{2p1h} \end{pmatrix}$$

Assume little effect from $\langle 1p2h| V |1p2h\rangle$

⇒ 0, etc.

Equivalent to ★

$$\left(\varepsilon_\alpha + \langle\alpha| V |\alpha\rangle + \langle\alpha| \Sigma^{(2)}(E) |\alpha\rangle\right)(\psi_\alpha) = E\,(\psi_\alpha)$$

Explains also depletion of single-particle strength!

Self-consistent treatment of $\Sigma^{(2)}$

- Self-consistent treatment for a finite system
- Keep approximation of discrete poles and diagonal self-energy

$$G(\alpha;E) = \sum_m \frac{|z_\alpha^{m+}|^2}{E - \varepsilon_{m\alpha}^+ + i\eta} + \sum_n \frac{|z_\alpha^{n-}|^2}{E - \varepsilon_{n\alpha}^- - i\eta}$$

- appropriate for closed-shell nuclei and atoms
- Second-order self-energy

$$\Sigma^{(2)}(\alpha;E) = \frac{1}{2} \sum_{\lambda,\epsilon,\nu} |\langle\alpha\lambda| V |\epsilon\nu\rangle|^2 \times \left(\sum_{m_1 m_2 n_3} \frac{|z_\epsilon^{m_1+}|^2 |z_\nu^{m_2+}|^2 |z_\lambda^{n_3-}|^2}{E - (\varepsilon_{m_1\epsilon}^+ + \varepsilon_{m_2\nu}^+ - \varepsilon_{n_3\lambda}^-) + i\eta} + \sum_{n_1 n_2 m_3} \frac{|z_\epsilon^{n_1-}|^2 |z_\nu^{n_2-}|^2 |z_\lambda^{m_3+}|^2}{E + (\varepsilon_{m_3\lambda}^+ - \varepsilon_{n_1\epsilon}^- - \varepsilon_{n_2\nu}^-) - i\eta} \right)$$

- First-order

$$\Sigma^{(1)}(\alpha) = \sum_\beta \langle\alpha\beta| V |\alpha\beta\rangle \left(\sum_n |z_\beta^{n-}|^2 \right)$$

- can be absorbed into new sp energies by rewriting DE

SCGF

- Treatment is like HF: determines self-consistent Green's functions (SCGF)
- Both first- and second-order self-energy depend on these solutions and must be updated
- Solve DE again etc. so iterative procedure
- Strictly speaking: cannot use only discrete poles (dimensionality)
- Two practical approaches
- Bin energy axis and sum strength in each bin; then update propagator by taking center and summed strength in each bin
- or Replace spectral distribution by a small number of poles chosen to reproduce lowest-order energy-weighted moments of spectral function
- or treat continuum properly!

Schematic model

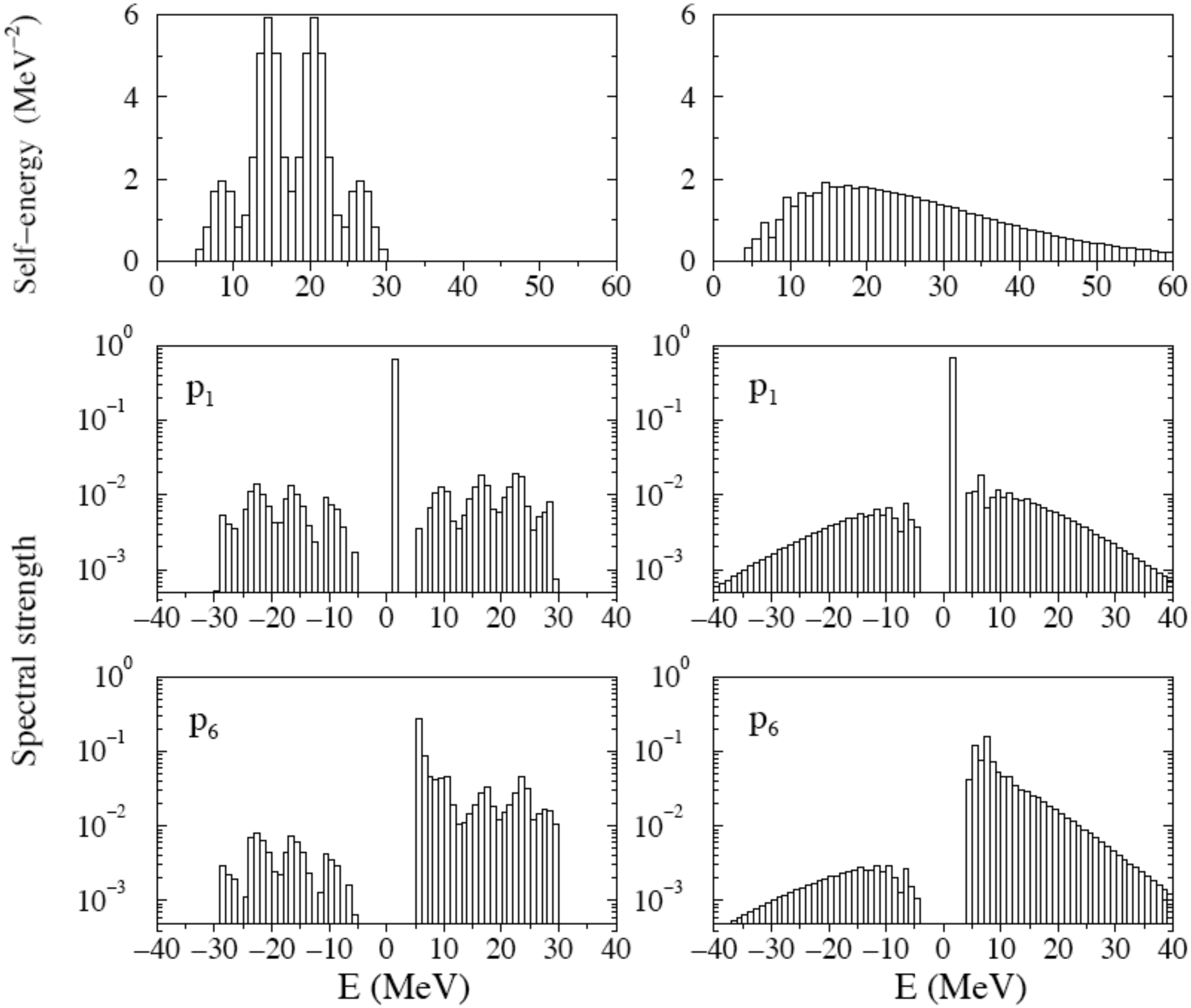
- Take M particle and M hole states with sp energies $\varepsilon_{h_i} = -\varepsilon_{p_i}$
- Keep sp energy fixed (neglect first-order self-energy)
- Assume constant interaction strength $|\langle\alpha\beta|V|\gamma\delta\rangle|^2 = |v|^2$
- With these assumptions $\Sigma(-E) = -\Sigma(E)$ is state-independent
- and there is exact ph symmetry $G(p_i; E) = -G(h_i; -E)$
- Example: M=6 $|v| = 0.75 \text{ MeV}$ and

$$\varepsilon_{p_i} = 2, 3, 4, 8, 9, 10 \text{ MeV}, \text{ for } i = 1, \ldots, 6$$

- mimicking two nuclear major shell above & below the Fermi level
- Solved iteratively with 0.1 MeV wide bins
- Illustrated for particle states 1 and 6 (collected in 1 MeV bins)

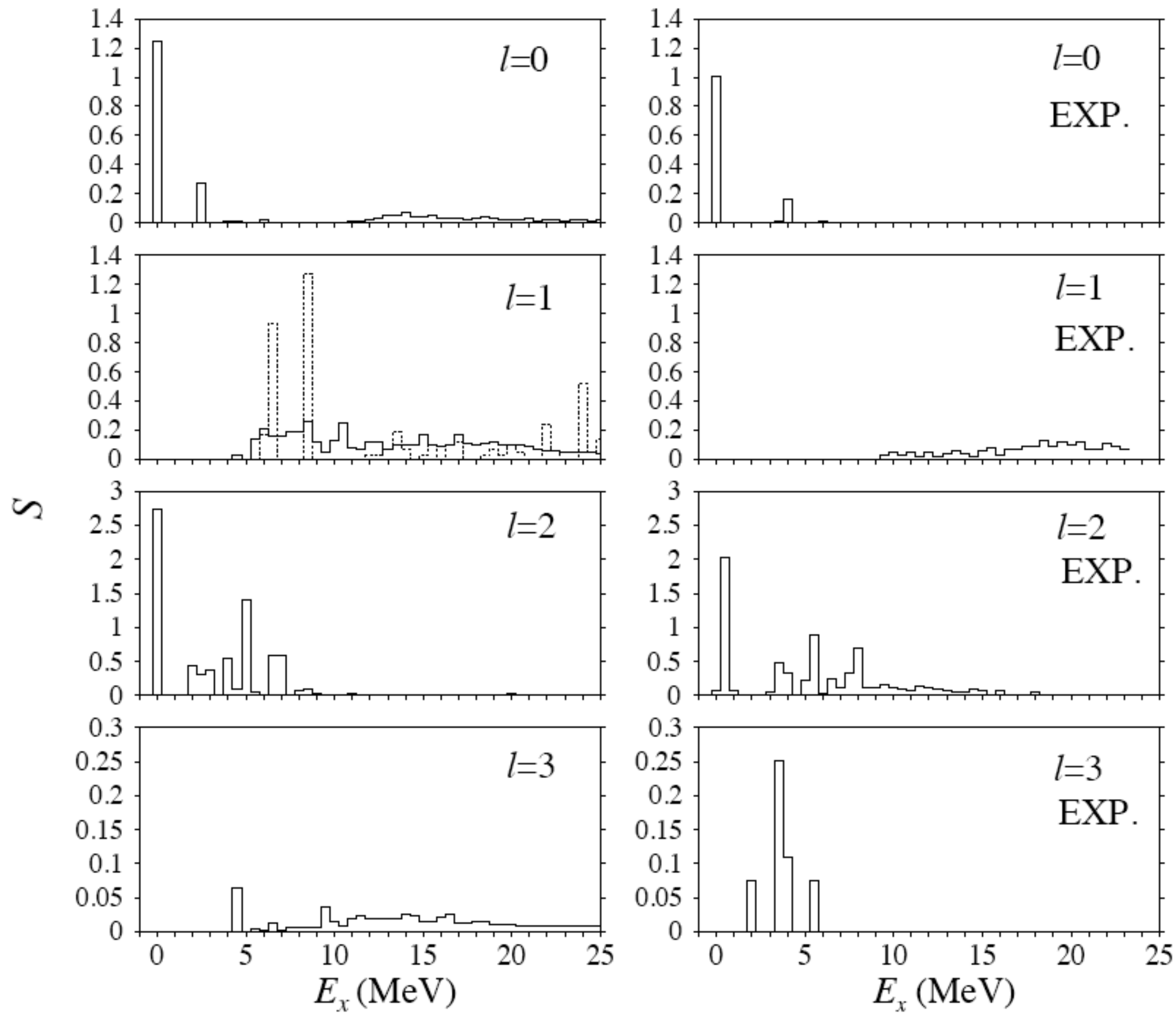
Plot: self-energy $\frac{1}{\pi}|\mathrm{Im}\ \Sigma(E)|$ and spectral functions

- Left: first iteration
- Right: SCGF
- First iteration:
  - p1 QP peak 64%
  - p6 fragmented
- SCGF
  - self-energy spread to larger energies
  - vanishing shell structure except near the Fermi energy
  - spectral functions have similar features



Nuclei

- Cannot use realistic NN interaction in second order
- Can be done in higher order (see later)
- Use approximate effective interactions in a limited model space
- ^{48}Ca protons "occupy" $0s\frac{1}{2}$, $0p\frac{3}{2}$, $0p\frac{1}{2}$, $0d\frac{5}{2}$, $0d\frac{3}{2}$ and $1s\frac{1}{2}$
- Qualitative success but improvement necessary including better L&SRC



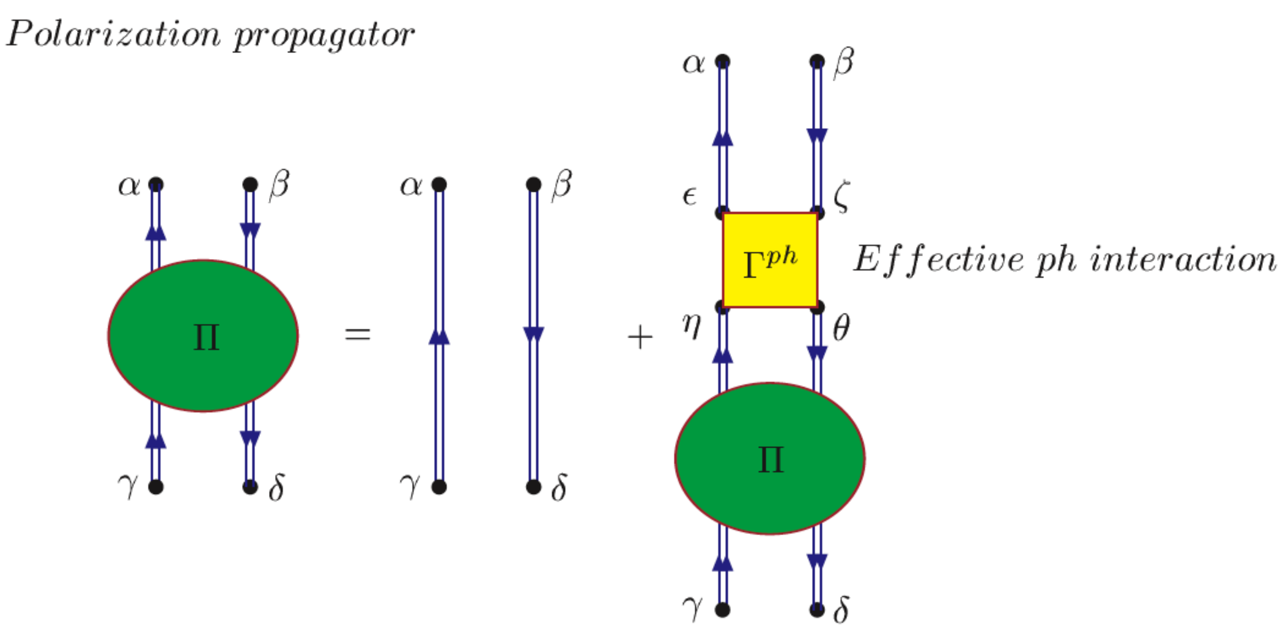
Van Neck, D., Waroquier, M. and Ryckebusch, J. (1991) *Nucl. Phys.* **A530**, 347.

FSI and (e,e´p) ⇔ analysis

$$\hat{O} = \sum_{\alpha\beta} \langle\alpha|\, O\, |\overline{\beta}\rangle\, a^\dagger_\alpha a_{\overline{\beta}}$$ Electron Scattering ⇒ one-body operator

$$\left|\langle\Psi^N_n|\, \hat{O}\, |\Psi^N_0\rangle\right|^2 = \sum_{\alpha\beta}\sum_{\gamma\delta} \langle\gamma|\, O\, |\overline{\delta}\rangle\, \langle\alpha|\, O\, |\overline{\beta}\rangle^*\, \langle\Psi^N_n|\, a^\dagger_\gamma a_{\overline{\delta}}\, |\Psi^N_0\rangle\, \langle\Psi^N_n|\, a^\dagger_\alpha a_{\overline{\beta}}\, |\Psi^N_0\rangle^*$$

Requires (imaginary part of) **exact** polarization propagator



Choose kinematics:

⇒ only first term

$$\langle\Psi^{N+1}_n|\, a^\dagger_\alpha\, |\Psi^N_0\rangle$$

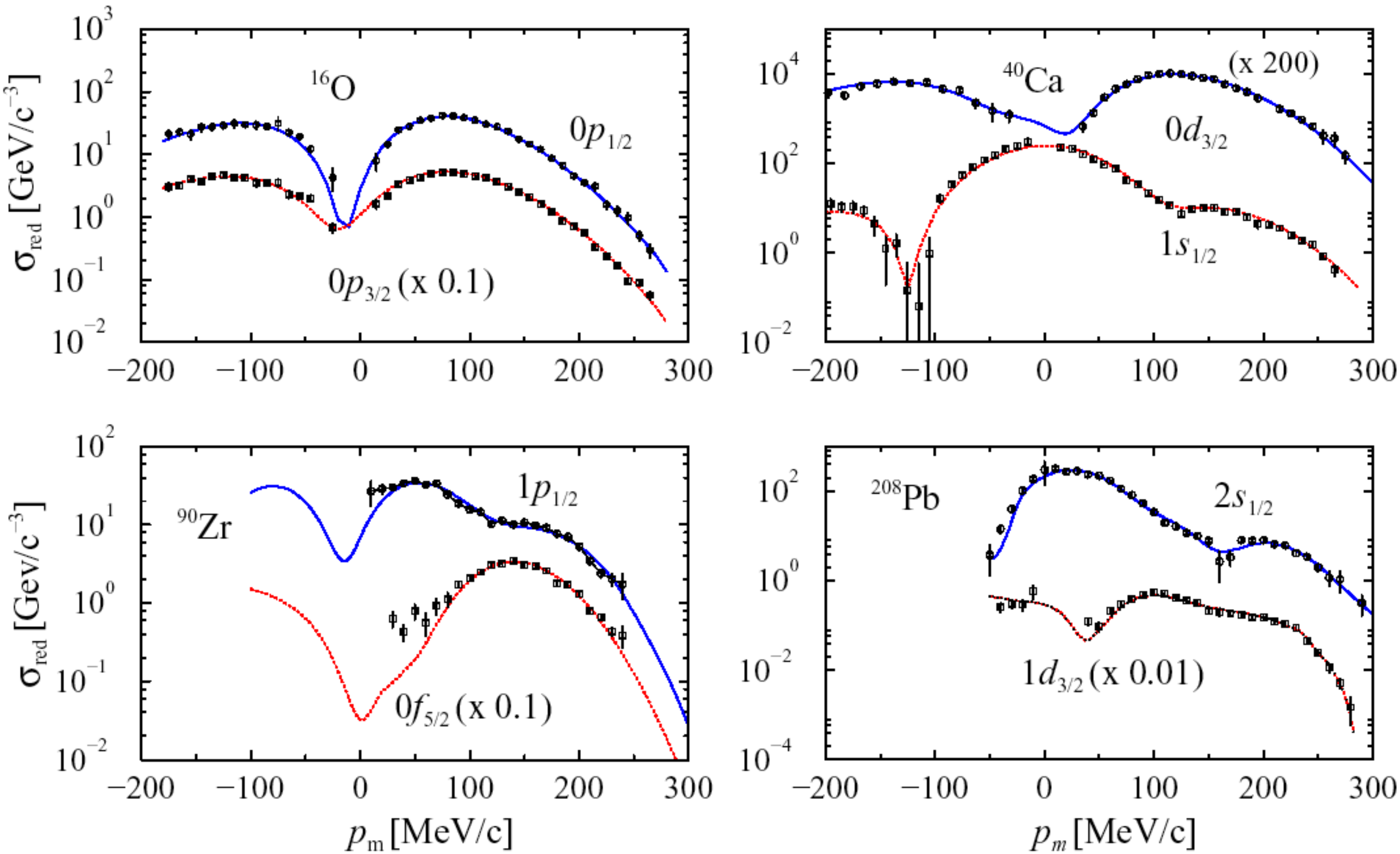
⇒ Elastic scattering (phenomenology)

$$\langle\Psi^{N-1}_m|\, a_\beta\, |\Psi^N_0\rangle$$

"Absolute" spectroscopic factors ✓? ⇒ Quasihole wave function

(e,e´p) cross sections for closed-shell nuclei

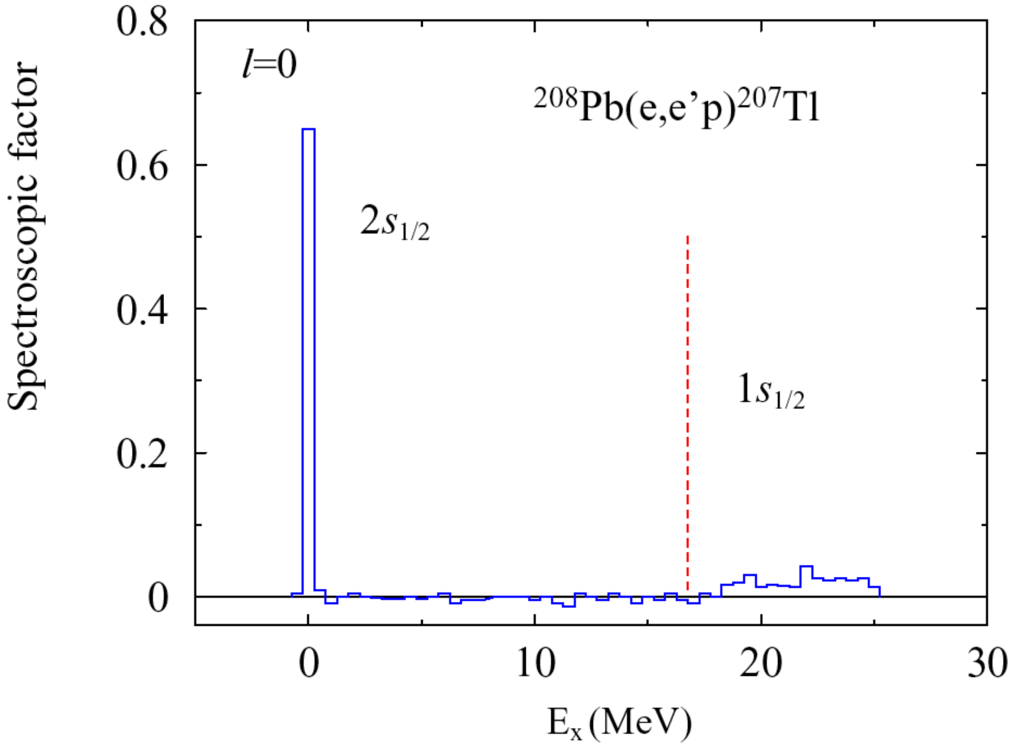
NIKHEF data, L. Lapikás, Nucl. Phys. A553, 297c (1993)

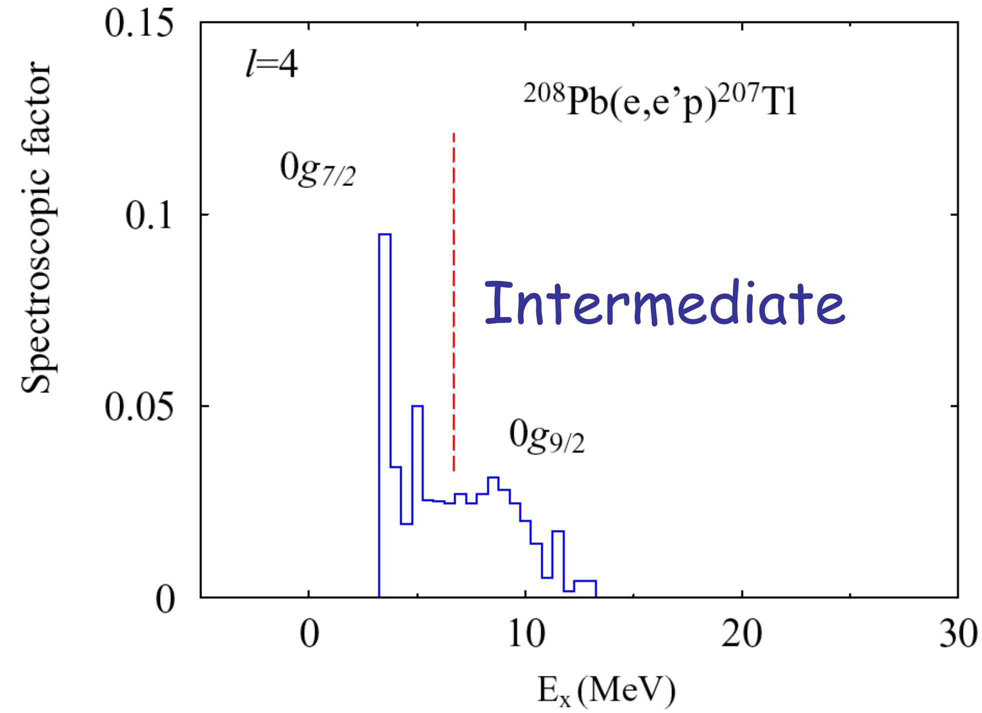


Normalization < 1

and …

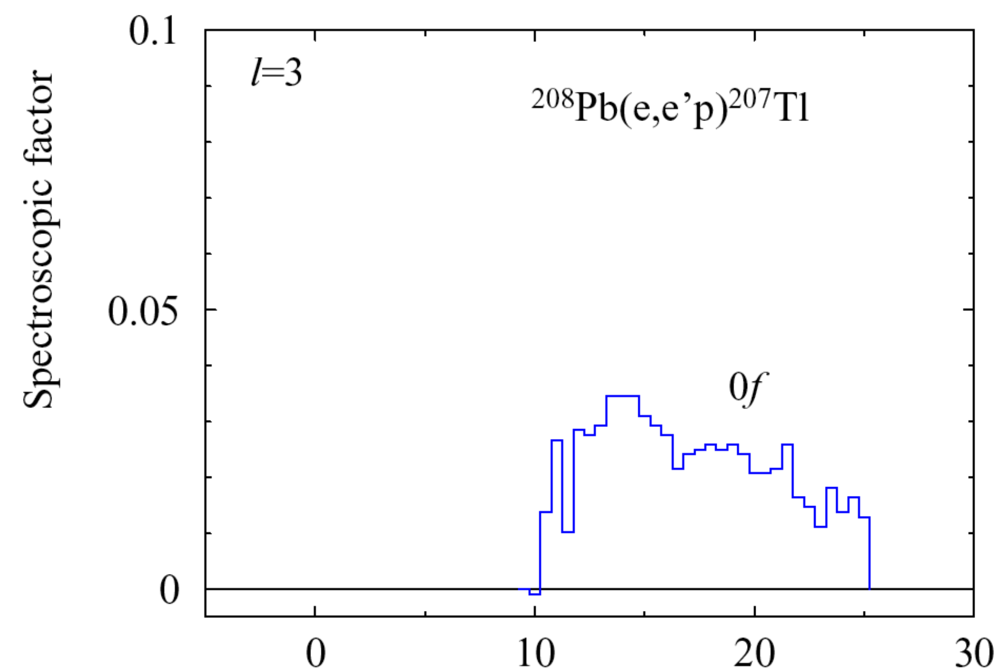
E. Quint, Ph.D.thesis NIKHEF, 1988





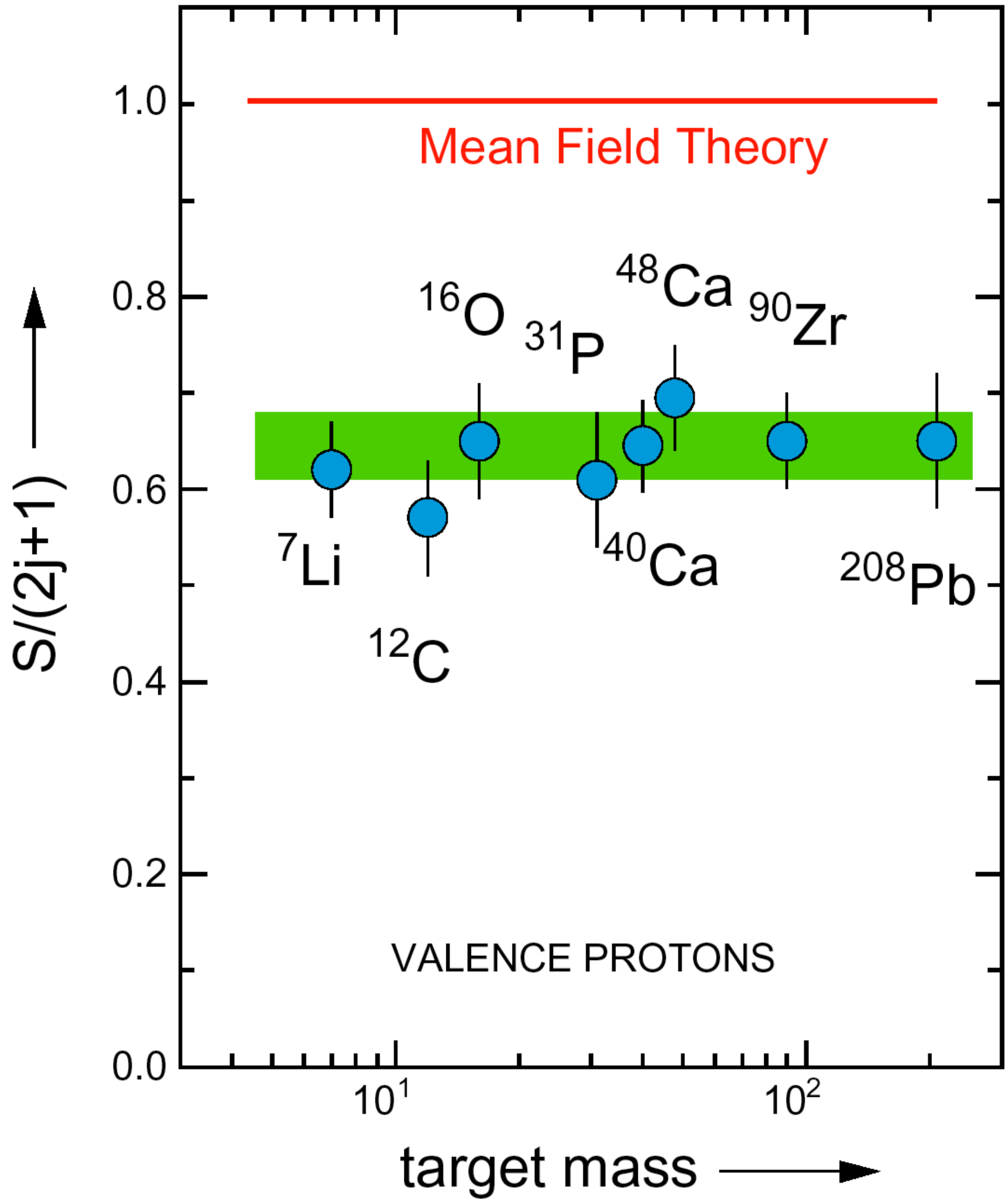
Quasihole strength or spectroscopic factor $Z(2s_{1/2})$ =0.65

$n(2s_{1/2}) = 0.75$
from elastic electron scattering



Strong fragmentation of deeply-bound states

Removal probability for valence protons from NIKHEF data Lapikás, NPA553,297c(1993)



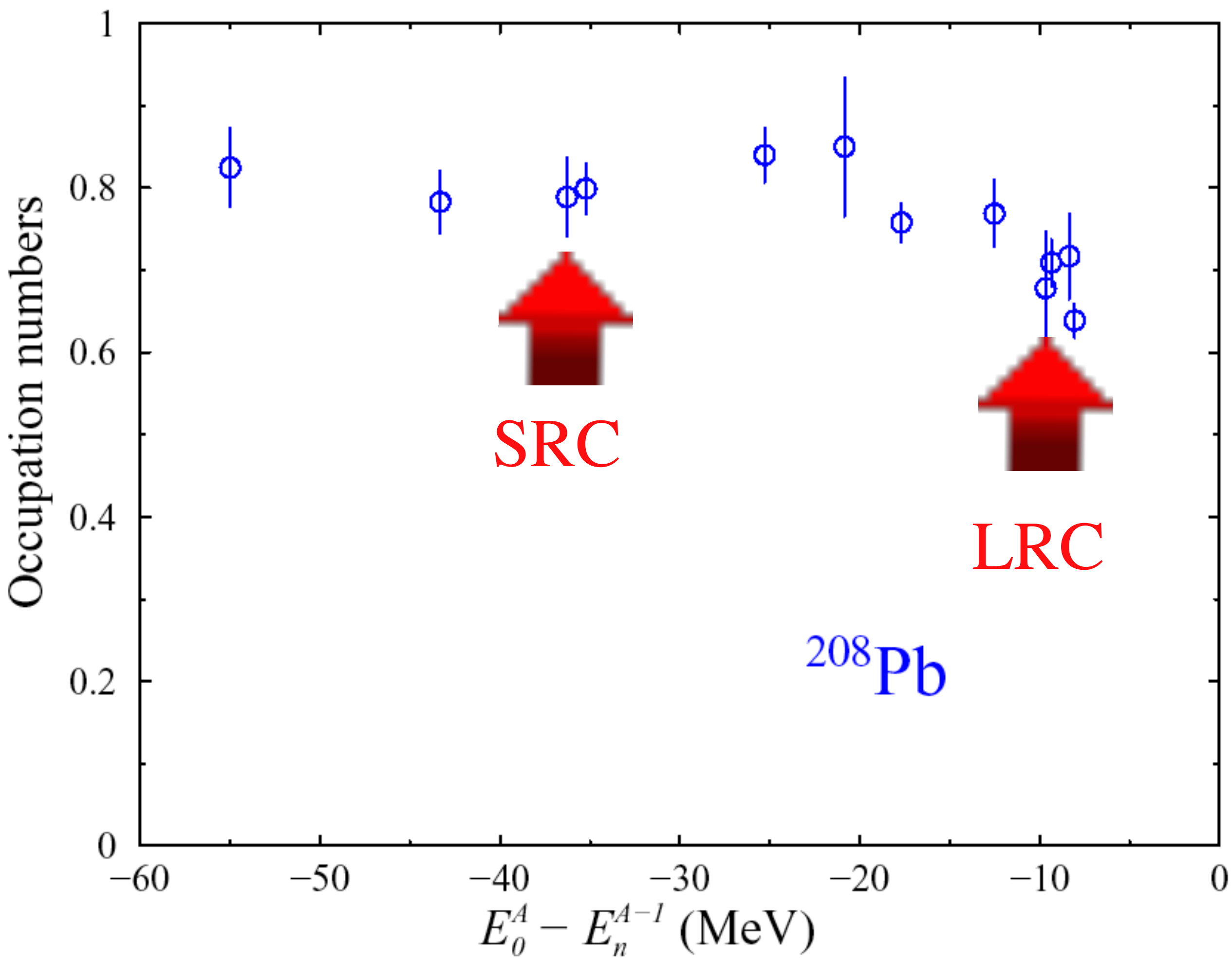
Note:
We have seen mostly data for removal of

valence protons

M. van Batenburg & L. Lapikás from ^{208}Pb (e,e´p) ^{207}Tl

NIKHEF group & W.D. to be published

Occupation of deeply-bound proton levels from **EXPERIMENT**



Up to 100 MeV missing energy and 270 MeV/c missing momentum

Covers the whole mean-field domain for the FIRST time!!

