Why do Green's functions?

- Properly executed --> answers an old question from Sir Denys Wilkinson: "What does a nucleon do in the nucleus?"

- Nucleon self-energy —> think of potential but energy dependent
- Nucleon self-energy —> elastic nucleon scattering data --> input for the analysis of many nuclear reactions
- Nucleon self-energy —> bound-state overlap functions with their normalization --> also used in the analysis of nuclear reactions --> for exotic nuclei only strongly interacting probes available
- Nucleon self-energy—>density distribution & E/A from V_{NN}
- Self-energy <--> data --> dispersive optical model (DOM)

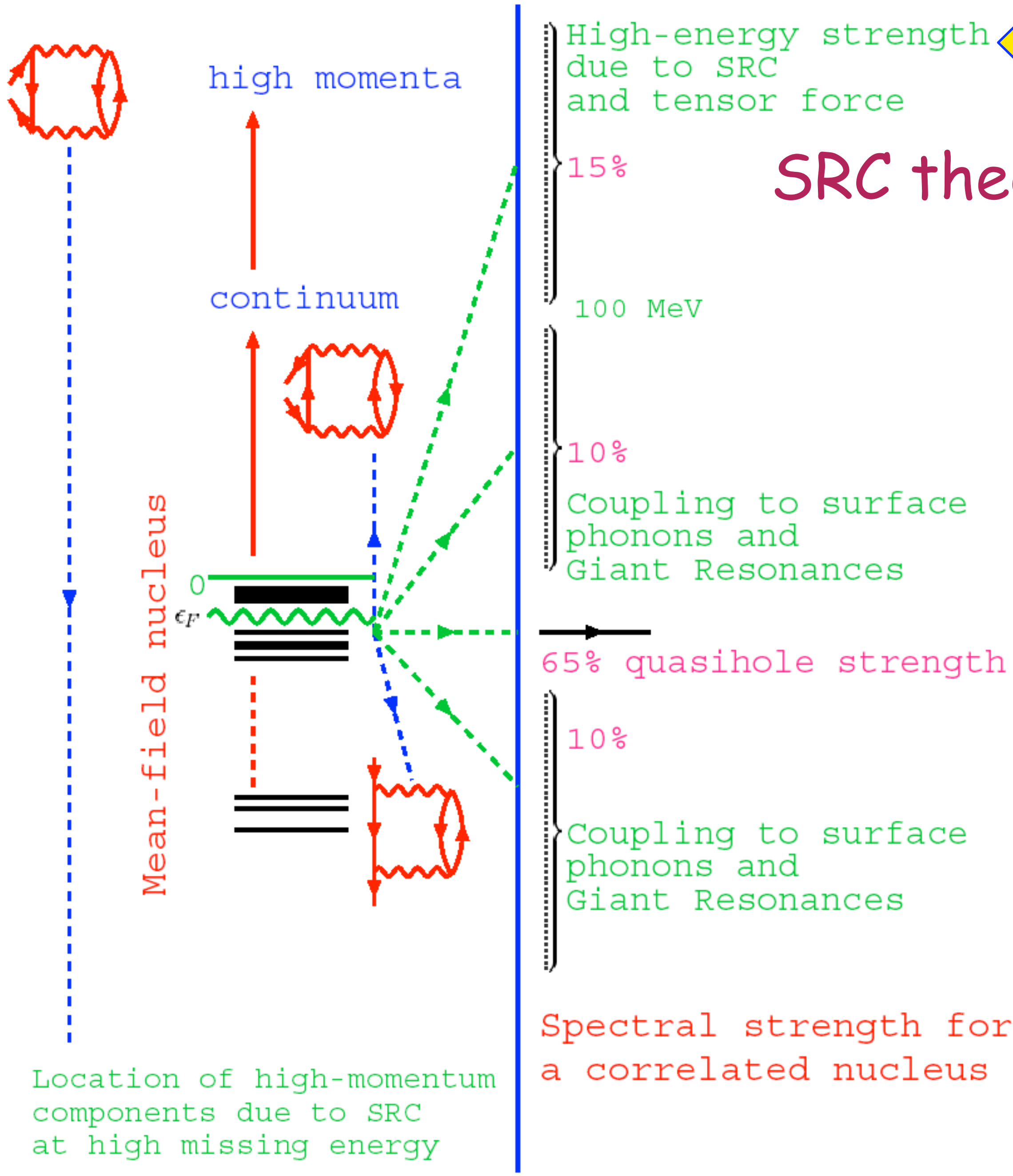
reactions and structure

Reviewed in Prog. Part. Nucl. Phys. 52 (2004) 377-496

Location of single-particle strength in closed-shell (stable) nuclei

For example: protons in ^{208}Pb

SRC



SRC theory

Elastic nucleon scattering

NIKHEF (e,e'p) data

L. Lapikás

Nucl. Phys. A553,297c (1993)

JLab E97-006

Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.

reactions and structure

Remarks

- Given a Hamiltonian, a perturbation expansion can be generated for the single-particle propagator
- Dyson equation determines propagator in terms of nucleon self-energy
- Self-energy is causal and obeys dispersion relations relating its real and imaginary part
- Data constrained self-energy acts as ideal interface between ab initio theory and experiment

Propagator / Green's function

- Lehmann representation

$$G_{\ell j}(k,k';E) = \sum_m \frac{\langle\Psi_0^A|\, a_{k\ell j}\, |\Psi_m^{A+1}\rangle \langle\Psi_m^{A+1}|\, a^\dagger_{k'\ell j}\, |\Psi_0^A\rangle}{E-(E_m^{A+1}-E_0^A)+i\eta} + \sum_n \frac{\langle\Psi_0^A|\, a^\dagger_{k'\ell j}\, |\Psi_n^{A-1}\rangle \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle}{E-(E_0^A-E_n^{A-1})-i\eta}$$

- Any other single-particle basis can be used
- Overlap functions --> numerator
- Corresponding eigenvalues --> denominator
- Spectral function

$$S_{\ell j}(k;E) = \frac{1}{\pi}\,\mathrm{Im}\; G_{\ell j}(k,k;E) \qquad E \le \varepsilon_F^-$$

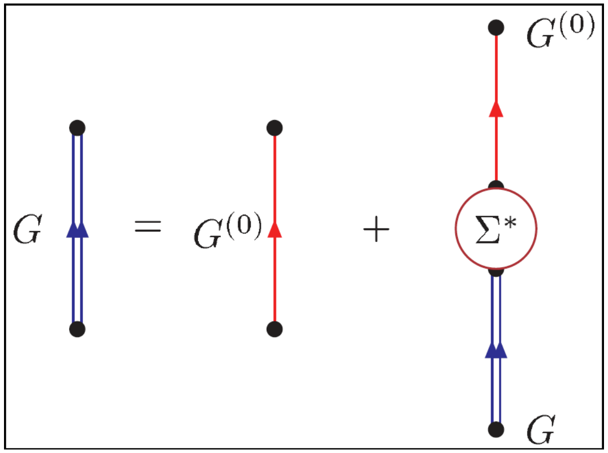
$$= \sum_n \left| \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle \right|^2 \delta(E-(E_0^A-E_n^{A-1}))$$

- Spectral strength in the continuum

$$S_{\ell j}(E) = \int_0^\infty dk\; k^2\; S_{\ell j}(k;E)$$

- Discrete transitions $\sqrt{S^n_{\ell j}}\;\phi^n_{\ell j}(k) = \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle$
- Positive energy —> see later

Propagator from Dyson Equation and "experiment"



Equivalent to …

Schrödinger-like equation with: $E_n^- = E_0^A - E_n^{A-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1

⇒ as extracted from (e,e'p) reaction

$$\frac{k^2}{2m}\phi_{\ell j}^n(k) + \int dq\ q^2\ \Sigma_{\ell j}^*(k,q;E_n^-)\ \phi_{\ell j}^n(q) = E_n^-\ \phi_{\ell j}^n(k)$$

Spectroscopic factor $\quad S_{\ell j}^n = \int dk\ k^2\ \left|\langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle\right|^2 < 1$

Dyson equation also yields $\left[\chi_{\ell j}^{elE}(r)\right]^* = \langle \Psi_{elE}^{A+1} | a_{r\ell j}^\dagger | \Psi_0^A \rangle$ for positive energies

Elastic scattering wave function for protons or neutrons

Dyson equation therefore provides:

Link between scattering and structure data from **dispersion relations**

reactions and structure

How is it done in practice?