Confirms predictions for depletion

Two effects associated with short-range correlations

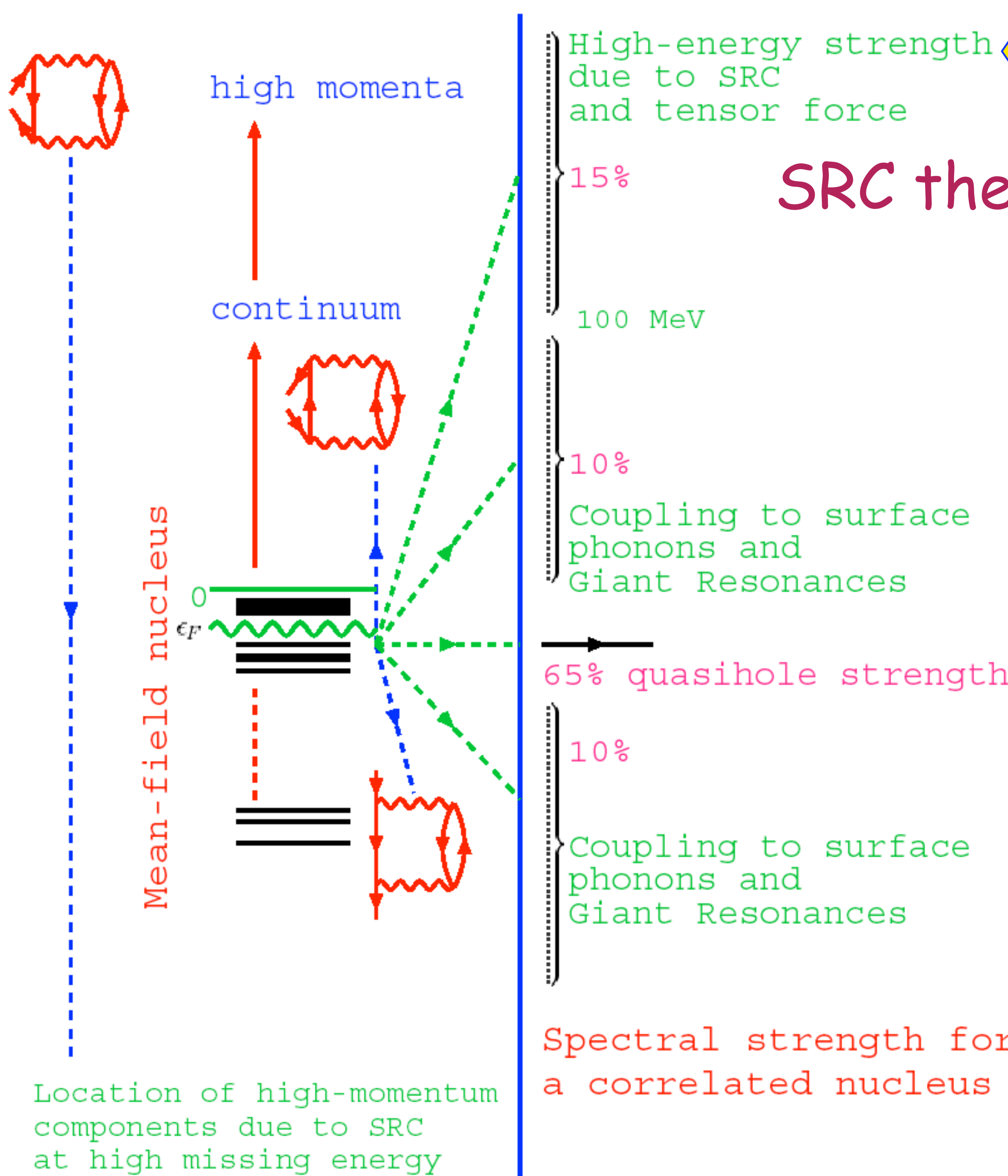
- Depletion of the Fermi sea

- Admixture of high-momentum components

Recent data confirm both aspects (predicted by nuclear matter results)

Reviewed in Prog. Part. Nucl. Phys. 52 (2004) 377-496

Location of single-particle strength in closed-shell (stable) nuclei



SRC theory

Elastic nucleon scattering

For example: protons in ^{208}Pb

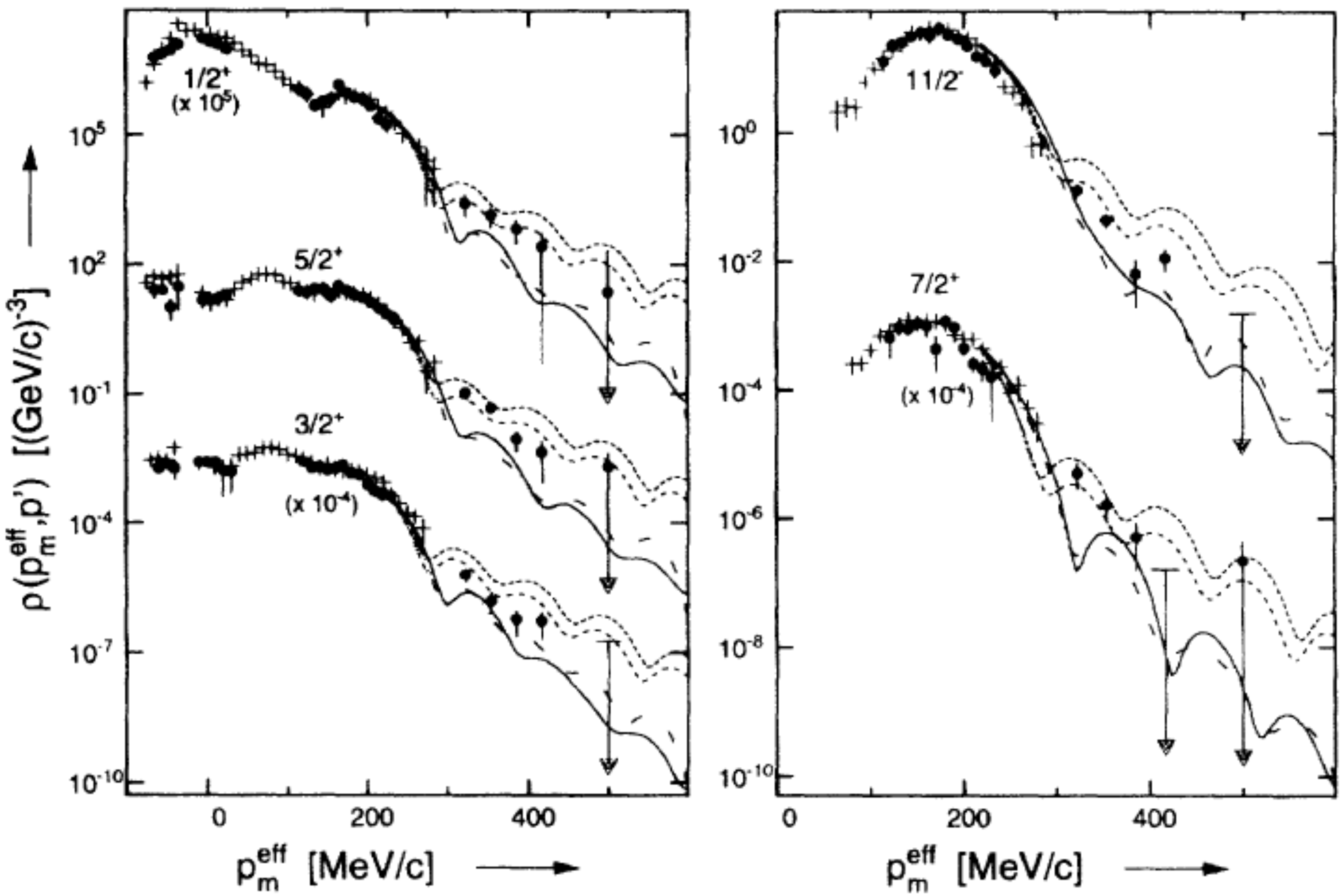
NIKHEF (e,e'p) data
L. Lapikás
Nucl. Phys. A553,297c (1993)

SRC

JLab E97-006

Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.

High-momenta near ε_F?

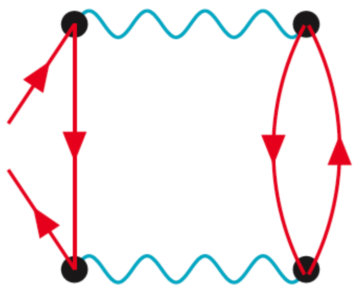


NO!

I. Bobeldijk et al., Phys. Rev. Lett. **73**, 2684 (1994)

Location of high-momentum components

high momenta



require specific intermediate states

External line **k** (large).

Intermediate holes < k_F, say total momentum ~ 0.

Momentum conservation: intermediate particle -**k**

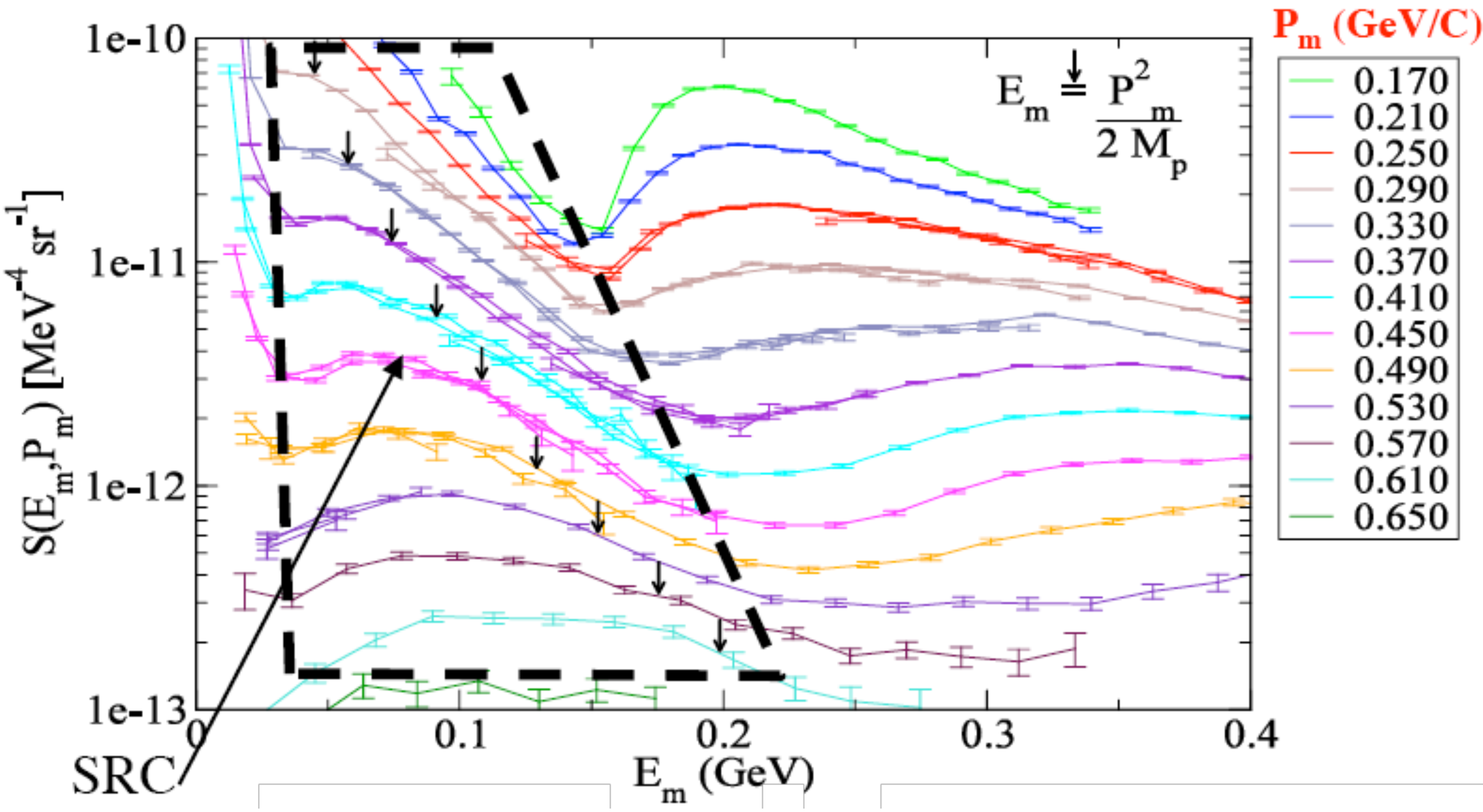
⇒ Energy intermediate state ~ $\langle \varepsilon_{2h} \rangle - \varepsilon(\boldsymbol{k})$

⇒ the higher k, the more negative the location of its strength

⇒ no high-momentum components near ε_F

High-momentum protons have been seen in nuclei!

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



^{12}C

- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)

$\Rightarrow$ ~0.6 protons for ^{12}C $\Rightarrow$ **~10%**

We now essentially "know" what all the protons are doing in the ground state of a "closed-shell" nucleus !!!

- Unique for a correlated many-body system
- Information available for electrons in atoms (Hartree-Fock)
- Not for electrons in solids
- Not for atoms in quantum liquids
- Not for quarks in nucleons

⇒ Demonstrates the value of the study of the nucleus

for its intrinsic interest

as a quantum many-body problem!

Schrödinger-like equation from DE

- Do for finite system with discrete bound states
- Appropriate Lehmann representation

$$
\begin{aligned}
G(\alpha,\beta;E) &= \sum_m \frac{\langle \Psi_0^N | a_\alpha | \Psi_m^{N+1}\rangle \langle \Psi_m^{N+1} | a_\beta^\dagger | \Psi_0^N \rangle}{E - (E_m^{N+1} - E_0^N) + i\eta} \\
&+ \int_{\varepsilon_T^+}^{\infty} d\tilde{E}_\mu^{N+1} \frac{\langle \Psi_0^N | a_\alpha | \Psi_\mu^{N+1}\rangle \langle \Psi_\mu^{N+1} | a_\beta^\dagger | \Psi_0^N \rangle}{E - \tilde{E}_\mu^{N+1} + i\eta} \\
&+ \sum_n \frac{\langle \Psi_0^N | a_\beta^\dagger | \Psi_n^{N-1}\rangle \langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle}{E - (E_0^N - E_n^{N-1}) - i\eta} \\
&+ \int_{-\infty}^{\varepsilon_T^-} d\tilde{E}_\nu^{N-1} \frac{\langle \Psi_0^N | a_\beta^\dagger | \Psi_\nu^{N-1}\rangle \langle \Psi_\nu^{N-1} | a_\alpha | \Psi_0^n \rangle}{E - \tilde{E}_\nu^{N-1} - i\eta}
\end{aligned}
$$

- using continuum thresholds $\varepsilon_T^{\pm}$
- and notation

$$
\tilde{E}_\mu^{N+1} = E_\mu^{N+1} - E_0^N
$$

$$
\tilde{E}_\nu^{N-1} = E_0^N - E_\nu^{N-1}
$$

SE from DE

- Noninteracting propagator: poles different from interacting one
- Take limits as for sp problem to obtain eigenvalue problem

$$\lim_{E\to\varepsilon_n^-}(E-\varepsilon_n^-)\left\{G(\alpha,\beta;E)=G^{(0)}(\alpha,\beta;E)+\sum_{\gamma\delta}G^{(0)}(\alpha,\gamma;E)\;\Sigma^*(\gamma,\delta;E)\;G(\delta,\beta;E)\right\}$$

- with $\varepsilon_n^- = E_0^N - E_n^{N-1}$
- and $z_\alpha^{n-} = \langle\Psi_n^{N-1}|\,a_\alpha\,|\Psi_0^N\rangle$
- as before $z_\alpha^{n-} = \sum_{\gamma,\delta} G^{(0)}(\alpha,\gamma;\varepsilon_n^-)\;\Sigma^*(\gamma,\delta;\varepsilon_n^-)\;z_\delta^{n-}$
- Rewrite in different sp basis (coordinate space)

$$z_{\boldsymbol{r}m}^{n-} = \sum_{m_1,m_2}\int d^3r_1\int d^3r_2\;G^{(0)}(\boldsymbol{r}m,\boldsymbol{r}_1m_1;\varepsilon_n^-)\;\Sigma^*(\boldsymbol{r}_1m_1,\boldsymbol{r}_2m_2;\varepsilon_n^-)\;z_{\boldsymbol{r}_2m_2}^{n-}$$

- employing basis transformation on self-energy and noninteracting propagator

Invert and remember

$$G^{(0)}(\boldsymbol{r}m_s,\boldsymbol{r}'m_s';E) = \langle\Phi_0^N|\, a_{\boldsymbol{r}m_s} \frac{1}{E-(\hat{H}_0 - E_{\Phi_0^N}) + i\eta} a^\dagger_{\boldsymbol{r}'m_s'}\,|\Phi_0^N\rangle$$
$$+ \langle\Phi_0^N|\, a^\dagger_{\boldsymbol{r}'m_s'} \frac{1}{E-(E_{\Phi_0^N} - \hat{H}_0) - i\eta} a_{\boldsymbol{r}m_s}\,|\Phi_0^N\rangle$$
$$= \sum_\alpha \left\{ \frac{\langle \boldsymbol{r}m_s|\alpha\rangle\langle\alpha|\boldsymbol{r}'m_s'\rangle\theta(\alpha-F)}{E-\varepsilon_\alpha+i\eta} + \frac{\langle \boldsymbol{r}m_s|\alpha\rangle\langle\alpha|\boldsymbol{r}'m_s'\rangle\theta(F-\alpha)}{E-\varepsilon_\alpha-i\eta} \right\}$$

- Rearrange by using

$$\sum_m \int d^3r\, \langle \boldsymbol{r}'m'|\, \varepsilon_n^- - H_0\, |\boldsymbol{r}m\rangle\, G^{(0)}(\boldsymbol{r}m, \boldsymbol{r}_1 m_1; \varepsilon_n^-) = \delta_{m',m_1}\delta(\boldsymbol{r}'-\boldsymbol{r}_1)$$

- same operation yields (U local and spin-independent)

$$\sum_m \int d^3r\ \langle \boldsymbol{r}'m'|\, \varepsilon_n^- - H_0\, |\boldsymbol{r}m\rangle\, z^{n-}_{\boldsymbol{r}m} = \left\{ \varepsilon_n^- + \frac{\hbar^2\nabla'^2}{2m} - U(\boldsymbol{r}') \right\} z^{n-}_{\boldsymbol{r}'m'}$$

- Combine: cancellation of auxiliary potential (as it should)

$$-\frac{\hbar^2\nabla^2}{2m} z^{n-}_{\boldsymbol{r}m} + \sum_{m_1} \int d^3r_1\ \Sigma'^*(\boldsymbol{r}m, \boldsymbol{r}_1 m_1; \varepsilon_n^-)\, z^{n-}_{\boldsymbol{r}_1 m_1} = \varepsilon_n^-\, z^{n-}_{\boldsymbol{r}m}$$

- Σ'^* does not contain auxiliary potential
- Like SE but energy dependent potential (energy in = energy out)

Quasiholes

- For quasihole solutions

$$S = \mid z_{\alpha_{qh}}^{n-} \mid^2 = \left(1 - \left. \frac{\partial \Sigma'^*(\alpha_{qh}, \alpha_{qh}; E)}{\partial E} \right|_{\varepsilon_n^-} \right)^{-1}$$

- Normalization of quasihole wave function is spectroscopic factor!