- Dyson equation for discrete states

$$\left[\frac{p_r^2}{2m}+\frac{\hbar^2\ell(\ell+1)}{2mr^2}\right]\psi_{\ell j}^n(r)+\int dr'\ r'^2\Sigma_{\ell j}(r,r';\varepsilon_n^-)\psi_{\ell j}^n(r')=\varepsilon_n^-\psi_{\ell j}^n(r)$$

- with $p_r=-\frac{i}{\hbar}\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)$

- Work with $u_{\ell j}^n(r)=r\psi_{\ell j}^n(r)$ so only 2nd derivative $\frac{d^2}{dr^2}$
- Put coordinate on equidistant grid $r_i=(i-1/2)\Delta \qquad i=1,2,...$
- Approximate 2nd derivative for i>1 by

$$u''(r_i)=[u(r_{i+1})+u(r_{i-1})-2u(r_i)]/\Delta^2$$

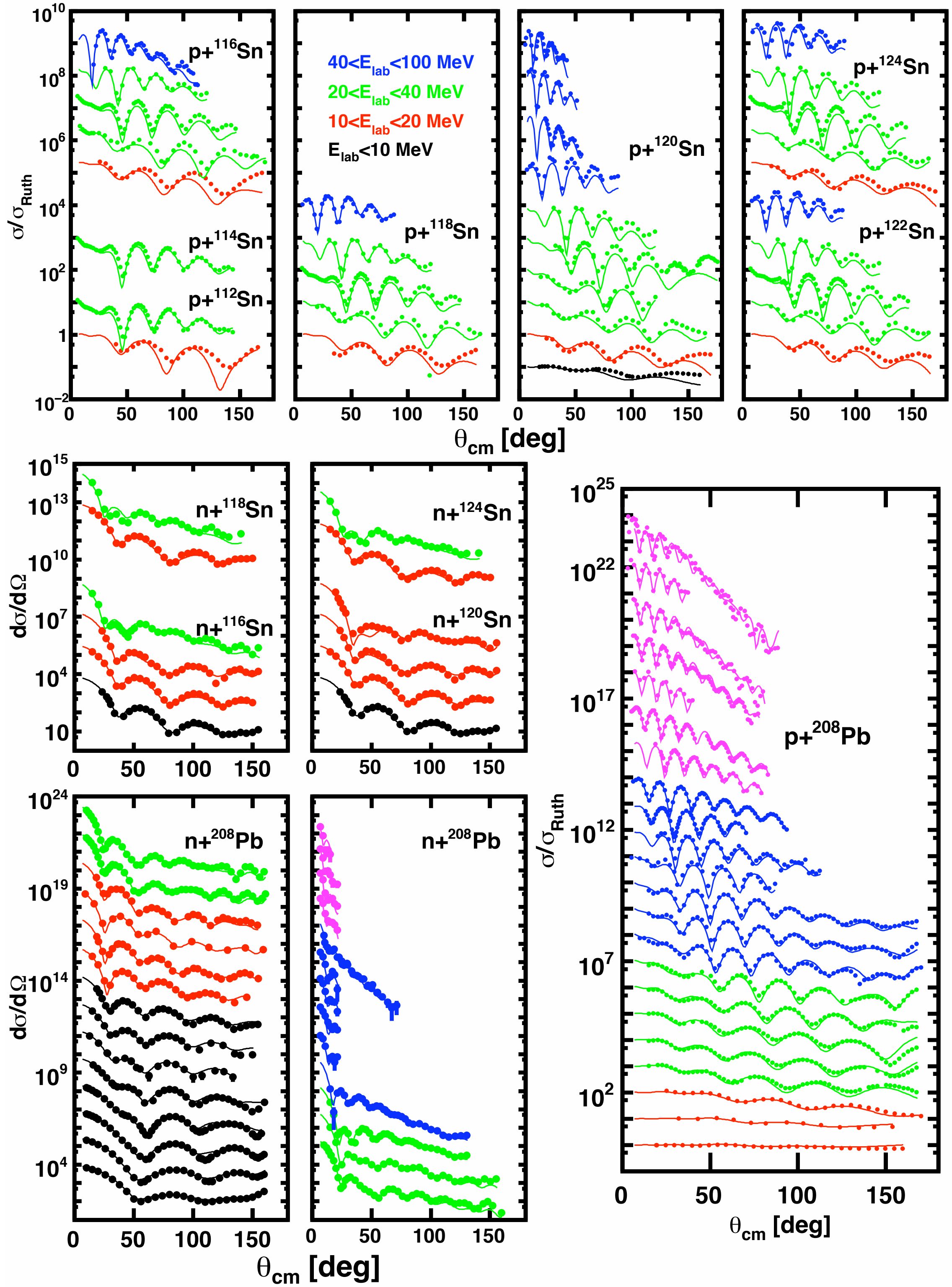
- First point $u''(r_1)=[u(r_2)+u(r_0)-2u(r_1)]/\Delta^2$
- Use "continuity" for r<0: for parity $\pm 1\Rightarrow u(-r)=\pm u(r)$
- using bc —> $\begin{aligned} u''(r_1) &= [u(r_2)-u(r_1)]/\delta^2 \Rightarrow + \\ u''(r_1) &= [u(r_2)-3u(r_1)]/\delta^2 \Rightarrow - \end{aligned}$ then diagonalize etc.

Propagator

- Same discretization

$$G_{\ell j}(r,r';E) = G^{(0)}_{\ell j}(r,r';E) + \int d\tilde{r}\ \tilde{r}^2 \int d\tilde{r}'\ \tilde{r}'^2 G^{(0)}_{\ell j}(r,\tilde{r};E)\Sigma_{\ell j}(\tilde{r},\tilde{r}';E)G_{\ell j}(\tilde{r}',r';E)$$

- Use matrix inversion now
- Then spectral amplitude $S_{\ell j}(r,r';E) = \frac{1}{\pi}\mathrm{Im}\ G_{\ell j}(r,r';E)$
- Spectral function $S_{\ell j}(r;E) = \frac{1}{\pi}\mathrm{Im}\ G_{\ell j}(r,r;E)$
- and so on



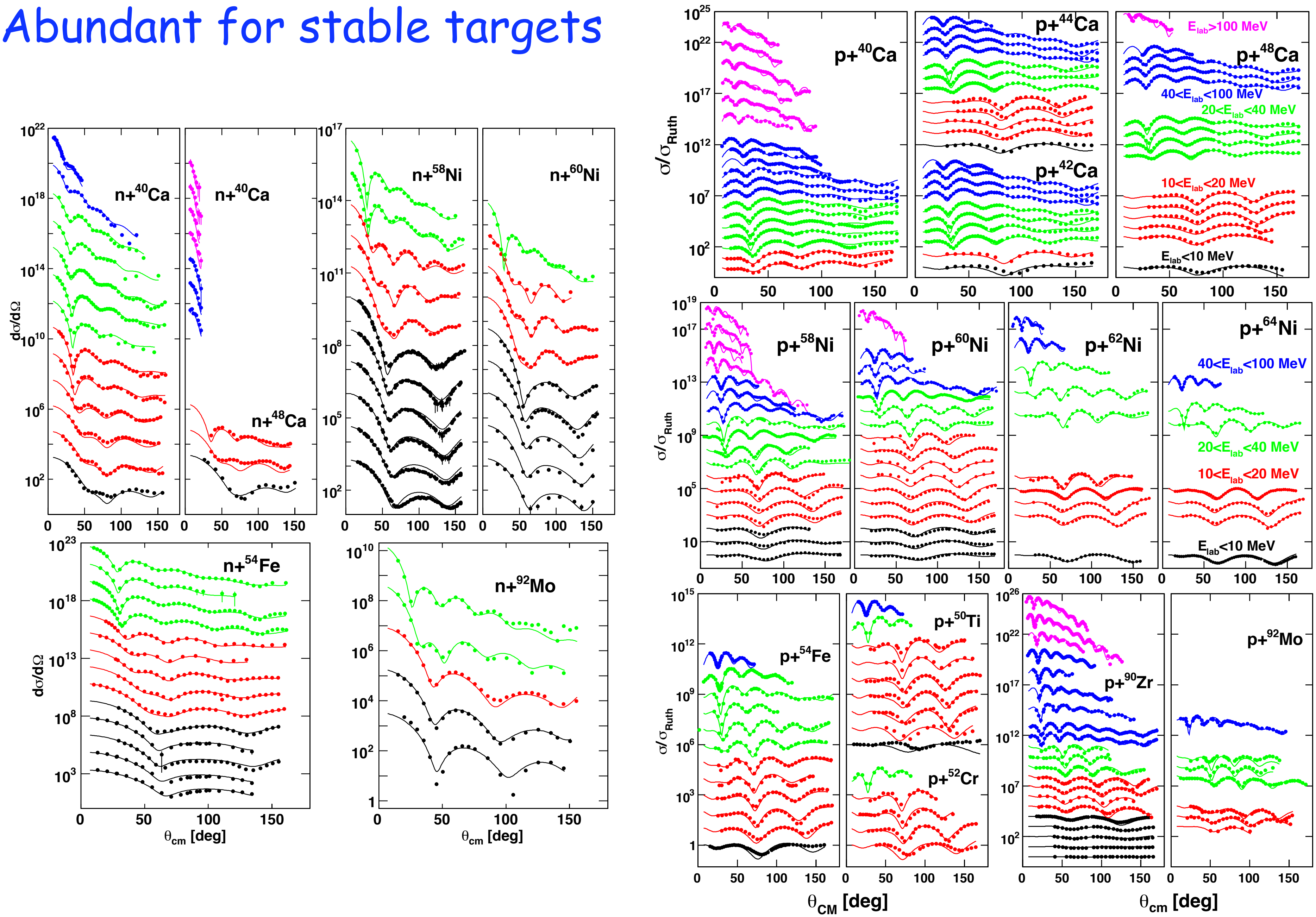
Recent local DOM analysis --> towards global

J. Mueller et al.
PRC83,064605 (2011), 1-32

reactions and structure

Elastic scattering data for protons and neutrons

- Abundant for stable targets

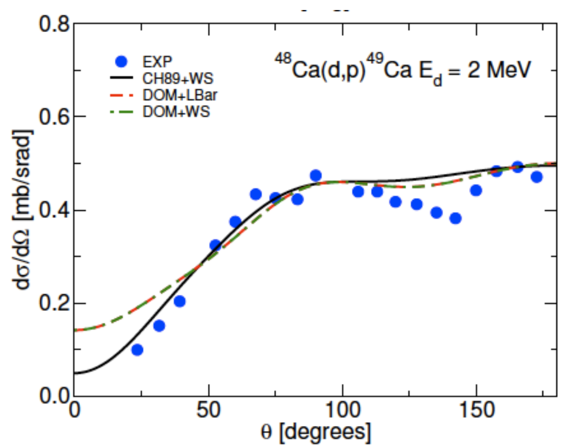


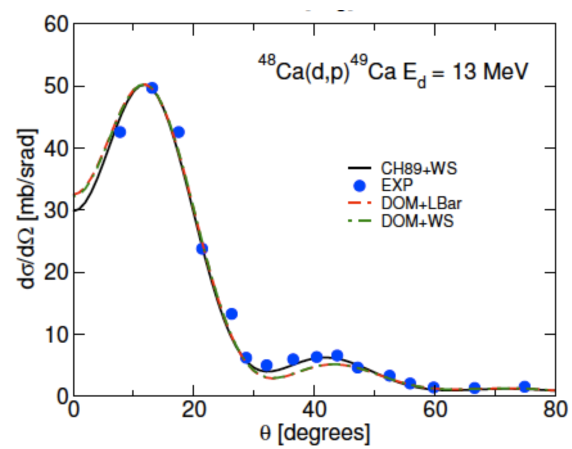
Local DOM ingredients and transfer reactions