Dispersive Optical Model

- Claude Mahaux end of 1980s
  - connect traditional optical potential to bound-state potential
  - crucial idea: use the dispersion relation for the nucleon self-energy
  - smart implementation: use it in its subtracted form
  - applied successfully to ^{40}Ca and ^{208}Pb in a limited energy window
  - employed traditional volume and surface absorption potentials and a local energy-dependent Hartree-Fock-like potential
  - Reviewed in Adv. Nucl. Phys. **20**, 1 (1991)
- Radiochemistry group at Washington University in St. Louis: Charity and Sobotka propose to use it for a sequence of Ca isotopes —> data-driven extrapolations to the drip line
  - First results 2006 PRL
  - Subsequently —> attention to data below the Fermi energy related to ground-state properties —> Dispersive Self-energy Method (**DSM**)

"Mahaux" analysis

C. Mahaux and R. Sartor, Adv. Nucl. Phys. **20**, 1 (1991)

Optical potential used to analyze elastic nucleon scattering data

Extend analysis employing the optical potential (*A*+1$\Rightarrow$ particle part of propagator) to include structure information related to the levels in *A*-1 ($\Rightarrow$ hole part of propagator)

Employ exact relation between real and imaginary part of self-energy (dispersion relation) and take advantage of empirical information concerning the imaginary part of the optical potential

Use subtracted dispersion relation (at E_F) and assume standard surface and volume contributions

(e,e′p) and DOM

- Analysis of (e,e′p) involves Woods-Saxon bound states and distorted waves subject to standard local optical potential
- DOM fits can be extended to include all the "bare" (e,e′p) cross section data by incorporating the DOM bound wave function and the relevant optical potential (with $Z \Rightarrow Z-1$)
- Thus yielding "consistent" information only fitted to data without any other intermediate step!!!

Employed equations for "local" implementation

$$\Sigma^*(\boldsymbol{r}m, \boldsymbol{r}'m'; E) \Rightarrow \mathcal{U}(r,E) = -\mathcal{V}(r,E) + V_{SO}(r) + V_C(r)$$
$$- \quad iW_V(E) f(r, r_V, a_V) + 4ia_S W_S(E) f'(r, r_S, a_S)$$

$$f(r, r_i, a_i) = \left[1 + \exp\left((r - r_i A^{1/3})/a_i\right)\right]^{-1}$$

Woods-Saxon form factor

$$\mathcal{V}(r,E) = V_{HF}(E) f(r, r_{HF}, a_{HF}) + \Delta\mathcal{V}(r,E)$$

"HF" includes "main" effect of nonlocality
$\Rightarrow$ k-mass

$$\Delta\mathcal{V}(r,E) = \Delta V_V(E) f(r, r_V, a_V) - 4a_s \Delta V_s(E) f'(r, r_S, a_S)$$

"Time" nonlocality
$\Rightarrow$ E-mass

$$\Delta V_i(E) = \frac{\mathcal{P}}{\pi} \int_{-\infty}^{\infty} dE' \; W_i(E') \left(\frac{1}{E' - E} - \frac{1}{E' - \varepsilon_F} \right)$$

Subtracted dispersion relation
equivalent to following page

Optical potential <--> nucleon self-energy

- e.g. Bell and Squires --> elastic T-matrix = reducible self-energy
- Mahaux and Sartor Adv. Nucl. Phys. **20**, 1 (1991)
  - relate dynamic (energy-dependent) real part to imaginary part
  - employ subtracted dispersion relation

General dispersion relation for self-energy:

$$\text{Re}\ \Sigma(E) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im}\ \Sigma(E')}{E-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im}\ \Sigma(E')}{E-E'}$$

Calculated at the Fermi energy $\varepsilon_F = \frac{1}{2}\left\{(E_0^{A+1} - E_0^A) + (E_0^A - E_0^{A-1})\right\}$

$$\text{Re}\ \Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im}\ \Sigma(E')}{\varepsilon_F-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im}\ \Sigma(E')}{\varepsilon_F-E'}$$

Subtract

$$\text{Re}\ \Sigma(E) = \text{Re}\ \Sigma^{\widetilde{HF}}(\varepsilon_F)$$

$$- \quad \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')} + \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')}$$

Locality and other approximations

Mahaux $V_{HF}(\boldsymbol{r}m, \boldsymbol{r}'m') = \mathrm{Re}\ \Sigma^*(\boldsymbol{r}m, \boldsymbol{r}'m'; \varepsilon_F) \Rightarrow V_{HF}(r; E) = U_{HF}(E) f(X_{HF})$

with
$$
\begin{aligned}
f(X_{HF}) &= [1 + \exp(X_{HF})]^{-1} \\
X_{HF} &= \frac{r - R_{HF}}{a_{HF}} \\
R_{HF} &= r_{HF} A^{1/3} \\
U_{HF}(E) &= U_{HF}(\varepsilon_F) + \left[1 - \frac{m^*_{HF}}{m}\right](E - \varepsilon_F)
\end{aligned}
$$

Dispersive part: - assumed large E contribution and m^*_{HF} correlated

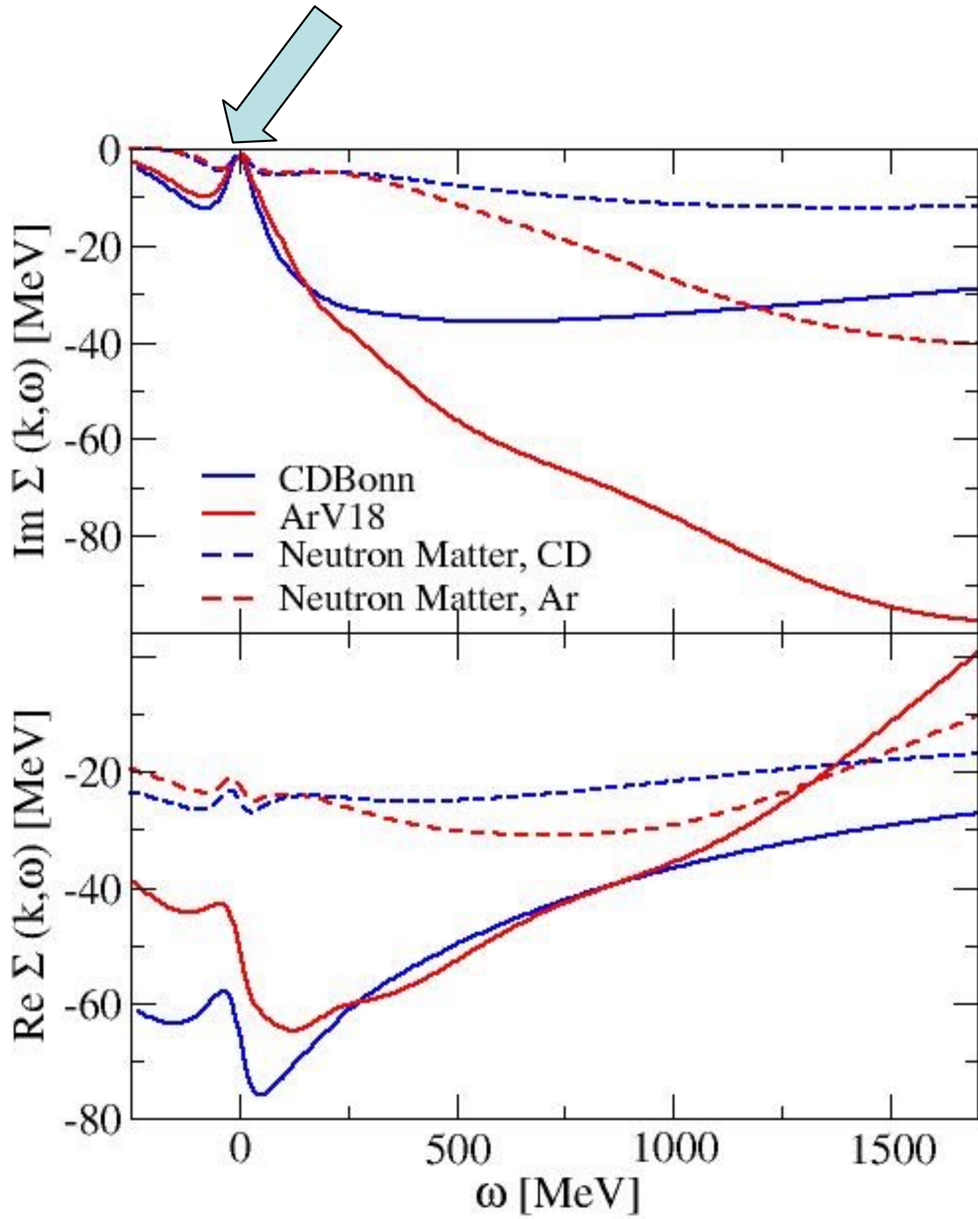
$\Rightarrow$ can use nuclear matter model

and introduces asymmetry in Im part

- nonlocality of Im Σ smooth

$\Rightarrow$ replace by local form identified with the imaginary part of the optical-model potential with volume and surface contributions

Infinite matter self-energy



Real and imaginary part of the (retarded) self-energy

- $k_F = 1.35\ fm^{-1}$
- T = 5 MeV
- $k = 1.14\ fm^{-1}$

Note differences due to NN interaction

Asymmetry w.r.t. the Fermi energy related to phase space for p and h

Approximations to solving Dyson equation

- No ℓj dependence of self-energy apart from standard spin-orbit
- Assumed form of "HF" potential fixed geometry
- Factorization of energy and radial dependence is **assumption**
- Imaginary part of self-energy at low-energy is spiky (poles)
  $\Rightarrow$ extra fragmentation at low energy (open-shell nuclei!)
- Expressions for occupation numbers "heuristic" ($\Rightarrow$ wrong for N or Z)
- Z-factors not useful except near ε_F (exact there)
- Division volume & surface "physical" but ...
- Volume terms from nuclear matter should also include asymmetry

Exact solution of Dyson equation

- Coordinate space technique employed for atoms can be employed to solve Dyson equation including any true nonlocality (Van Neck)
- Yields

$$S_h(\alpha,\beta;E) = \sum_n \langle\Psi_0^N|\, a_\beta^\dagger \,|\Psi_n^{N-1}\rangle \langle\Psi_n^{N-1}|\, a_\alpha \,|\Psi_0^N\rangle\, \delta\left(E - (E_0^N - E_n^{N-1})\right)$$

spectral density (spectral function for $\alpha = \beta$) and therefore

$$n_{\beta\alpha} = \int_{-\infty}^{\varepsilon_F^-} dE\; S_h(\alpha,\beta;E) = \sum_n \langle\Psi_0^N|\, a_\beta^\dagger \,|\Psi_n^{N-1}\rangle \langle\Psi_n^{N-1}|\, a_\alpha \,|\Psi_0^N\rangle = \langle\Psi_0^N|\, a_\beta^\dagger a_\alpha \,|\Psi_0^N\rangle$$

the one-body density matrix including occupation numbers ($\alpha = \beta$)

and last but not least

$$E_0^N = \frac{1}{2}\left(\sum_{\alpha,\beta} \langle\alpha|\, T \,|\beta\rangle\, n_{\alpha\beta} + \sum_\alpha \int_{-\infty}^{\varepsilon_F^-} dE\; E\; S_h(\alpha;E)\right)$$

the ground state energy $\Rightarrow$ useful constraints (includes also Z & N)

Combined analysis of protons in ^{40}Ca and ^{48}Ca

Charity, Sobotka, & WD nucl-ex/0605026

Phys. Rev. Lett. 97, 162503 (2006)

Goal: Extract asymmetry dependence

$\delta = (N - Z)/A$

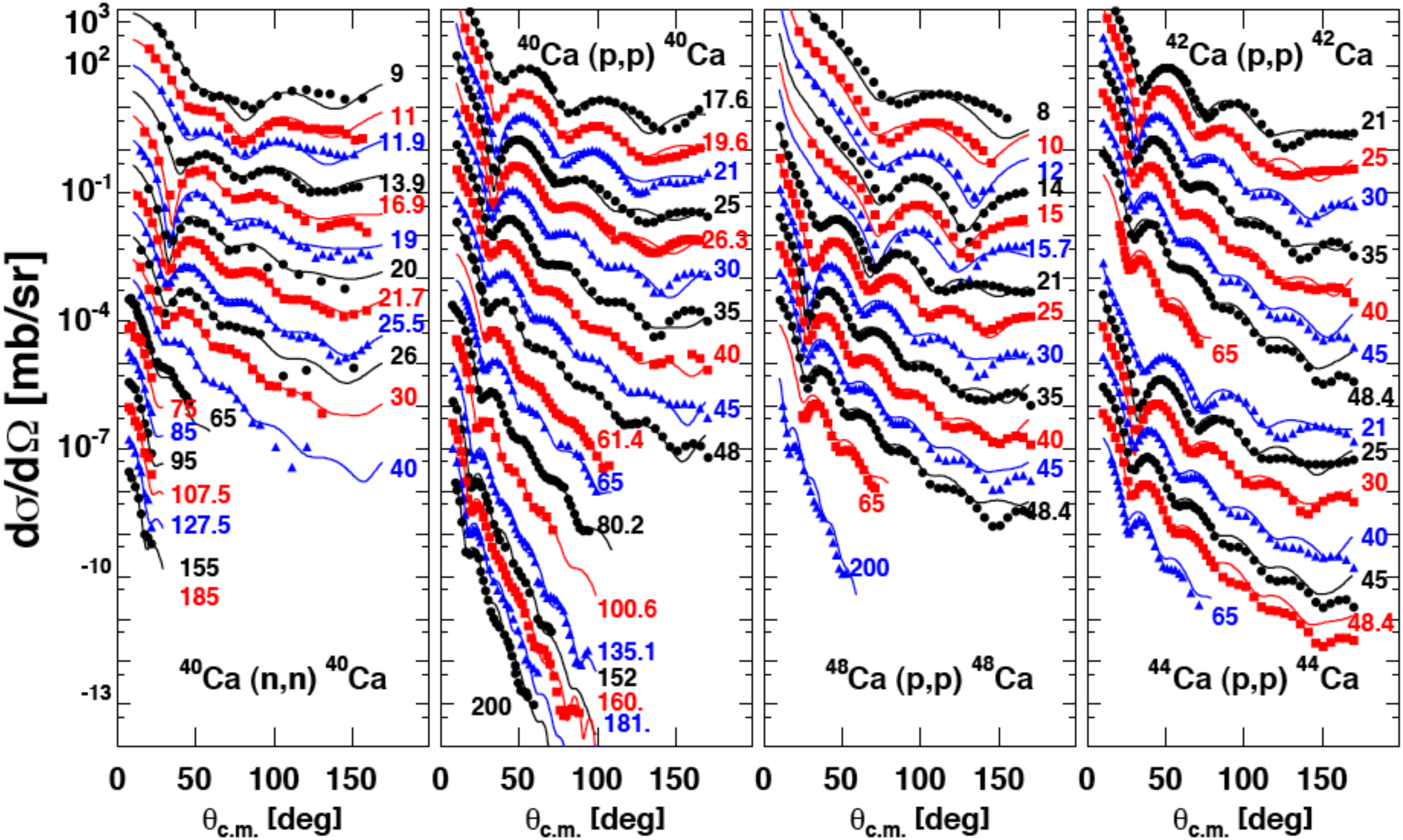
$\Rightarrow$ Predict large asymmetry properties ^{60}Ca

Features of simultaneous fit to ^{40}Ca and ^{48}Ca data

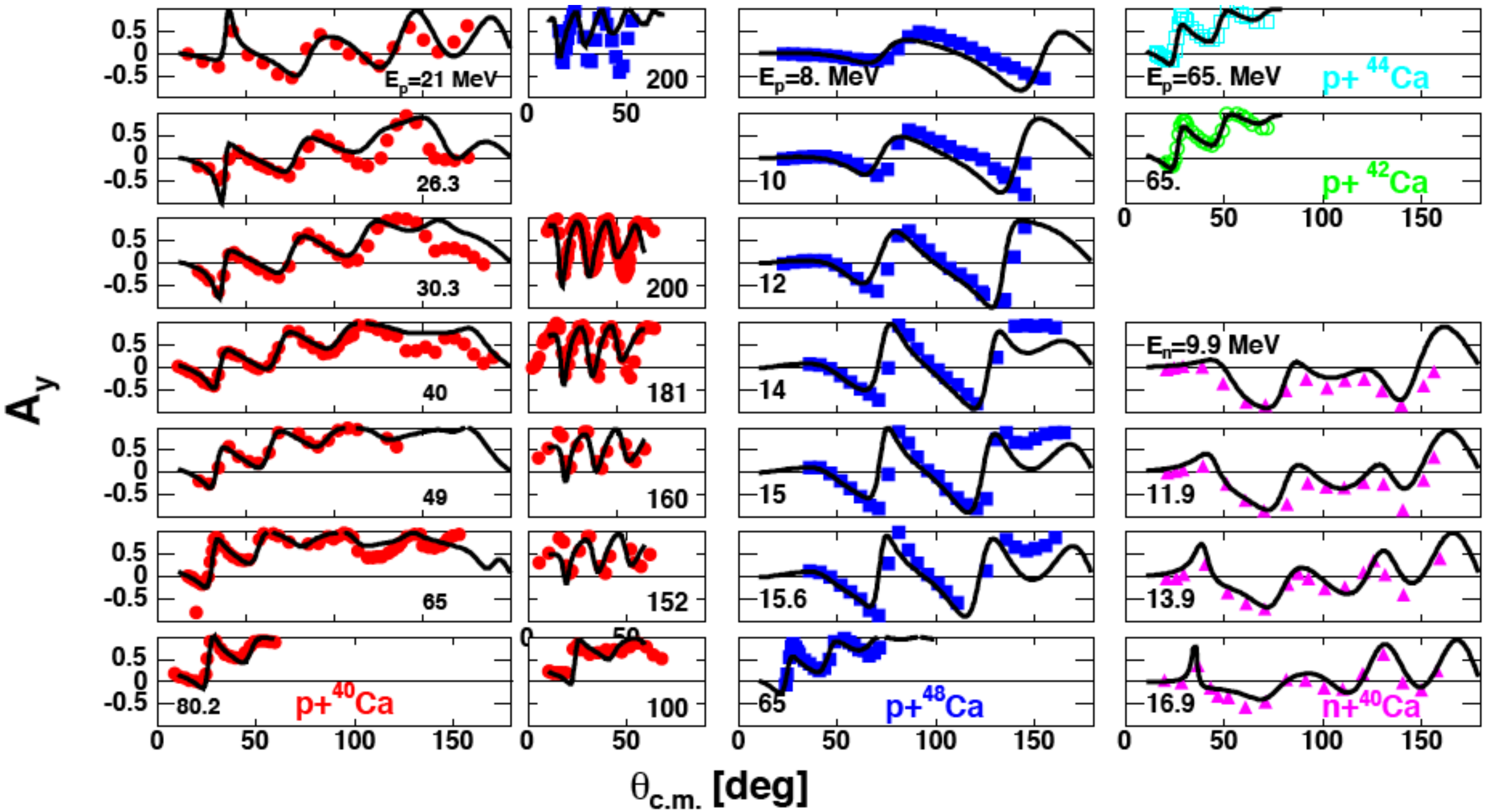
- Surface contribution assumed symmetric around ε_F
  Represents coupling to low-lying collective states (GR)
- Volume term asymmetric w.r.t. ε_F taken from nuclear matter
- Geometric parameters r_i and a_i fit but the same for both nuclei
- Decay (in energy) of surface term identical also
- Possible to keep volume term the same (consistent with exp) and independent of asymmetry
- "HF" and surface parameters different and can be extrapolated to larger asymmetry

- Surface potential stronger and narrower around ε_F for ^{48}Ca
- Both elastic scattering and (e,e'p) data included in fit

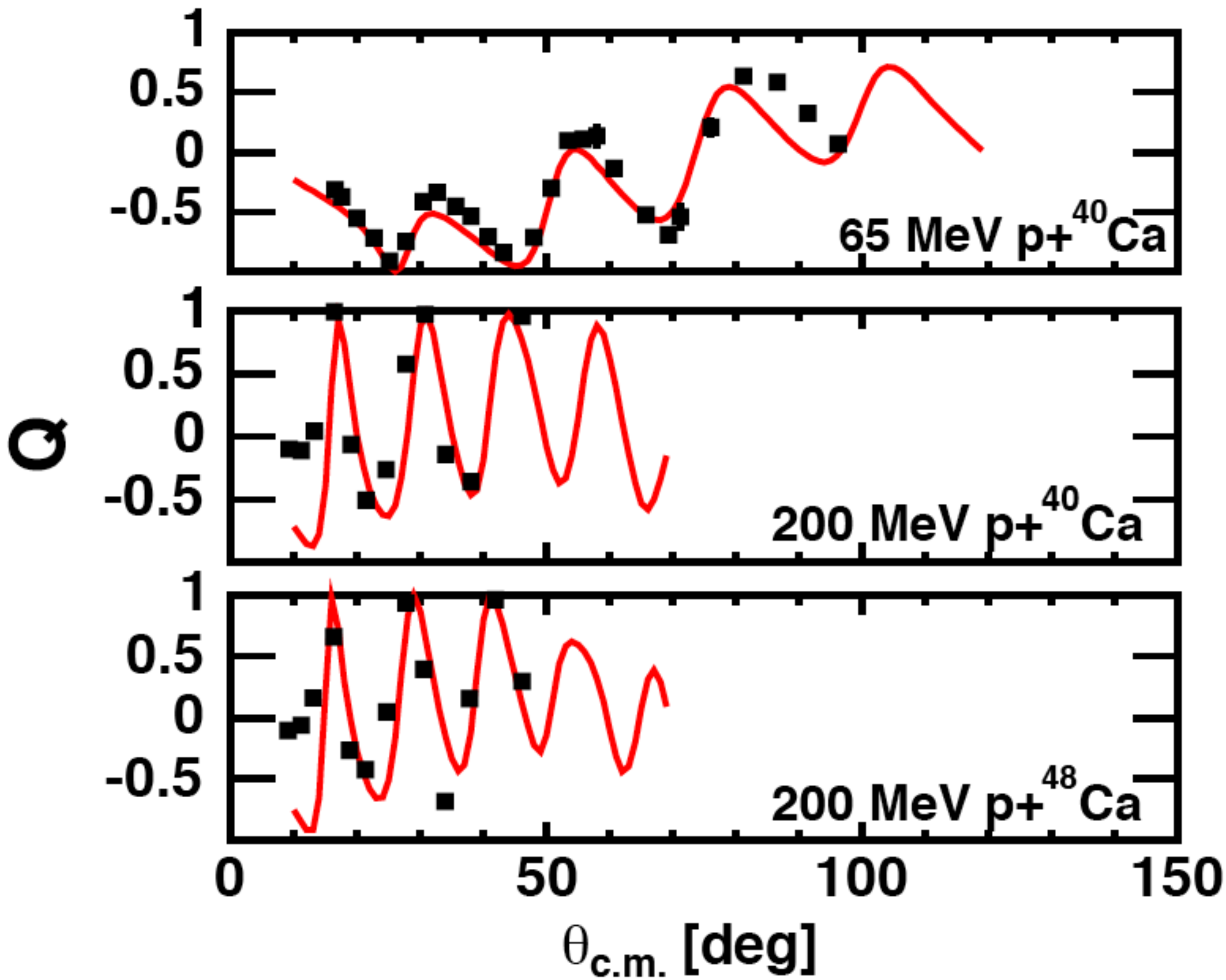
Fit and predictions of n & p elastic scattering cross sections



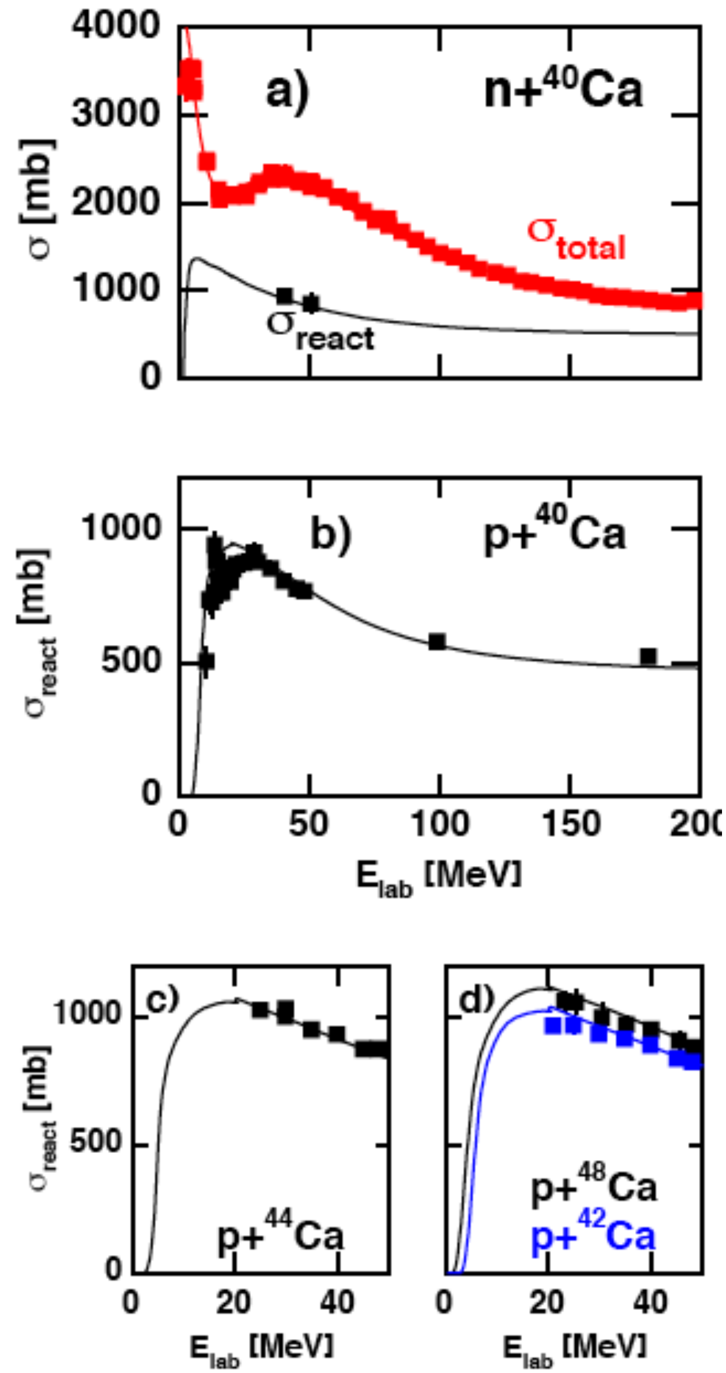
Present fit and predictions of polarization data



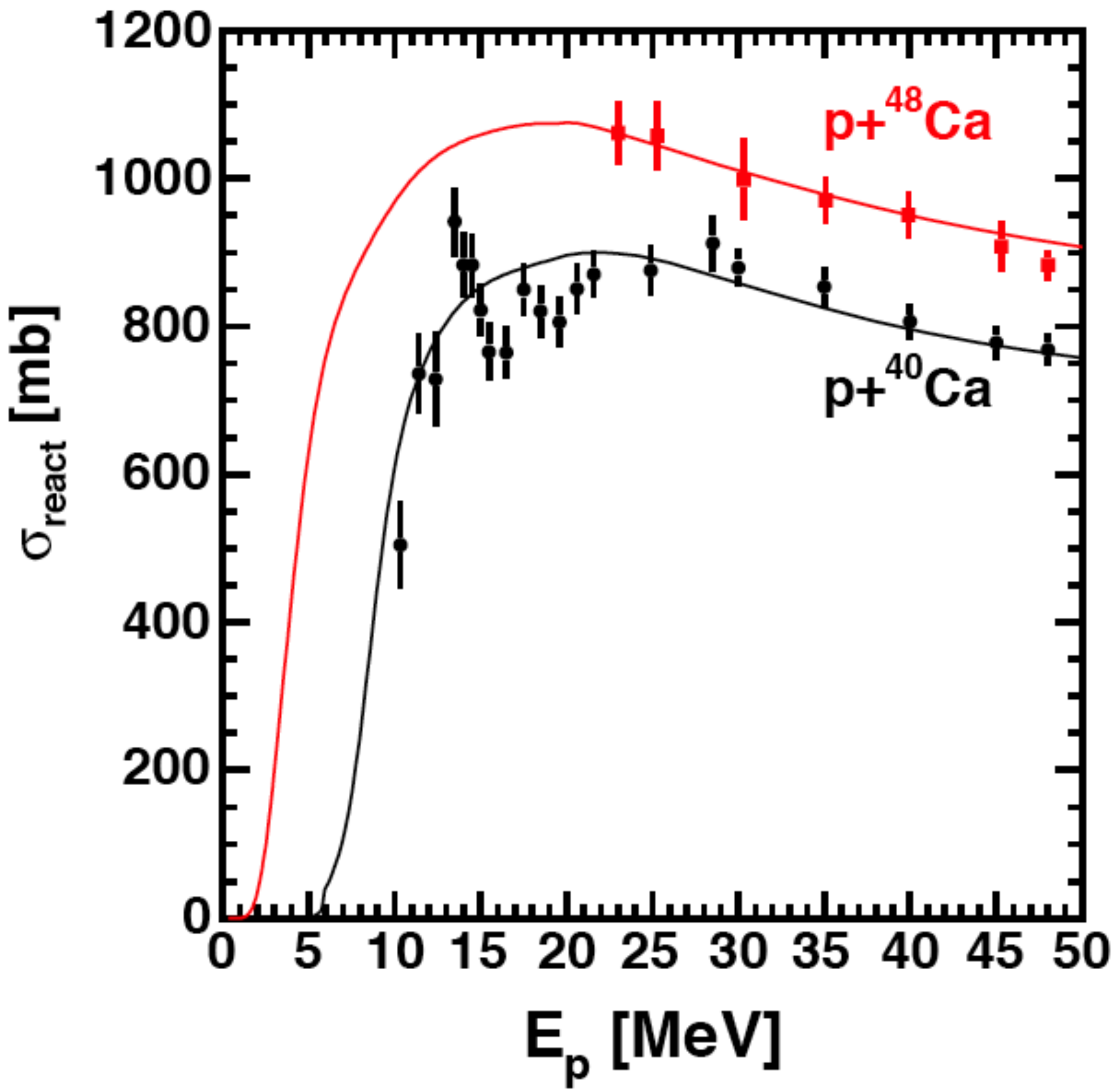
Spin rotation parameter (not fitted)



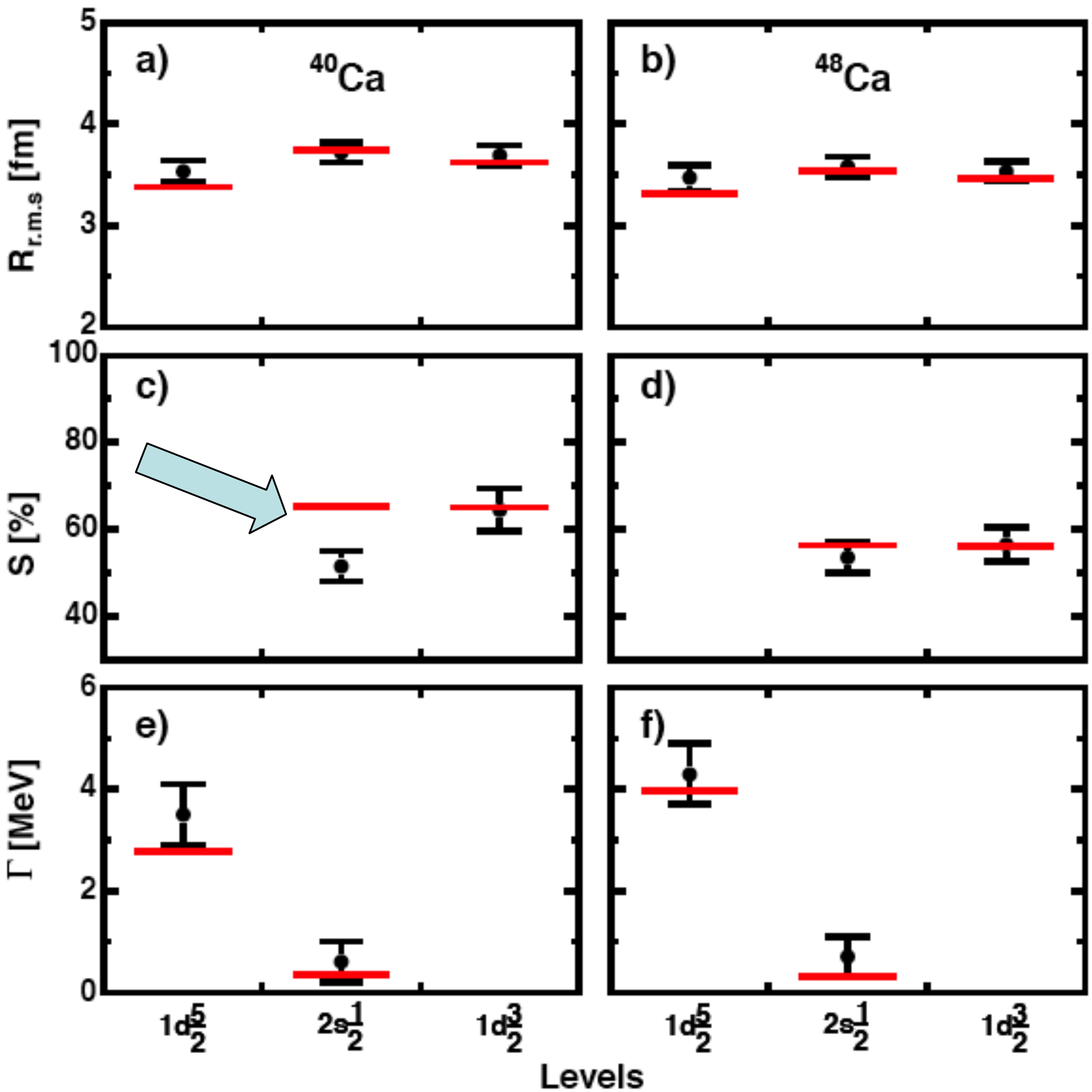
Fit and predictions
Of reaction cross sections