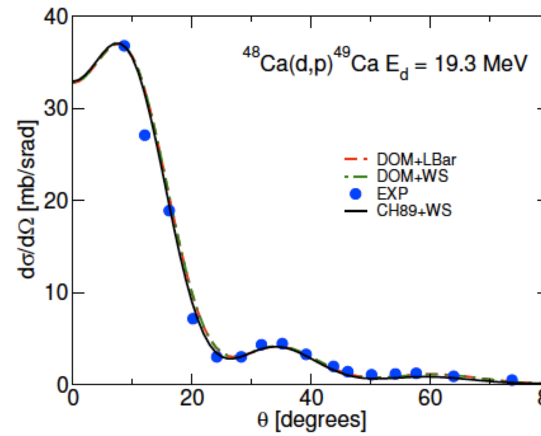
- Overlap function
- p and n optical potential
- ADWA (developed by Ron Johnson)
- MSU-WashU:-->
- ^{40,48}Ca,^{132}Sn,^{208}Pb(d,p)

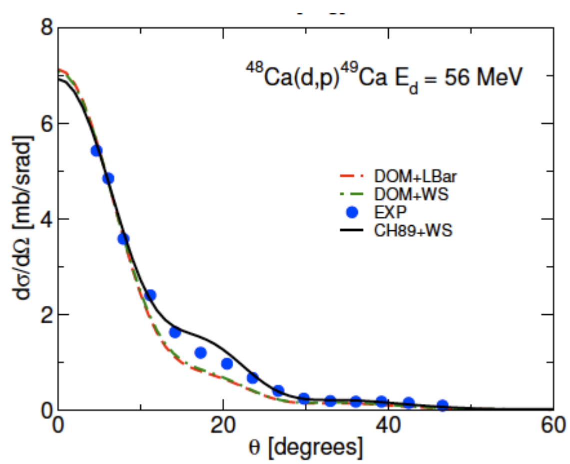
N. B. Nguyen, S. J. Waldecker, F. M. Nuñes, R. J. Charity, and W. H. Dickhoff

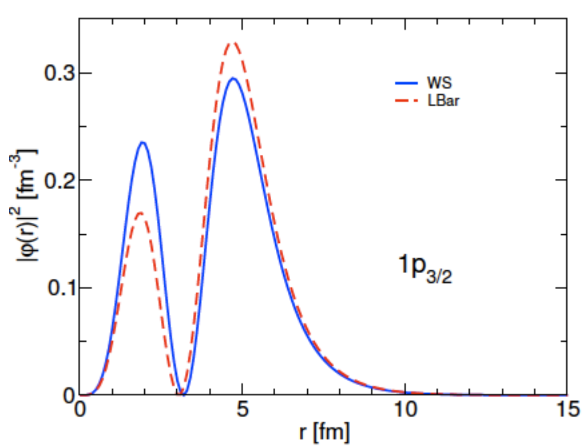
Phys. Rev. C84, 044611 (2011), 1-9











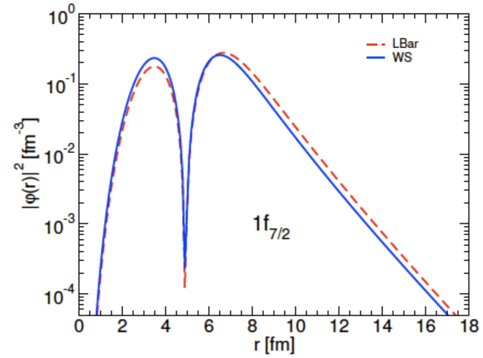
E	CH+ws	DOM
2	0.94	0.72
13	0.82	0.67
19.3	0.77	0.68
56	1.1	0.70

reactions and structure

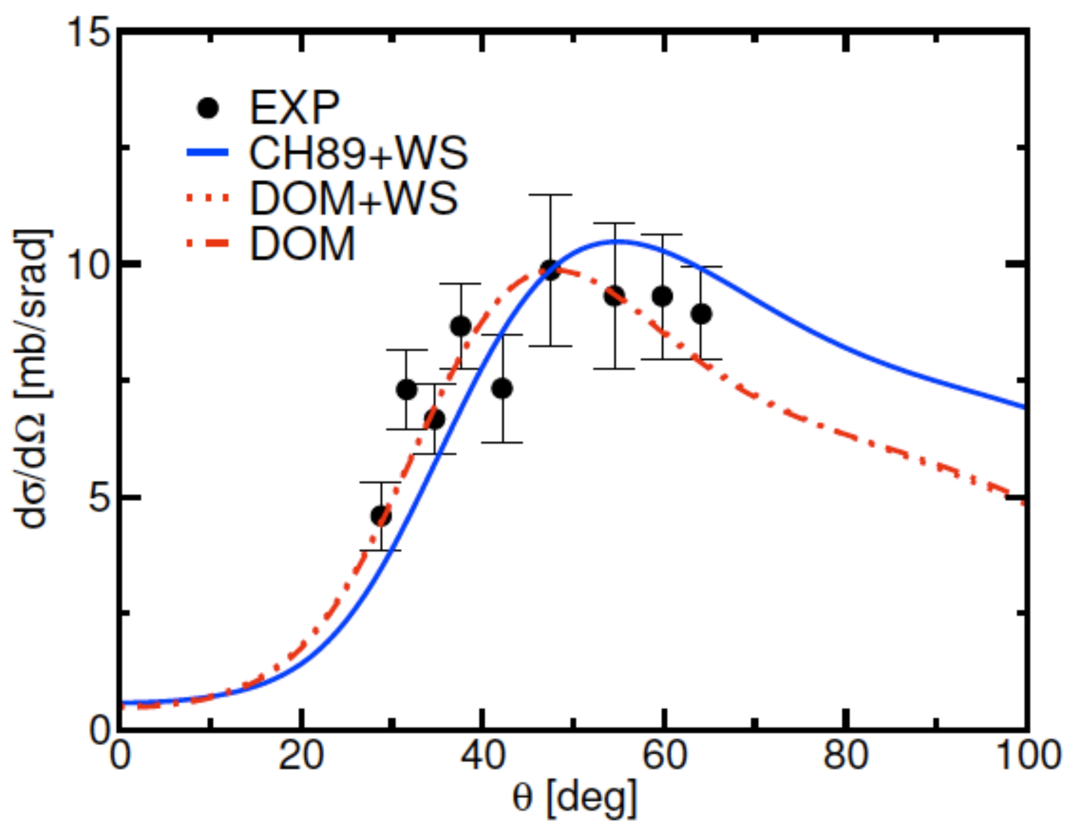
"SPECTROSCOPIC FACTORS"

$^{132}Sn(d,p)$

- Does it work when the potentials are extrapolated?



- Data: K.L. Jones et al., Nature 465, 454 (2010)
- E_d = 9.46 MeV $^{132}Sn(d,p)^{133}Sn$
- CH89+ws --> $S_{1f7/2}$ =1.1
- DOM --> $S_{1f7/2}$ =0.72



Propagator in principle generates

- Elastic scattering cross sections for p and n
- Including all polarization observables
- Total cross sections for n
- Reaction cross sections for p and n
- Overlap functions for adding p or n to bound states in Z+1 or N+1
- Plus normalization --> spectroscopic factor
- Overlap function for removing p or n with normalization
- Hole spectral function including high-momentum description
- One-body density matrix; occupation numbers; natural orbits
- Charge density
- Neutron distribution
- p and n distorted waves
- Contribution to the energy of the ground state from V_{NN}