Fit of reaction cross sections



Fit to (e,e´p) data

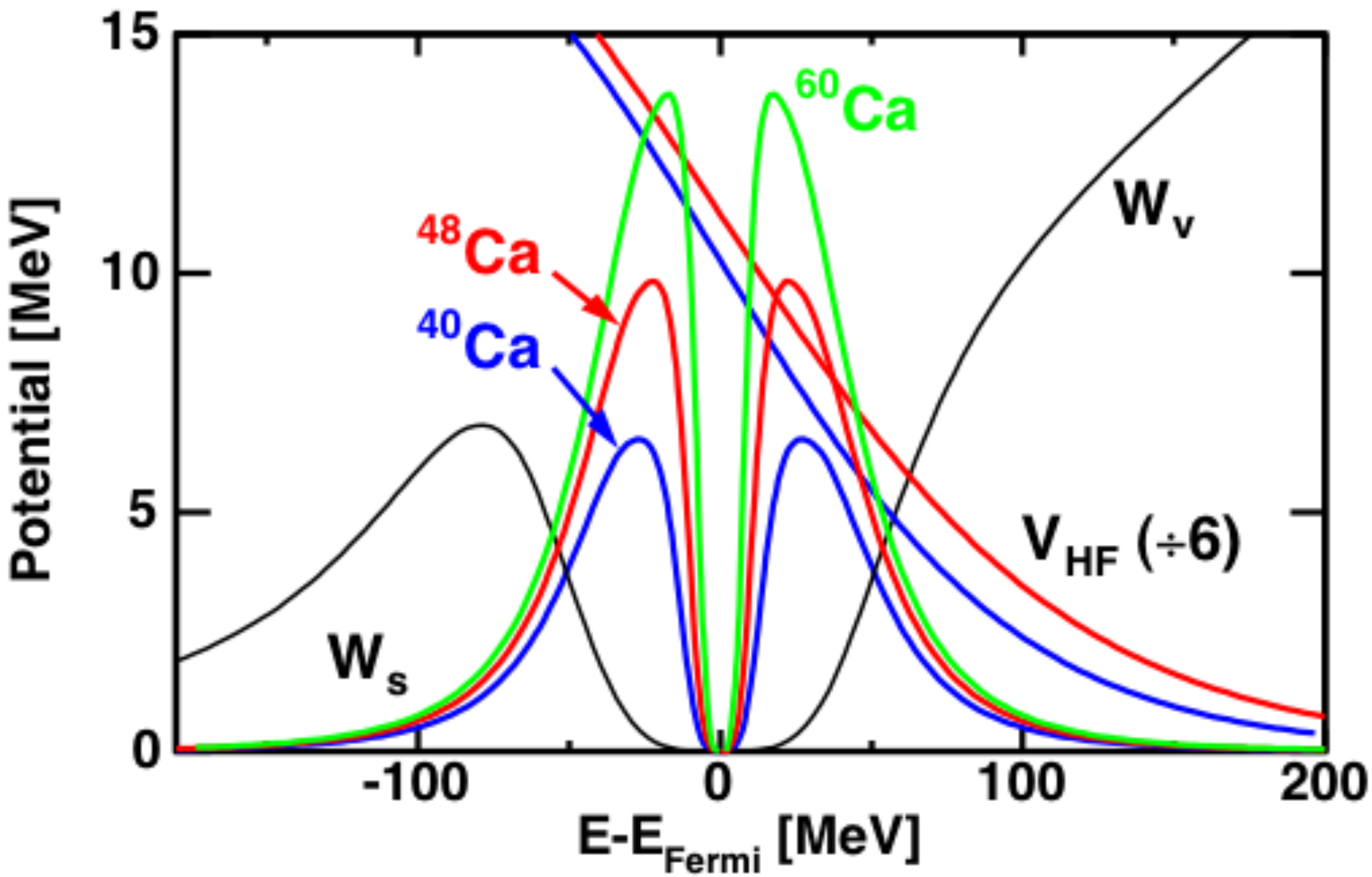


radii of
bound state
wave functions

spectroscopic
factors

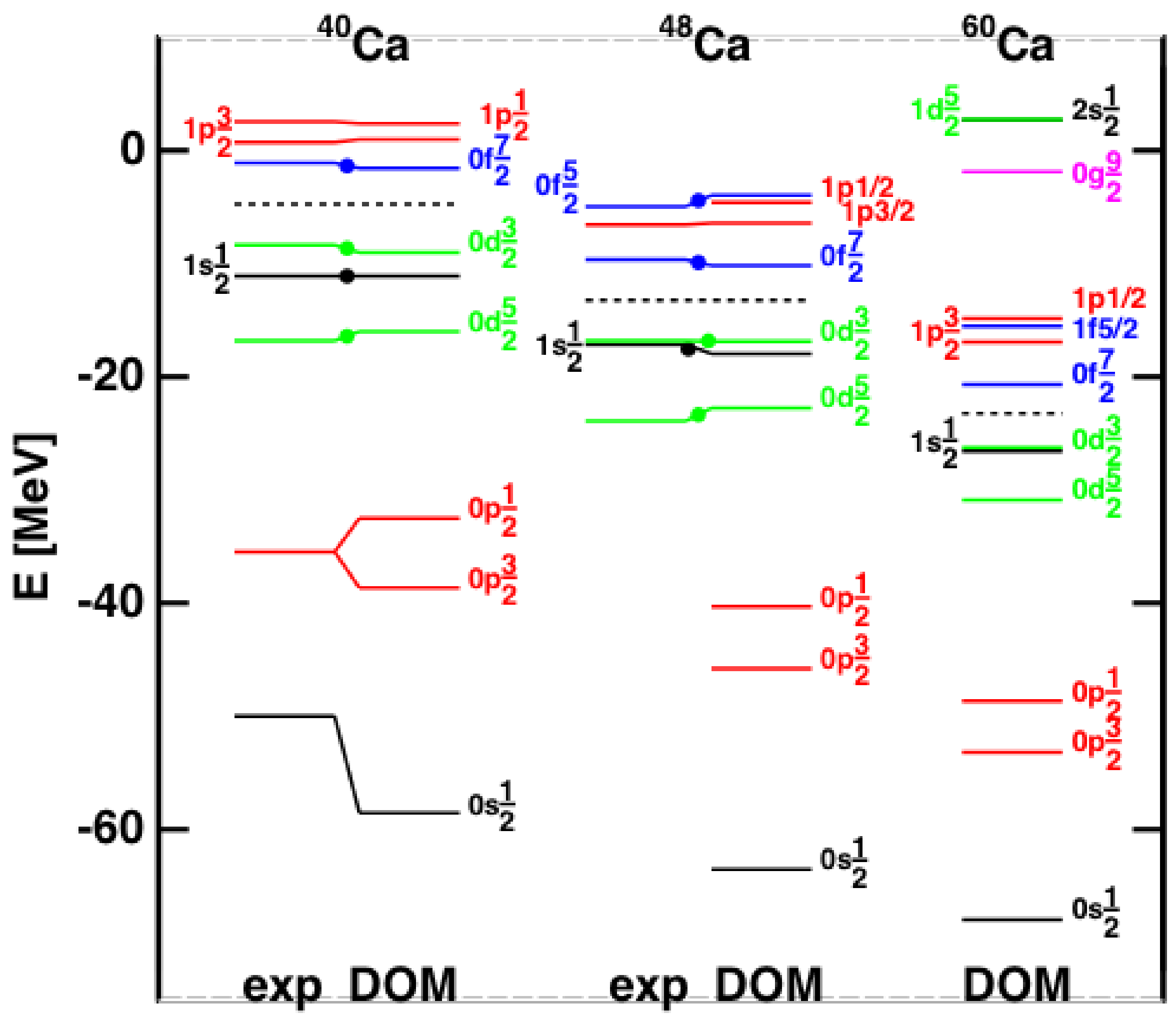
widths of strength
distribution

Potentials



15
10
5
0
-100
0
100
200
Potential [MeV]
E-E$_{Fermi}$ [MeV]
^{60}Ca
^{48}Ca
^{40}Ca
W_v
W_s
V_{HF} (÷6)

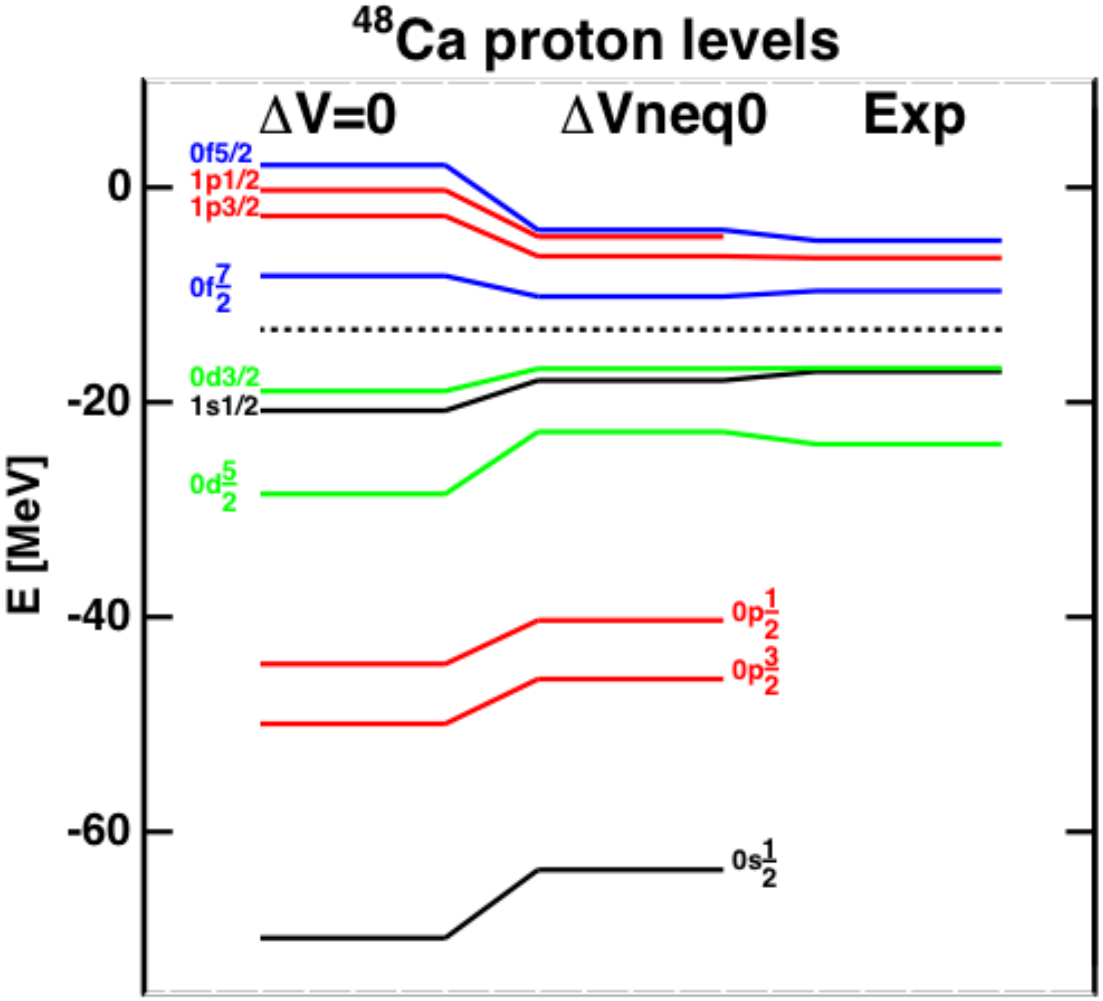
Proton single-particle structure and asymmetry



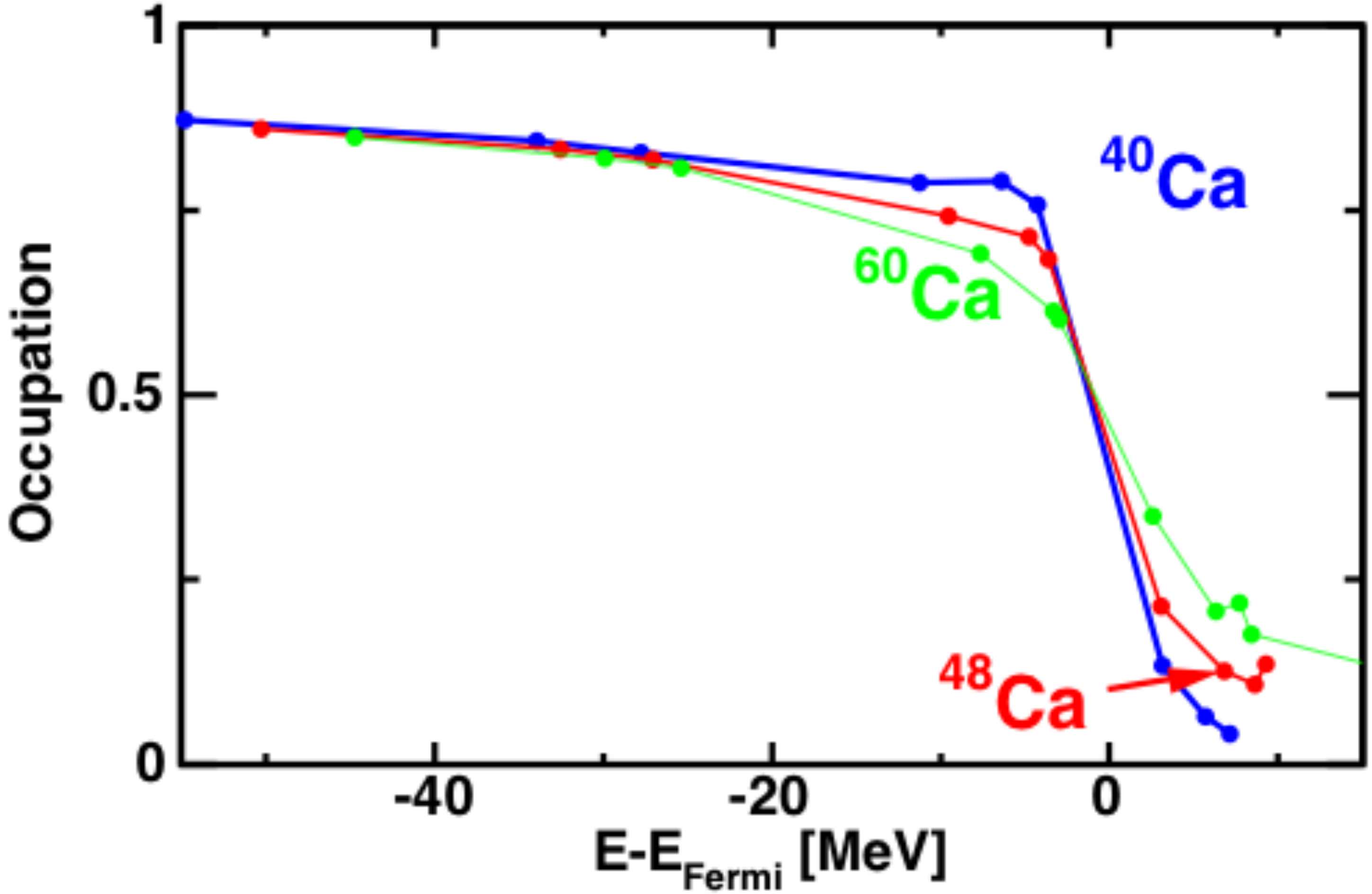
Pairing of protons on account of pn correlations?!

Increased correlations with increasing asymmetry!