DOM improvements

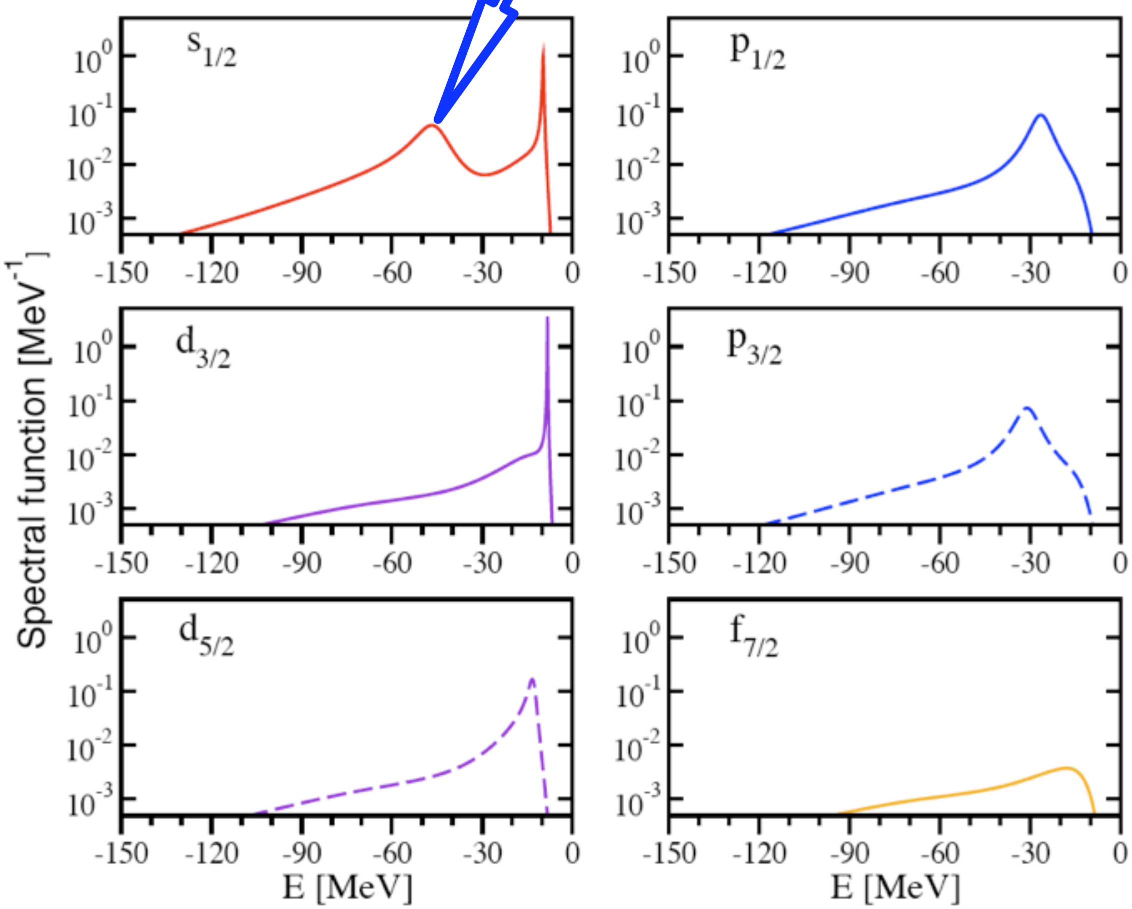
- Replace local energy-dependent HF potential by non-local (energy-independent potential) in order to calculate more properties below the Fermi energy like the charge density and spectral functions --> PRC82, 054306 (2010)

DOModel --> DOMethod-->DSelf-energyMethod

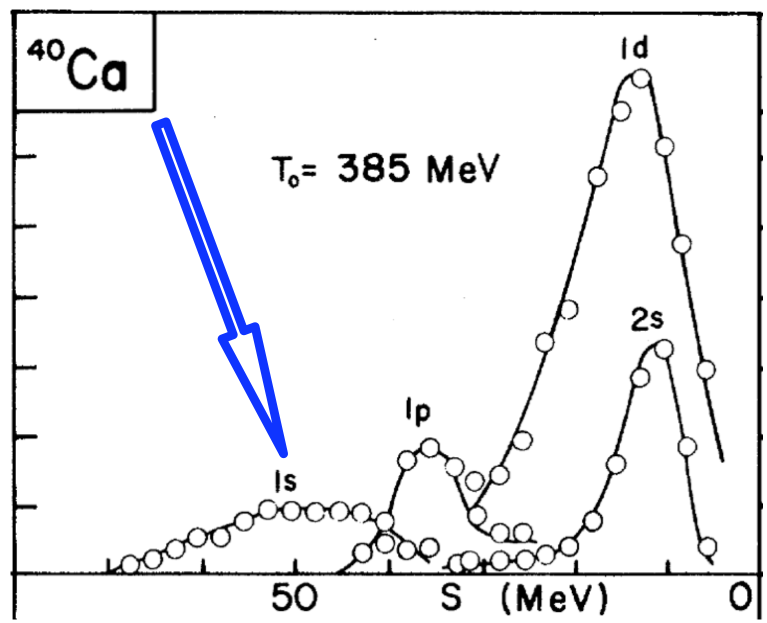
Below ε_F

Recent theoretical development: nonlocal "HF" self-energy --> below the Fermi energy
WD, Van Neck, Charity, Sobotka, Waldecker, PRC82, 054306 (2010)

^{40}Ca spectral function



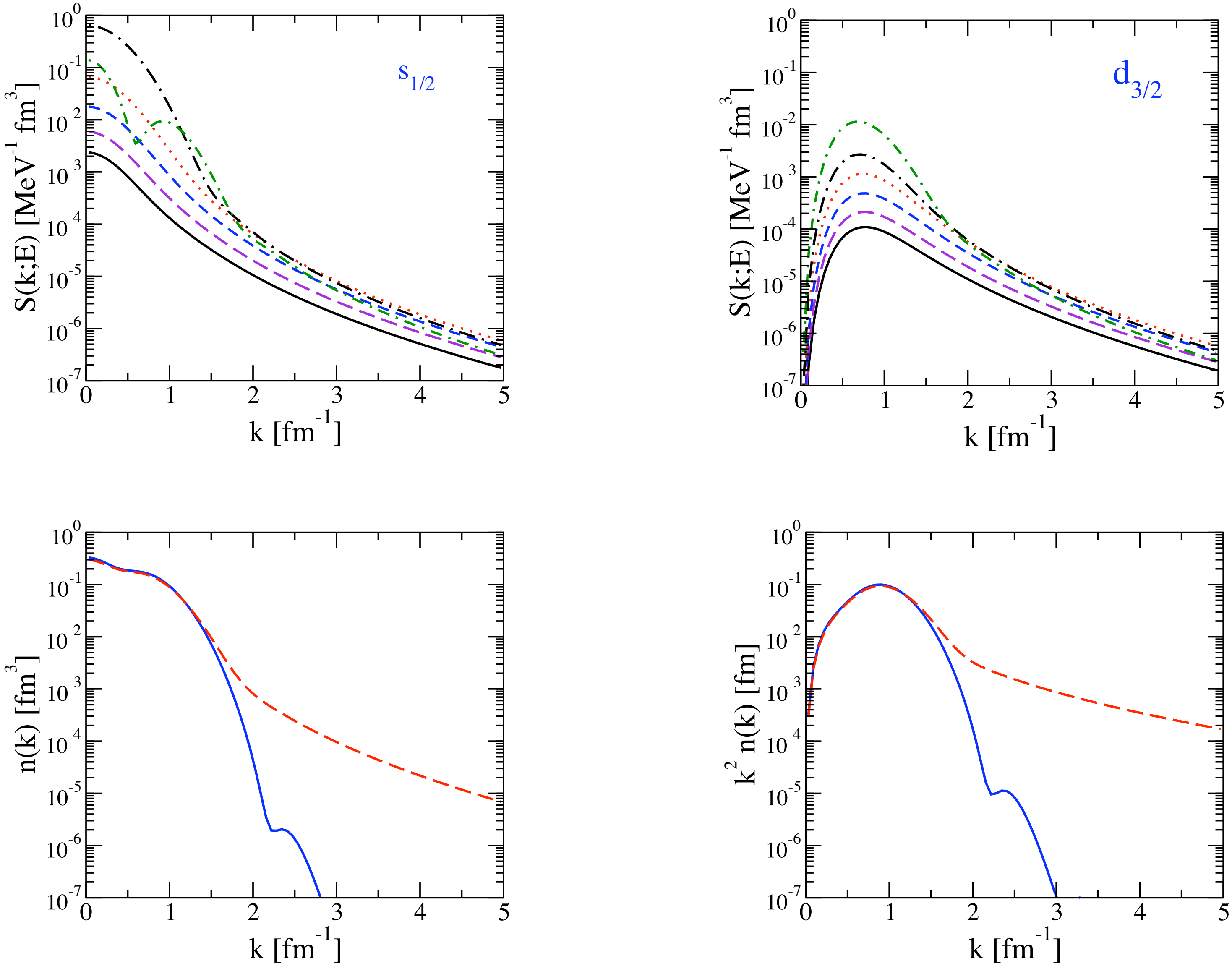
Old (p,2p) data from Liverpool



Spectral functions and momentum distributions

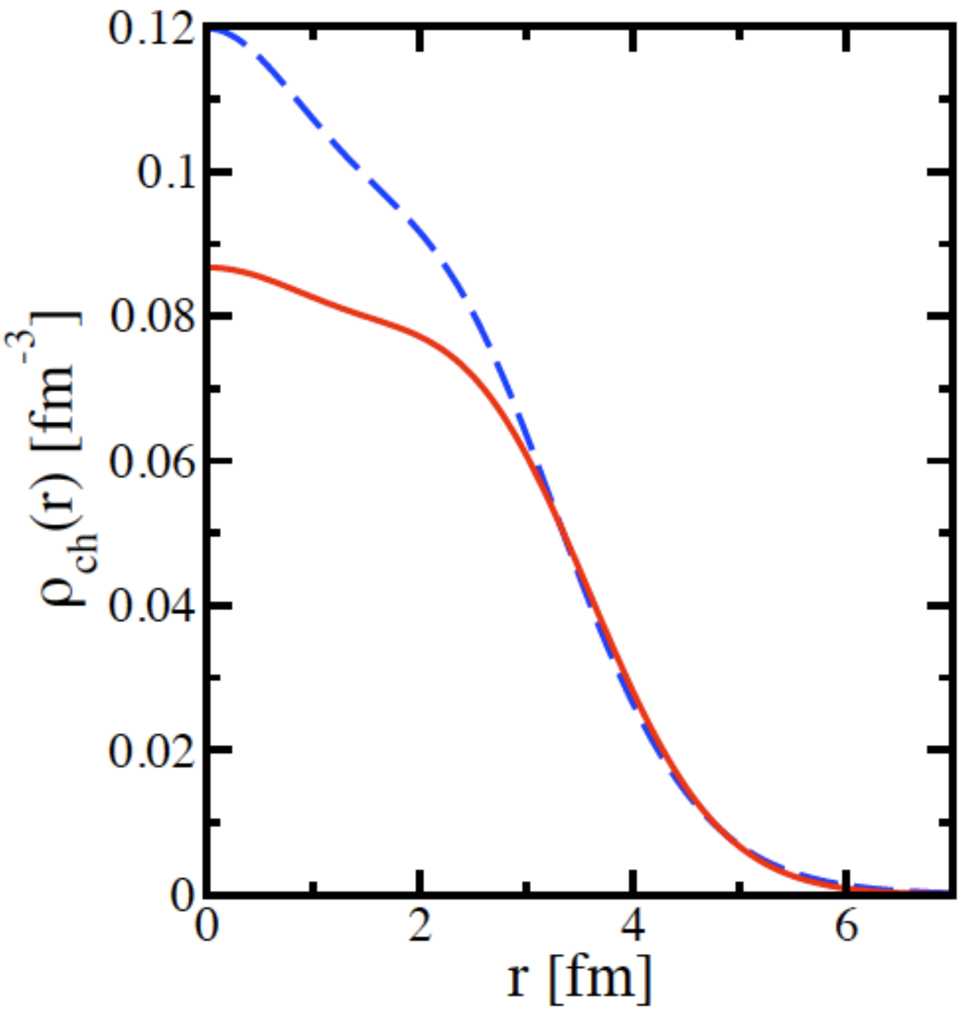
- ^{40}Ca

PRC 82, 054306 (2010)



Understanding/Calculating Self-energy

Charge density



Not a good reproduction of charge density even though mean square radius was fitted.

Related to local representation of the imaginary part of the self-energy --> independent of angular momentum --> must be abandoned to represent particle number correctly as well.

DOM extensions linked to ab initio FRPA

- Employ microscopic FRPA calculations of the nucleon self-energy to gain insight into future improvements of the DOM -->

S. J. Waldecker, C. Barbieri and W. H. Dickhoff

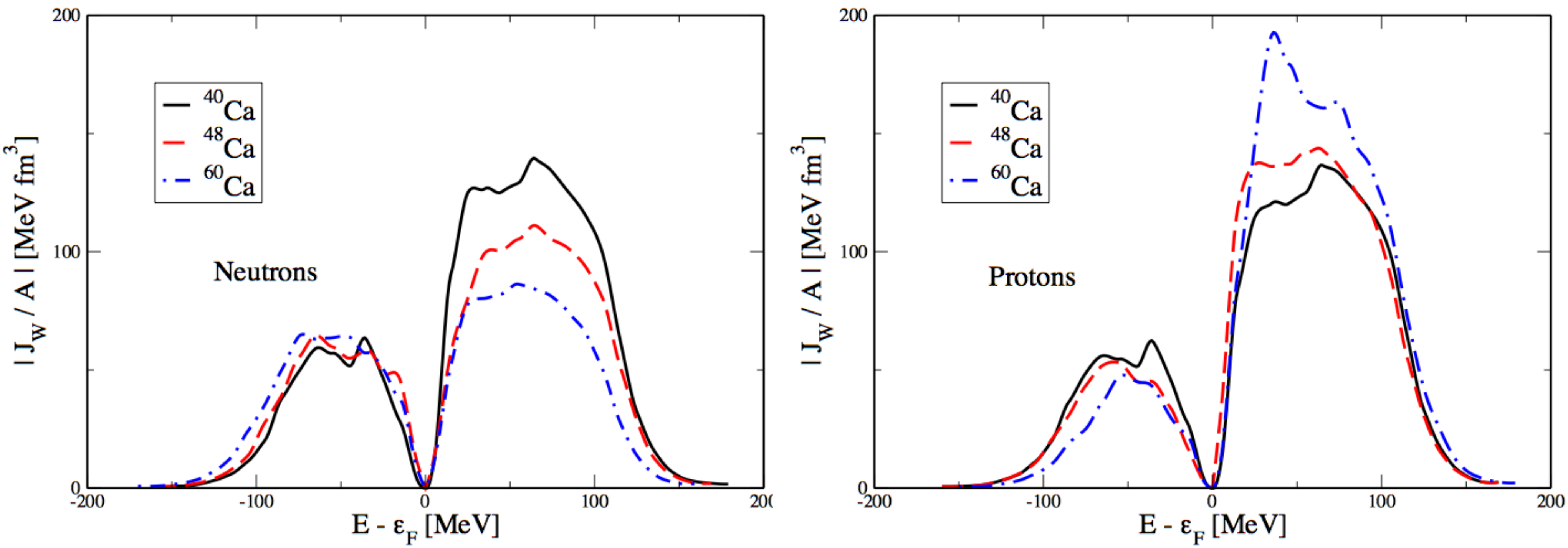
Phys. Rev. C84, 034616 (2011), 1-11

- FRPA = Faddeev RPA --> Barbieri for a recent application see e.g. PRL103,202502(2009)
- Most important conclusions
  - Ab initio self-energy has imaginary part with a substantial non-locality
  - Tensor force already operative for low-energy imaginary part
  - Absorption above and below Fermi energy not symmetric