Polarization effect on sp energies



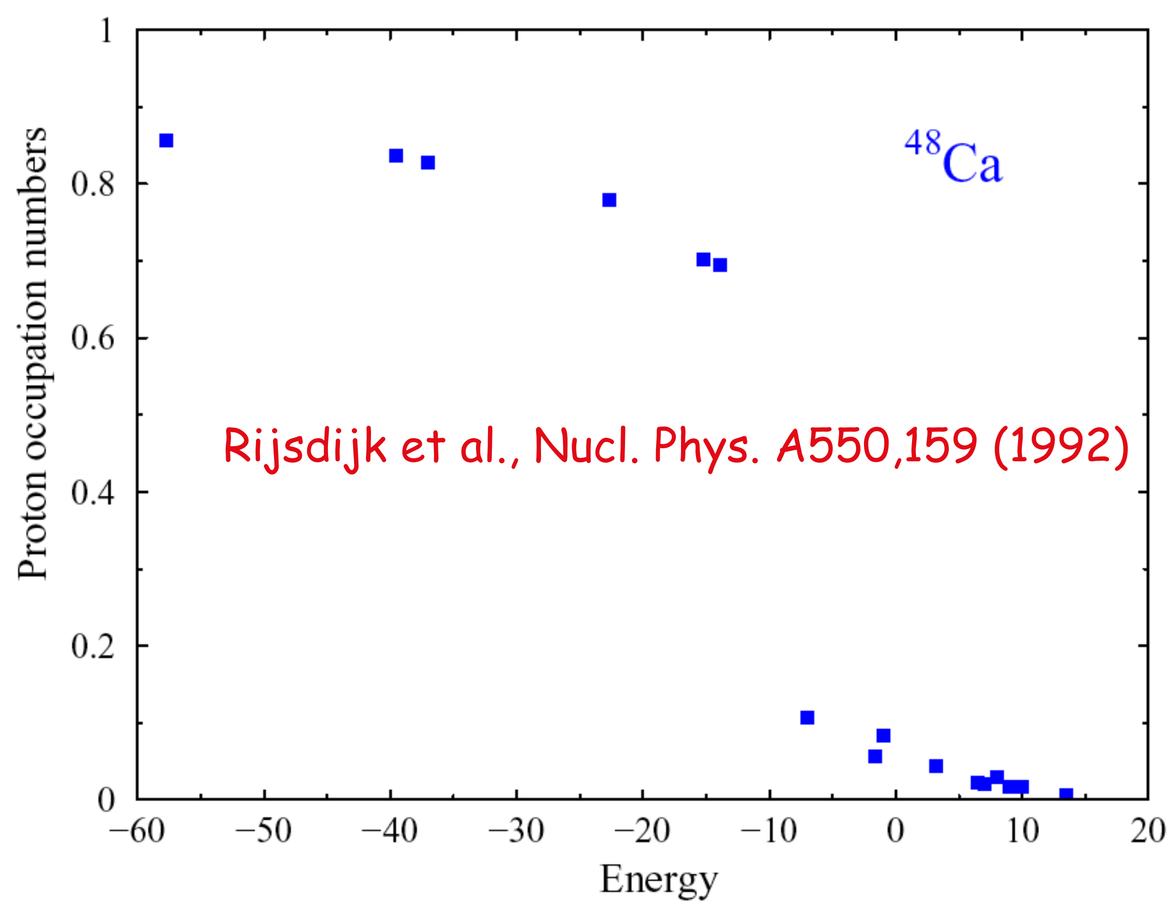
Occupation numbers



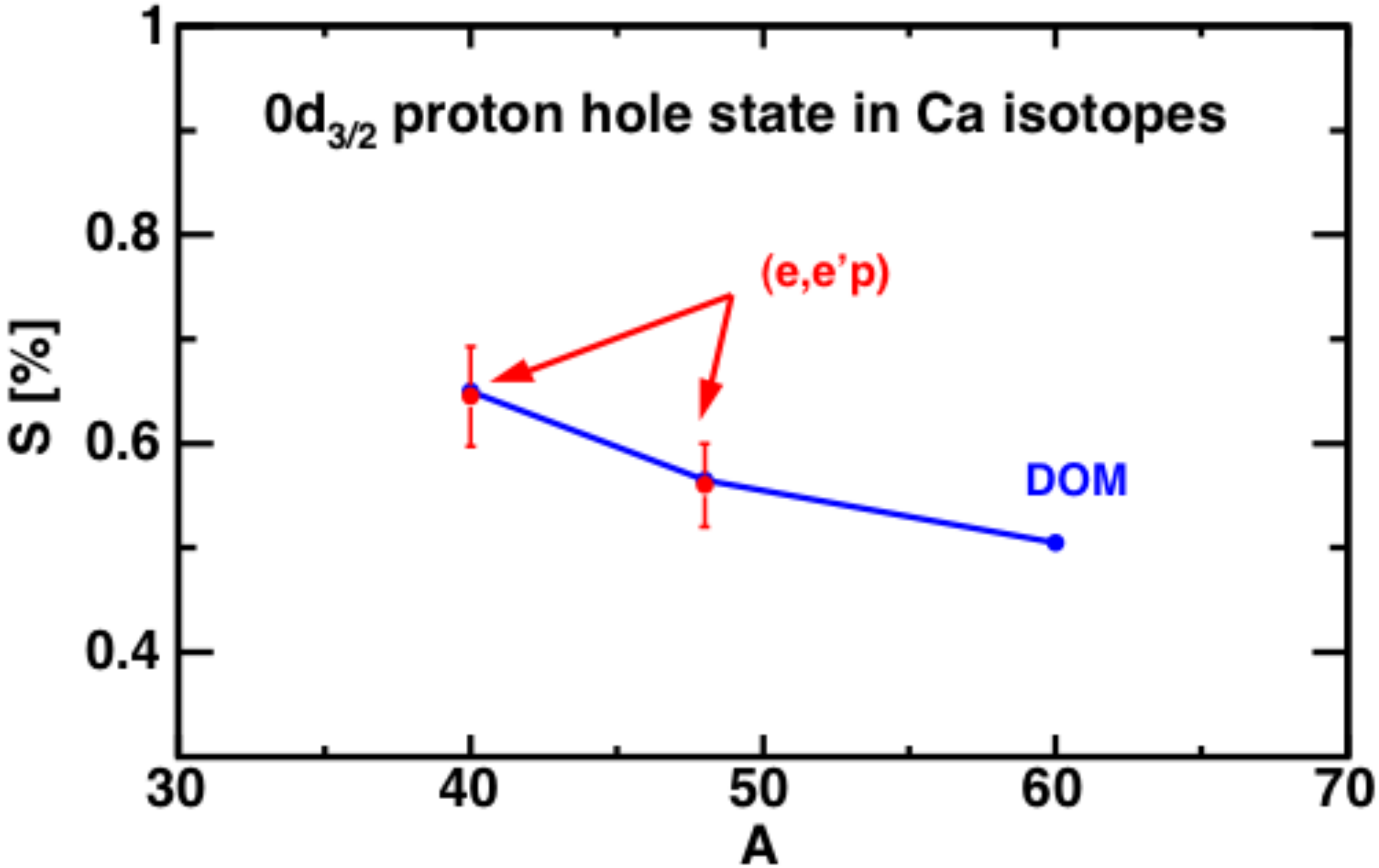
Occupation numbers from low-energy correlations from theoretical work

Shell	n(α)
$0s_{1/2}$	0.968
$0p_{3/2}$	0.956
$0p_{1/2}$	0.951
$0d_{5/2}$	0.925
$0d_{3/2}$	0.885
$1s_{1/2}$	0.860
$0f_{7/2}$	0.063
$0f_{5/2}$	0.044
$0p_{3/2}$	0.031
$0p_{1/2}$	0.028

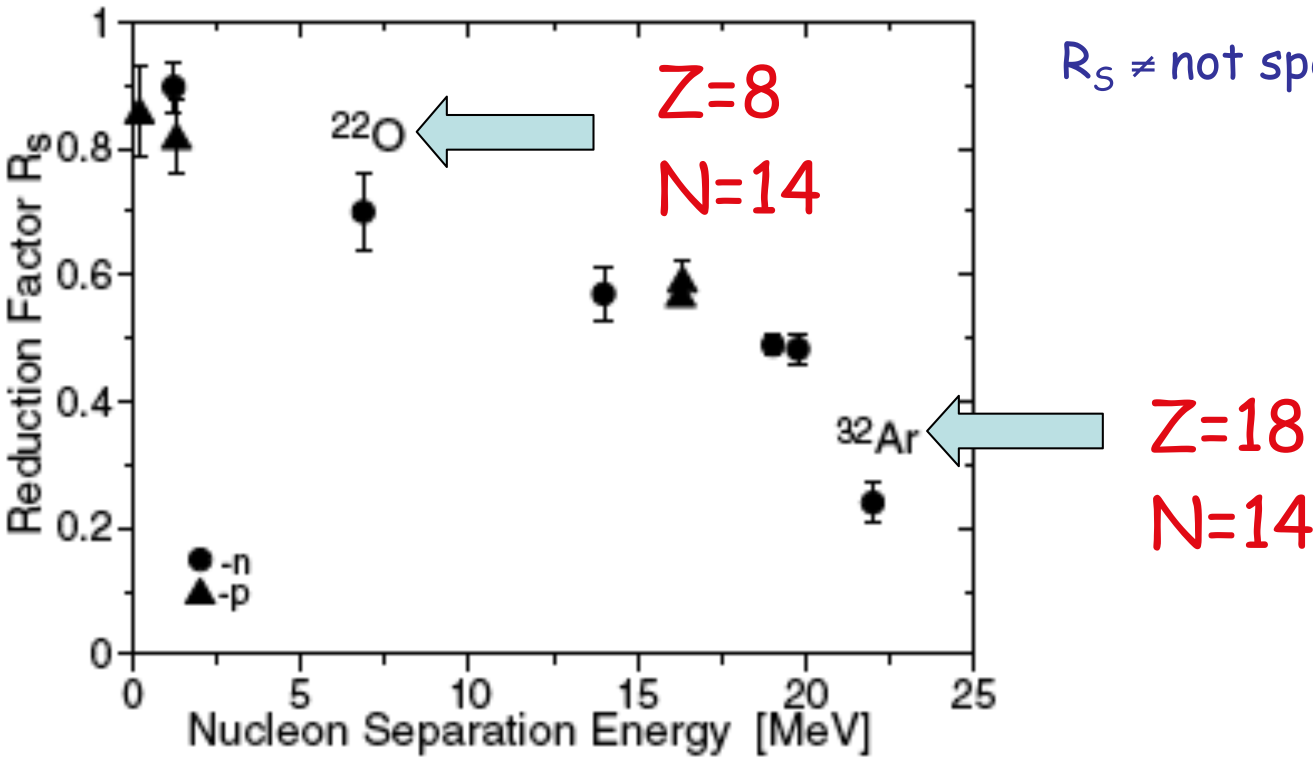
Including SRC depletion effect by treating energy dependence of G-matrix



Spectroscopic factor



A. Gade et al. Phys. Rev. Lett. 93, 042501 (2004)



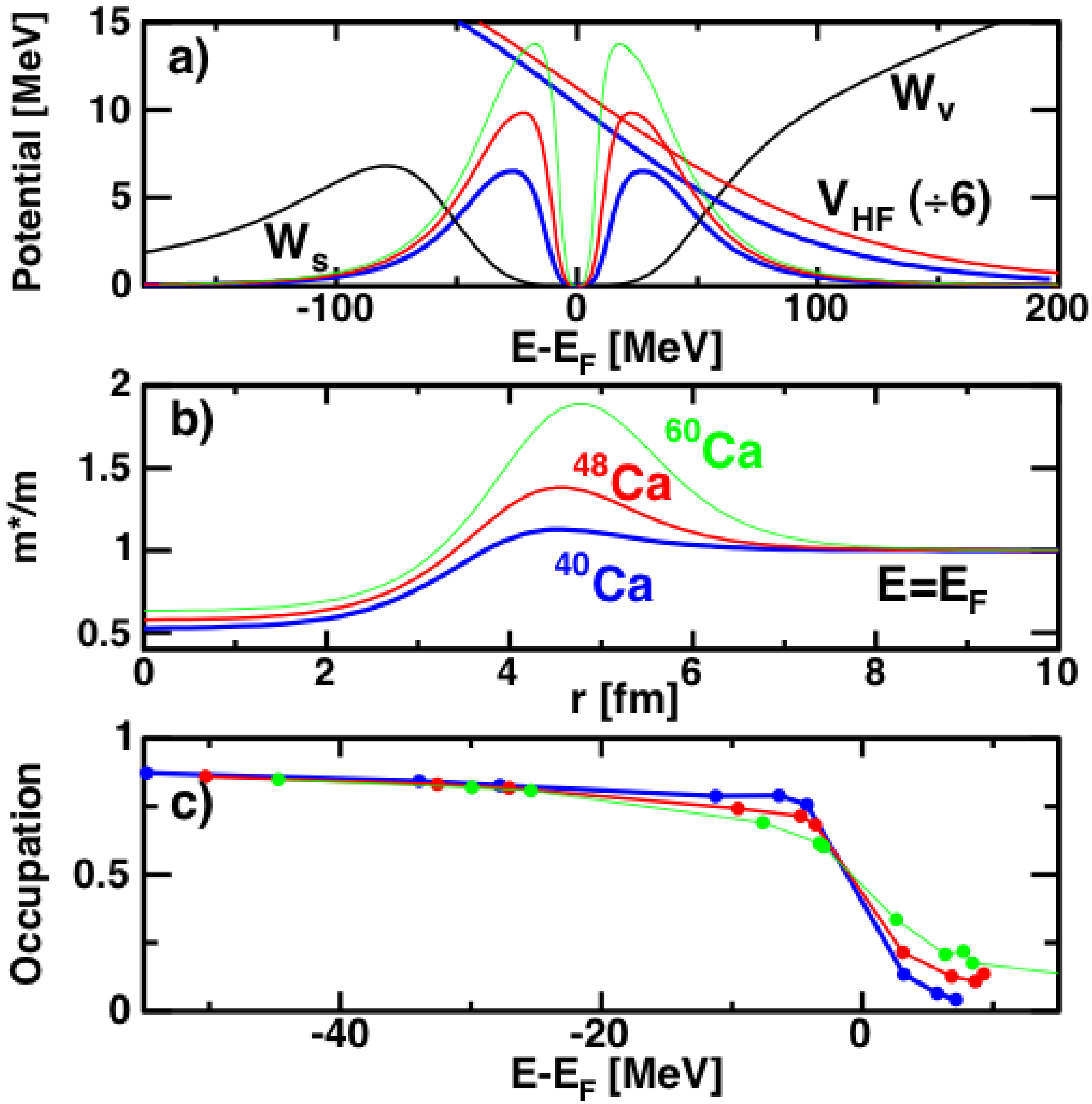
$R_S \neq$ not spectroscopic factor

neutrons more correlated with increasing proton number and accompanying increasing separation energy.

Parameters

TABLE I: Values of the fitted parameters

$r_{HF} = 1.16$ fm	$a_{HF} = .67$ fm
$r_s = 1.19$ fm	$a_s = 0.61$ fm
$r_v = 1.36$ fm	$a_v = a_{HF}$
$r_{so} = 0.97$ fm	$a_{so} = 0.67$
$V_{so} = 6.57$ MeV	
$C_s = 0.015$ MeV^{-1}	$B_s^2 = 35.03$ MeV
$\Delta B = 14.84$ MeV	r_C(fixed) $= 1.31$ fm
$A_v = 9.95$ MeV	$B_v = 57.84$ MeV
A_{HF} (40) $= 61.55$ MeV	A_{HF} (48) $= 67.41$ MeV
B_{HF} (40) $=0.624$	B_{HF} (48) $= 0.574$
A_s^1 (40) $= 10.83$ MeV	A_s^1 (48) $= 14.94$ MeV
B_s^1 (40) $= 15.57$ MeV	B_s^1 (48) $= 12.25$ MeV



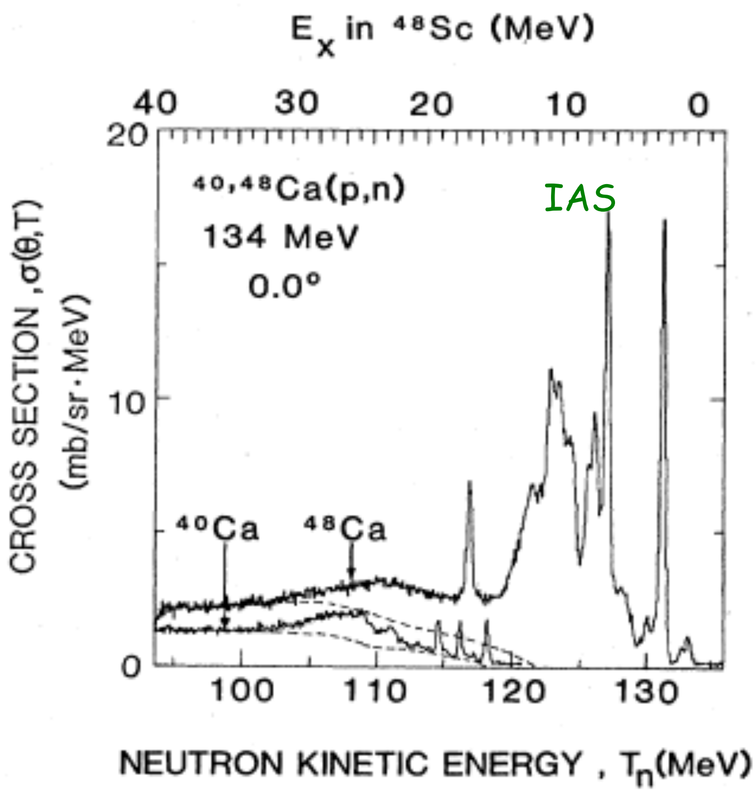
Potentials

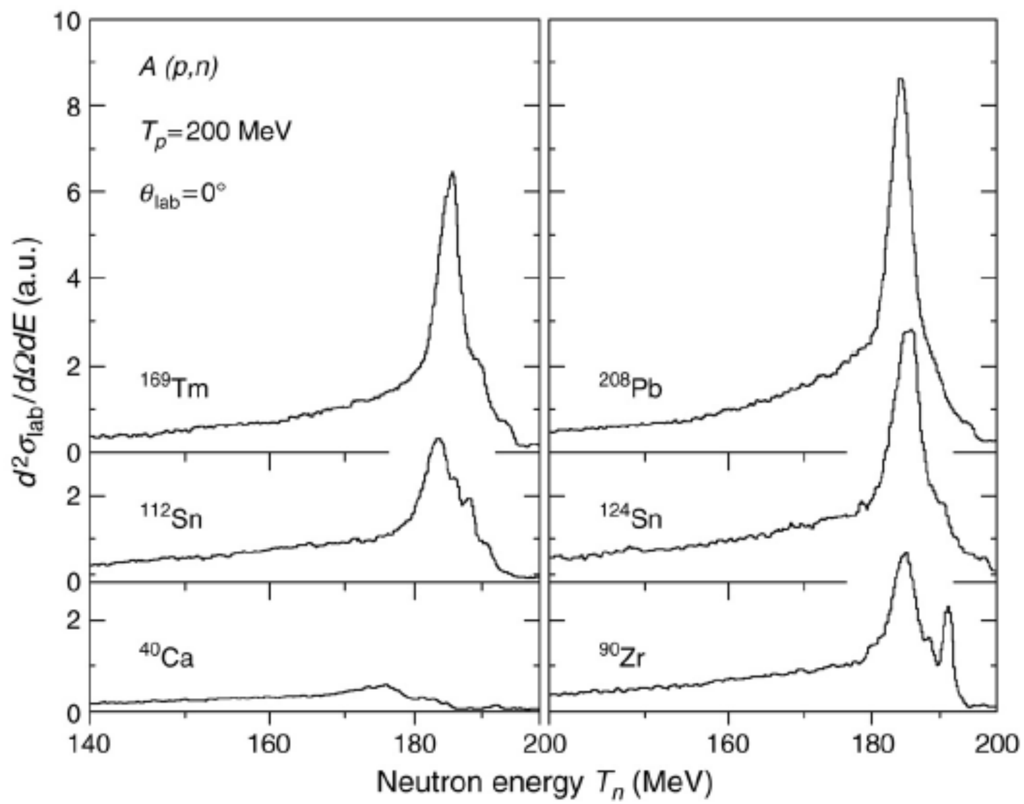
Effective mass

Occupation numbers

Phys. Rev. C76, 044314 (2007)

What's the physics? GT resonance?