Volume integrals from microscopic FRPA
relevant up to ~ 75 MeV

Volume integral for local imaginary potentials

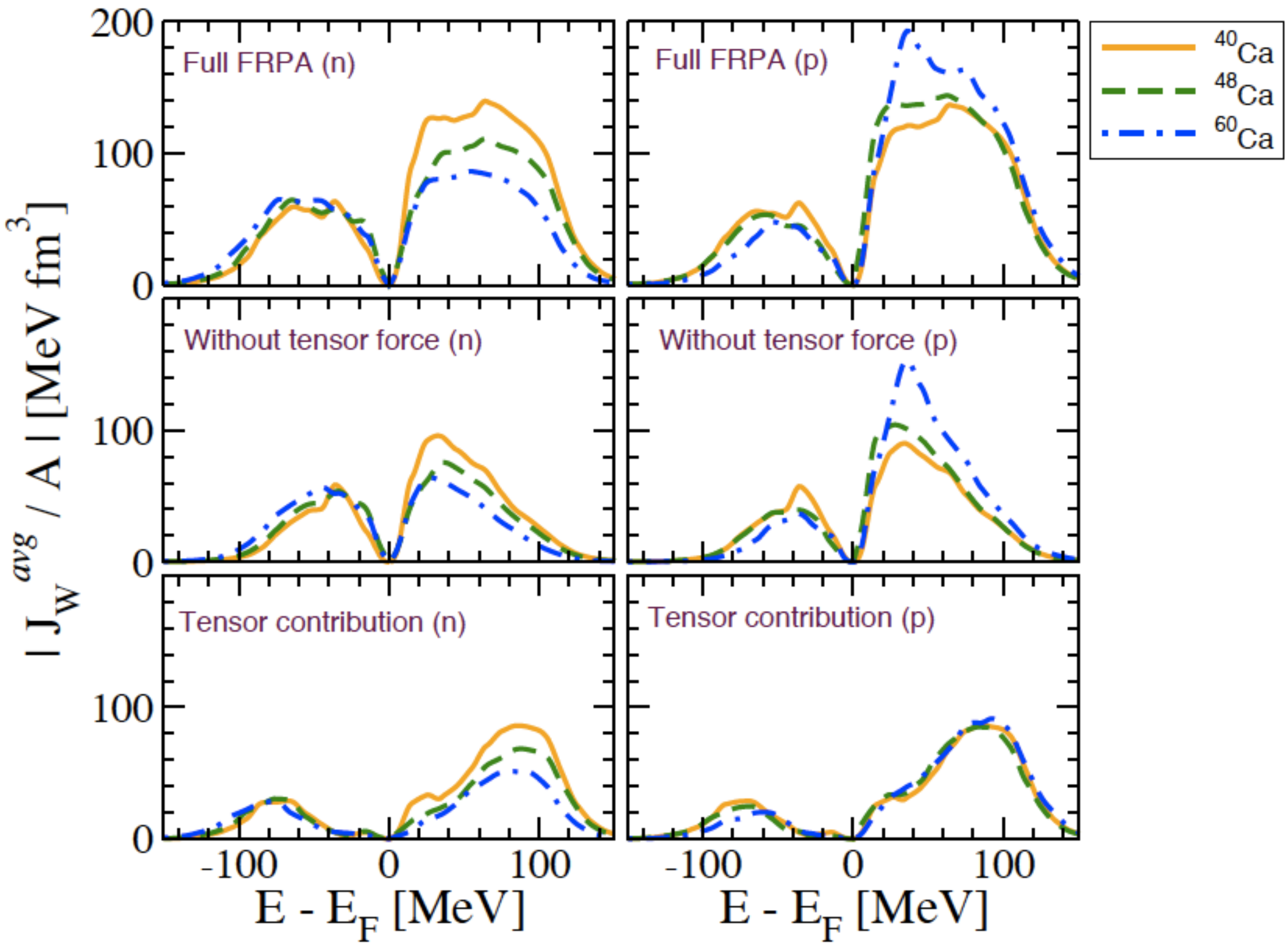
$$J_W(E) = 4\pi \int dr\ r^2\ W(r,E)$$



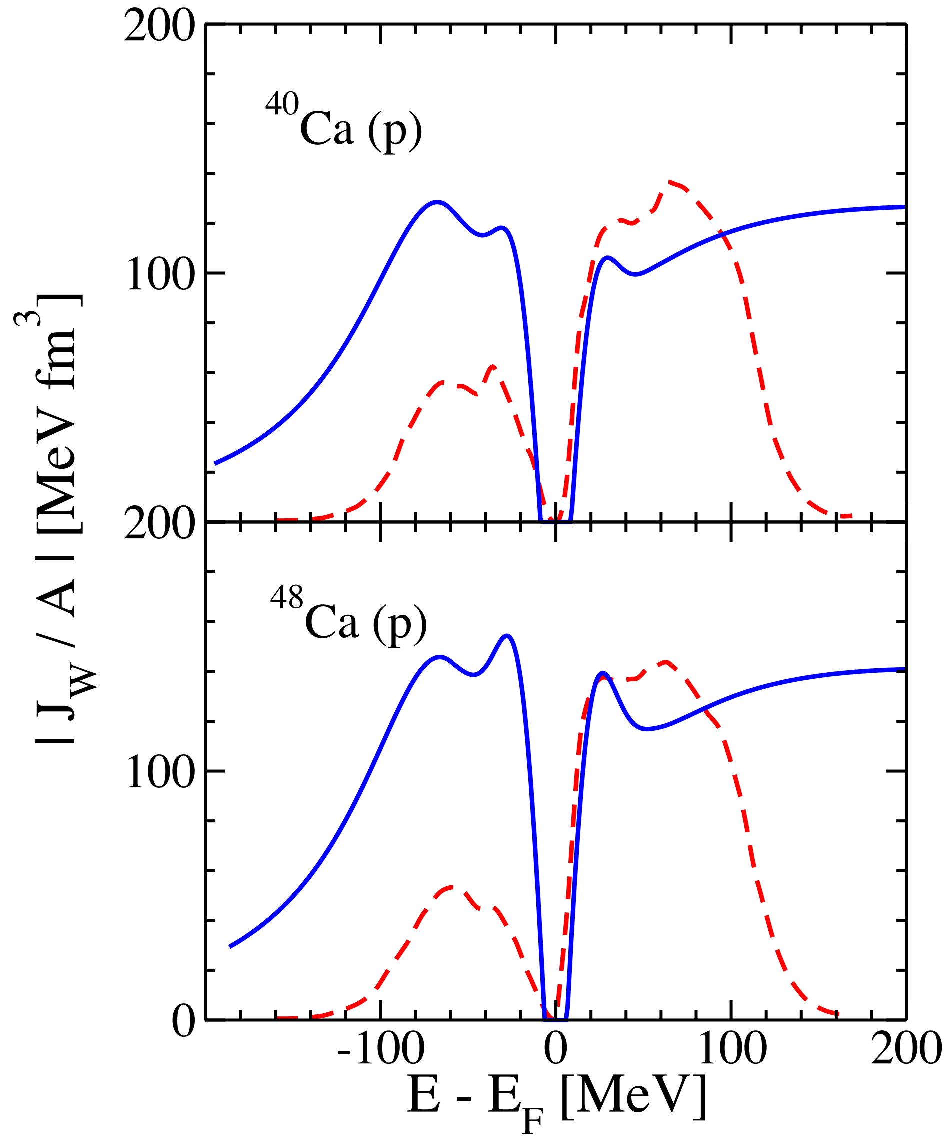
Microscopic potentials: nonlocal --> depend strongly on ℓ

Here averaged

Tensor force



Comparison with DOM for ^{40,48}Ca



DOM extensions linked to ab initio treatment of SRC

- Employ microscopic calculations of the nucleon self-energy to gain insight into future improvements of the DOM -->

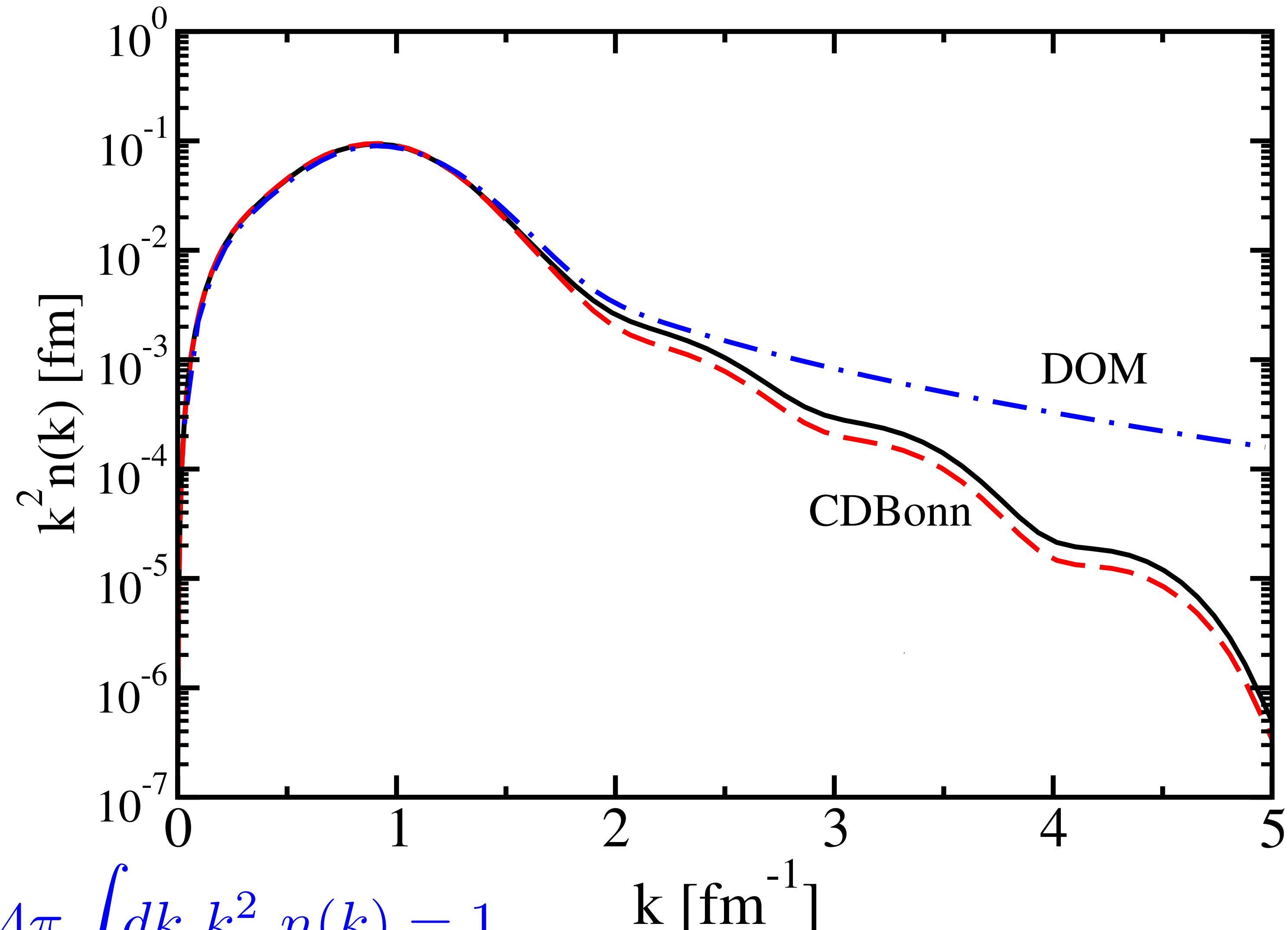
  H. Dussan, S. J. Waldecker, W. H. Dickhoff, H. Müther, and A. Polls

  Phys. Rev. C84, 044319 (2011), 1-16

- CDBonn --> self-energy in momentum space for ^{40}Ca
- Most important conclusions
  - Volume absorption below the Fermi energy is also nonlocal
  - Reaction cross section comparable with DOM above ~ 80 MeV

Ab initio with CDBonn for ^{40}Ca

- Dussan et al. PRC84, 044319 (2011); spectral functions available



$$4\pi \int dk\ k^2\ n(k) = 1$$

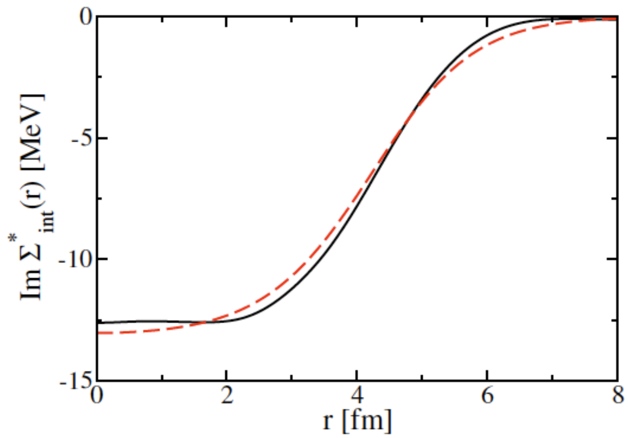
Understanding/Calculating Self-energy

Non-locality of imaginary part

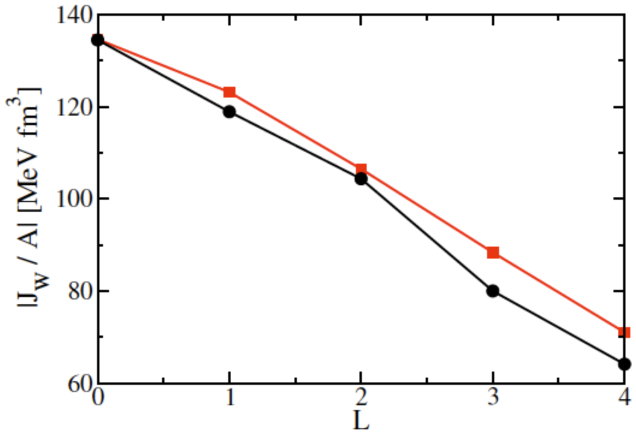
- Fit non-local imaginary part for ℓ=0

$$W_{NL}(\boldsymbol{r},\boldsymbol{r}') = W_0 \sqrt{f(r)}\sqrt{f(r')} H\left(\frac{\boldsymbol{r}-\boldsymbol{r}'}{\beta}\right)$$

- Integrate over one radial variable



- Predict volume integrals for higher ℓ

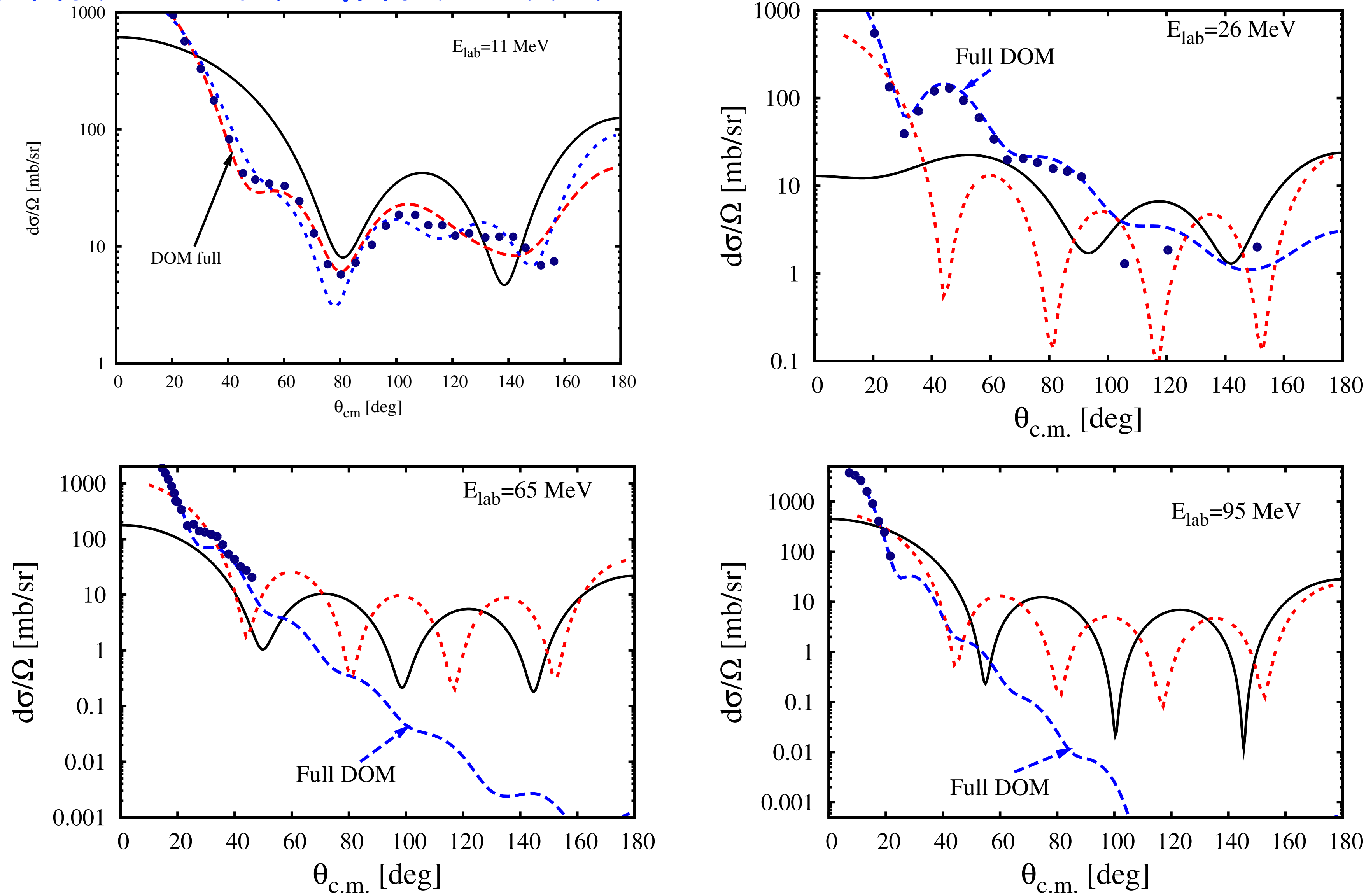


Parameters

Energy MeV	W_0	r_0	a_0	β	$\lvert J_W/A \rvert$	$\lvert J_W/A \rvert$ CDBonn
-76	36.30	0.90	0.90	1.33	193	193
49	6.51	1.25	0.91	1.43	73	73
65	13.21	1.27	0.70	1.29	135	135
81	23.90	1.22	0.67	1.21	215	215

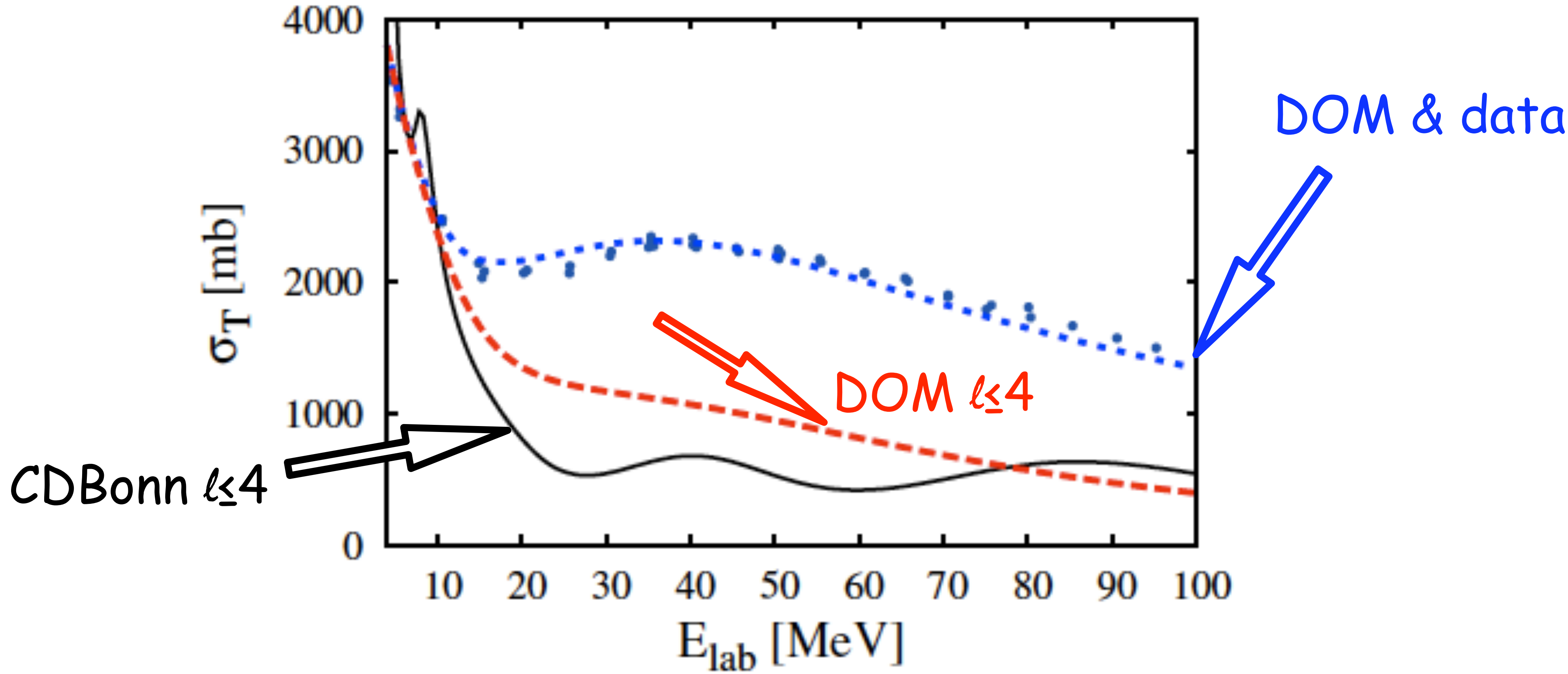
Ab initio description of elastic scattering

- Must be done much better



Ab initio calculation of elastic scattering n+^{40}Ca

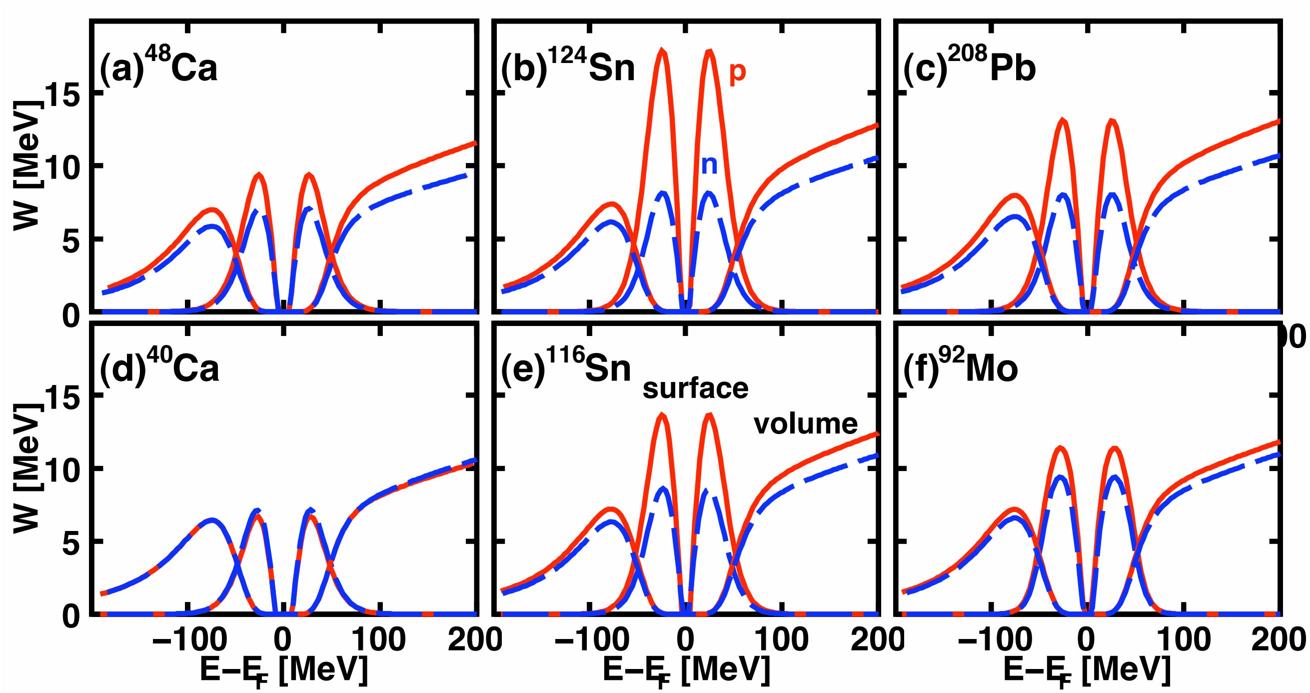
- ONLY treatment of short-range and tensor correlations