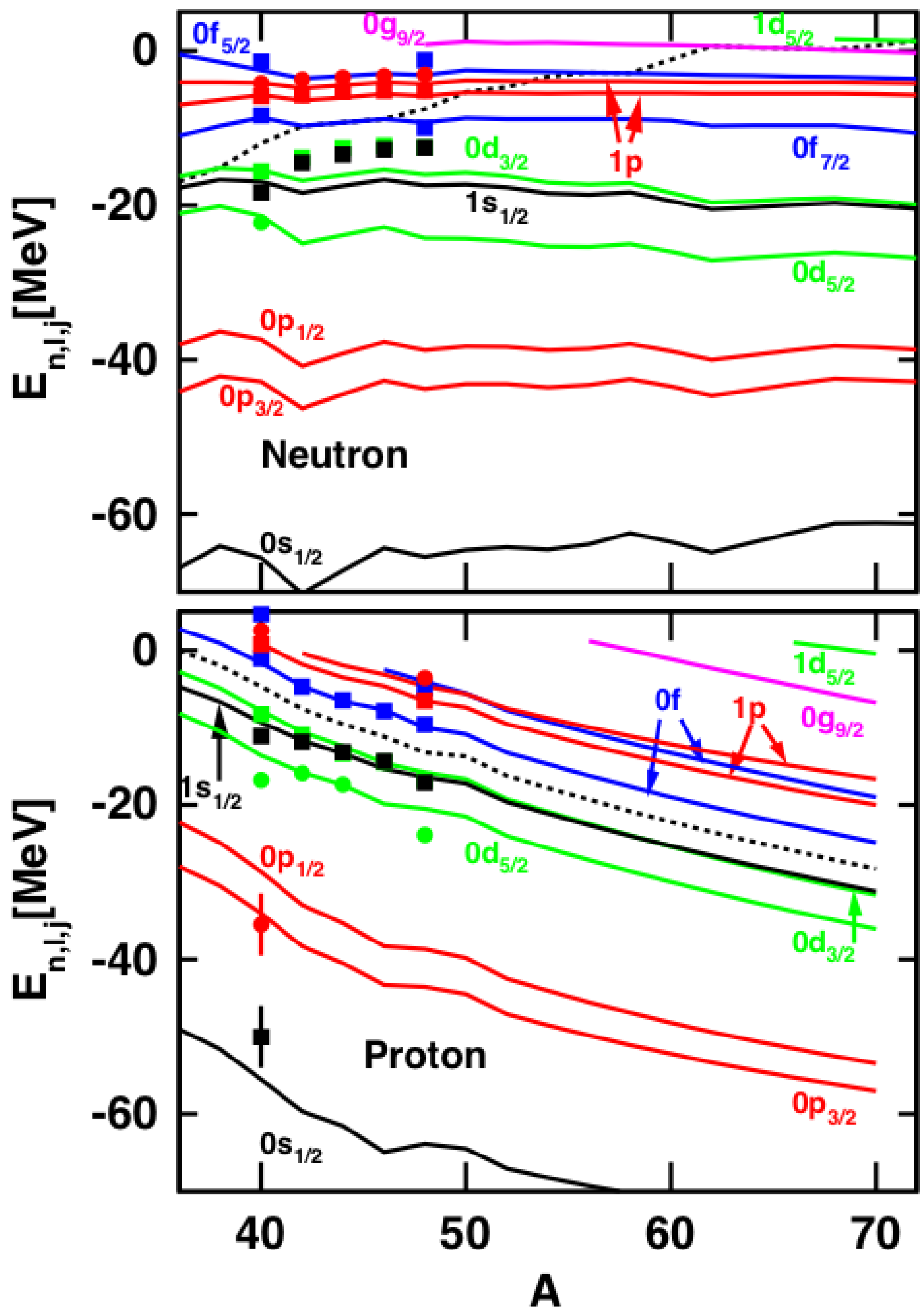


PRC31,1161(1985)

NPA369,258(1981)

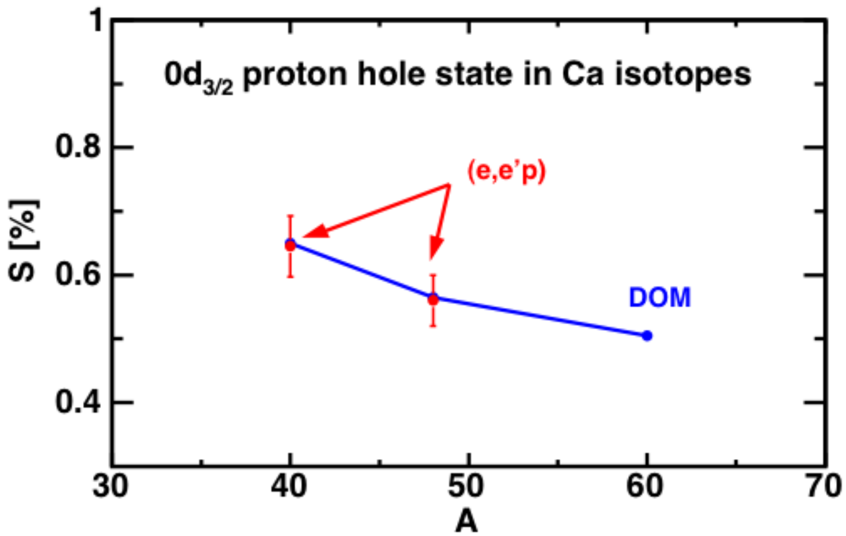
More on this next DOM lecture

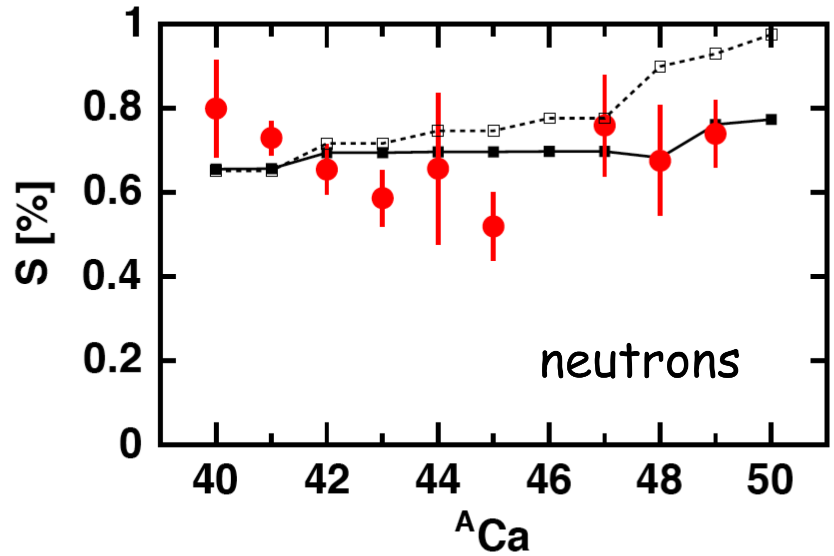
Extrapolation for large N of sp levels



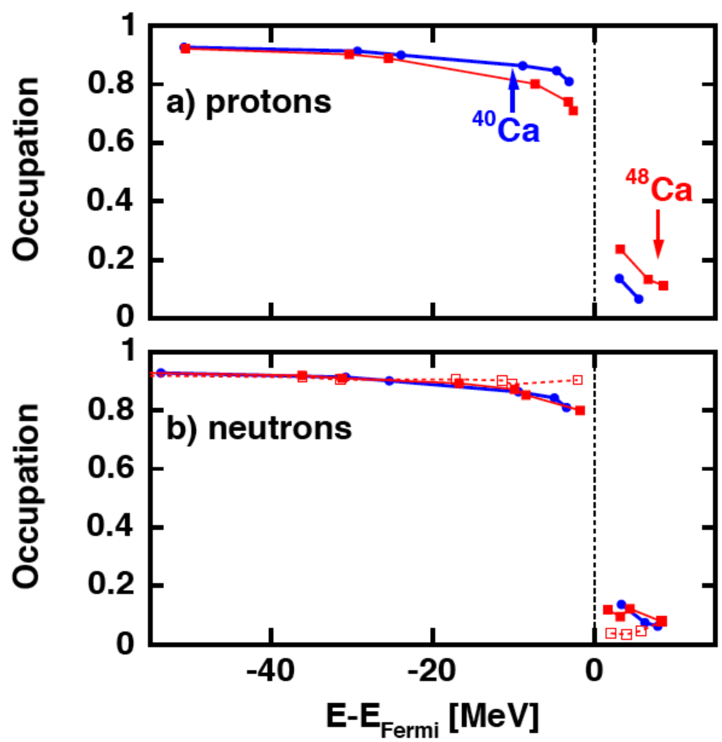
Old ^{48}Ca(p,pn) data
J.W.Watson et al.
Phys. Rev. C26,961 (1982)
~ consistent with DOM

Spectroscopic factors as a function of δ





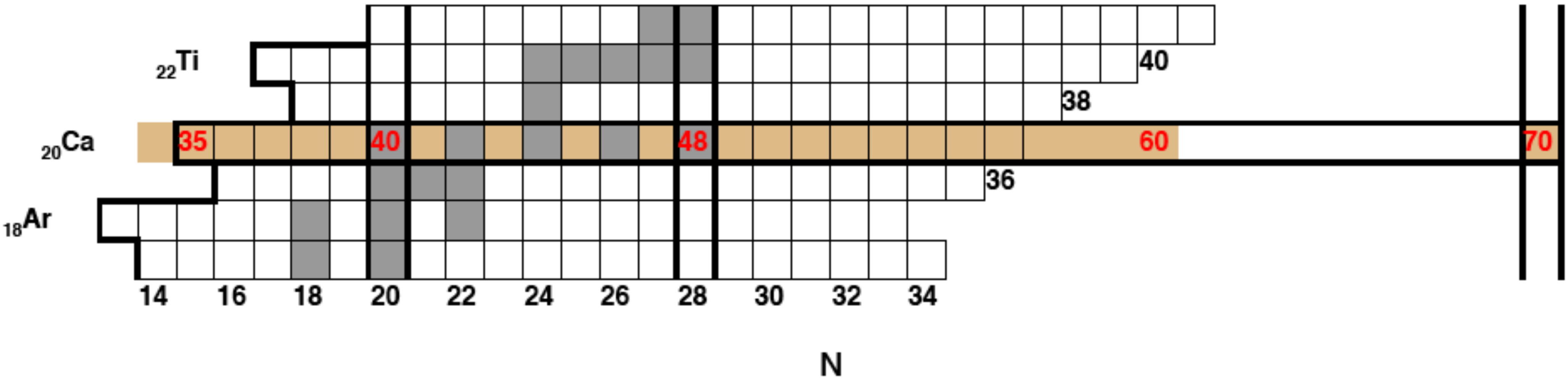
Occupation numbers



Protons more correlated with δ

Neutrons not much change

Driplines



Proton dripline wrong by 1

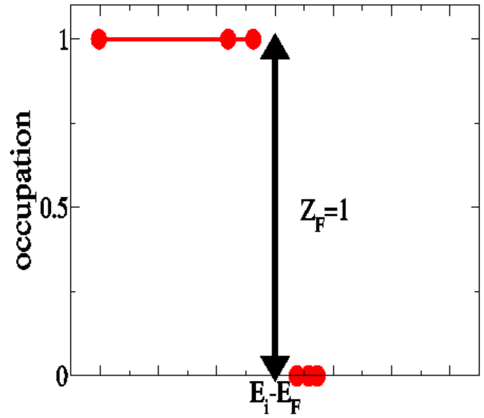
Neutron dripline more complicated:

^{60}Ca and ^{70}Ca particle bound
Intermediate isotopes unbound
Reef?

Correlations in ...

Atoms

weak correlations

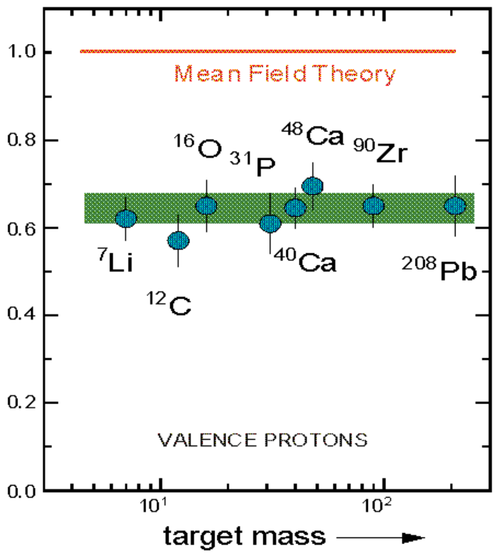


electrons in Ne
Data from (e,2e)

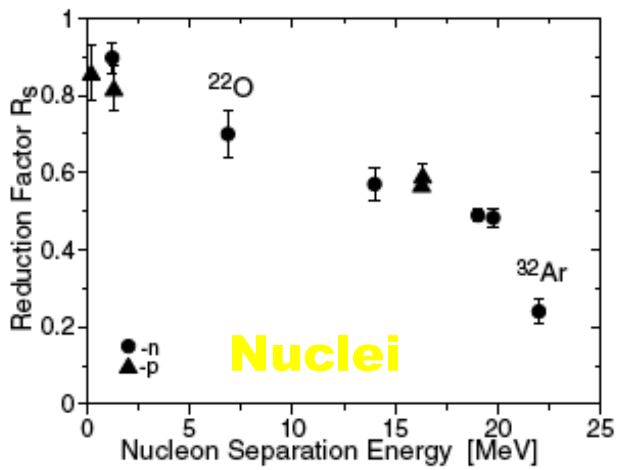
(e,e'p)

DOM

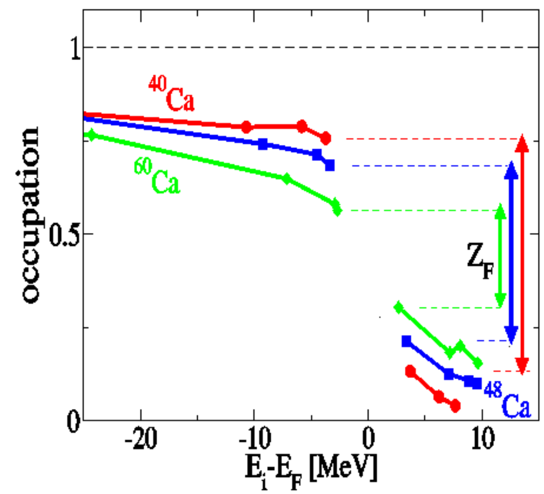
Z_F



Z_F

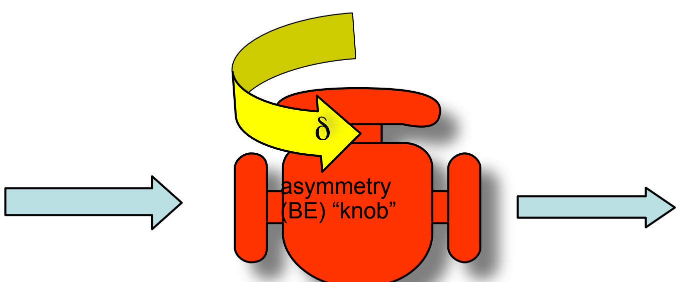


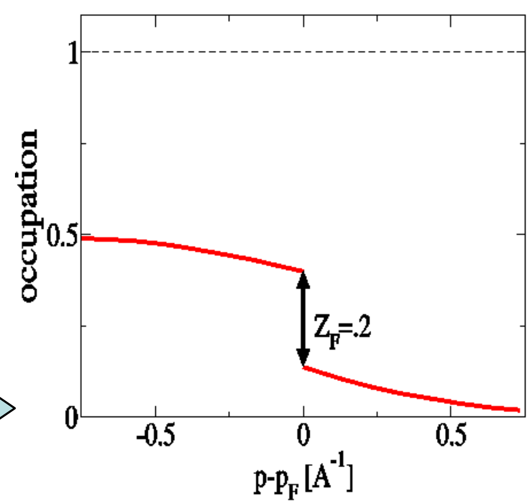
Neutron-proton asymmetry



protons in stable
closed-shell nuclei

protons in Ca





Liquid 3He

very strong correlations
Data from (n,n')

Outlook

- Explore the underlying physics
- More experimental information from elastic nucleon scattering is important!
    - lots of informative experiments to be done with radioactive beams
- Neutron experiments on ^{48}Ca and ^{48}Ca(p,d) in the ^{47}Ca continuum
- Data-driven extrapolations to the neutron dripline

- More DOM analysis requires nonlocal potentials —>
- Exact solution of the Dyson equation with nonlocal potentials (next time)