DOM predictions

- Use non-local "HF" potential and dispersive DOM potential to extrapolate to unstable Sn isotopes and predict (e.g.) properties of the last proton (based on the analysis of elastic scattering data on STABLE Sn nuclei)

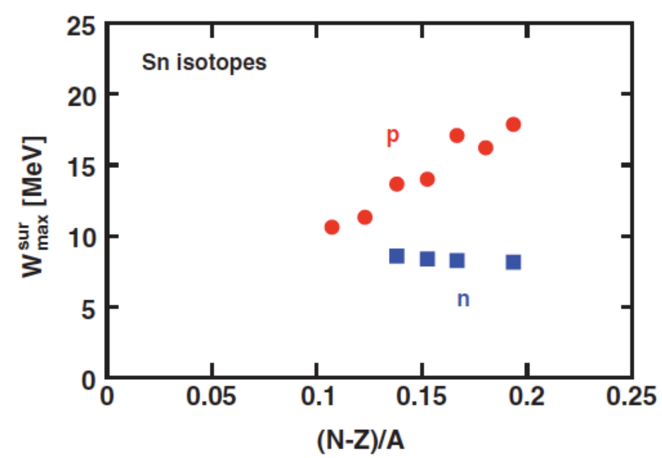
Asymmetry dependence of imaginary potentials



- Volume -> small asymmetry dependence determined in ^{208}Pb $W_{volume} = W^0_{volume} \pm \frac{N-Z}{A} W^1_{volume}$

- Neutron surface -> no strong dependencies on A or (N-Z)/A

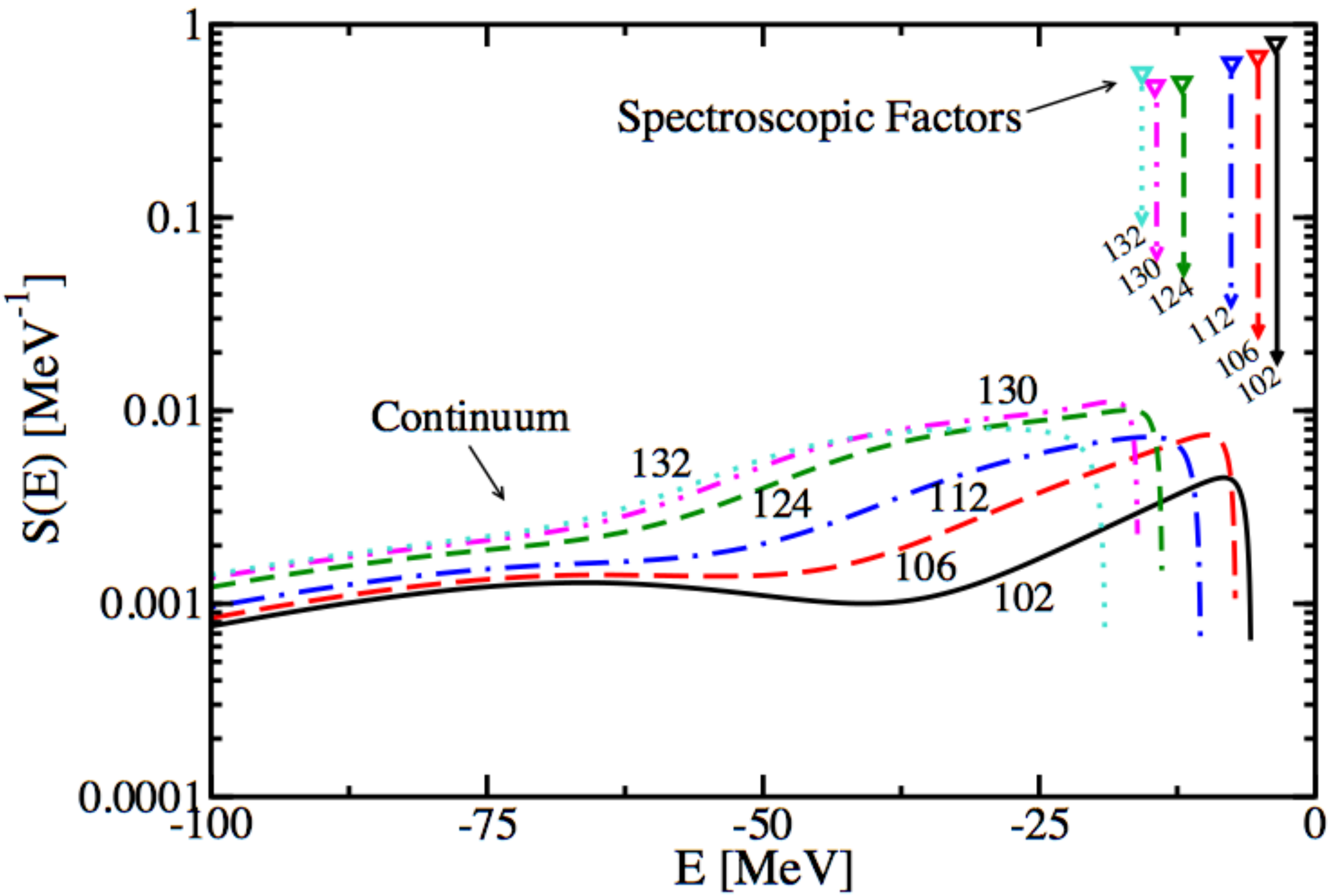
- Proton surface absorption -> increases with increasing neutron number



reactions and structure

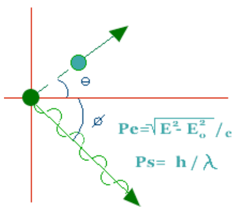
Last proton in Sn nuclei ($g_{9/2}$)

Spectral function for different Sn isotopes



Sn	S	n
102	0.80	0.91
106	0.68	0.85
112	0.63	0.83
124	0.50	0.78
130	0.48	0.78
132	0.56	0.81

People involved



Wim Dickhoff
Bob Charity
Lee Sobotka
Helber Dussan
Seth Waldecker
Hossein Mahzoon
Dong Ding

Carlo Barbieri, Surrey
Arnau Rios, Surrey
Arturo Polls, Barcelona
Dimitri Van Neck, Ghent
Herbert Müther, Tübingen

N.B. Nguyen & F. Nuñes

reactions and structure

Dispersive Optical Model

- Claude Mahaux 1980s
  - connect traditional optical potential to bound-state potential
  - crucial idea: use the dispersion relation for the nucleon self-energy
  - smart implementation: use it in its subtracted form
  - applied successfully to ^{40}Ca and ^{208}Pb in a limited energy window
  - employed traditional volume and surface absorption potentials and a local energy-dependent Hartree-Fock-like potential
  - Reviewed in Adv. Nucl. Phys. **20**, 1 (1991)
- Radiochemistry group at Washington University in St. Louis: Charity and Sobotka propose to use it for a sequence of Ca isotopes —> data-driven extrapolations to the drip line
  - First results 2006 PRL
  - Subsequently —> attention to data below the Fermi energy related to ground-state properties —> Dispersive Self-energy Method (**DSM**)

Optical potential <--> nucleon self-energy

- e.g. Bell and Squires --> elastic T-matrix = reducible self-energy
- Mahaux and Sartor Adv. Nucl. Phys. **20**, 1 (1991)
  - relate dynamic (energy-dependent) real part to imaginary part
  - employ subtracted dispersion relation

General dispersion relation for self-energy:

$$\mathrm{Re}\ \Sigma(E) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{E-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{E-E'}$$

Calculated at the Fermi energy $\varepsilon_F = \frac{1}{2}\left\{(E_0^{A+1} - E_0^A) + (E_0^A - E_0^{A-1})\right\}$

$$\mathrm{Re}\ \Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{\varepsilon_F-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{\varepsilon_F-E'}$$

Subtract

$$\mathrm{Re}\ \Sigma(E) = \mathrm{Re}\ \Sigma^{\widetilde{HF}}(\varepsilon_F)$$

$$- \quad \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')} + \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')}$$

Nonlocal DOM implementation PRL112,162503(2014)

- Particle number --> **nonlocal** imaginary part
- Microscopic FRPA & SRC --> different nonlocal properties above and below the Fermi energy
- Include charge density in fit
- Describe high-momentum nucleons <--> (e,e′p) data from JLab

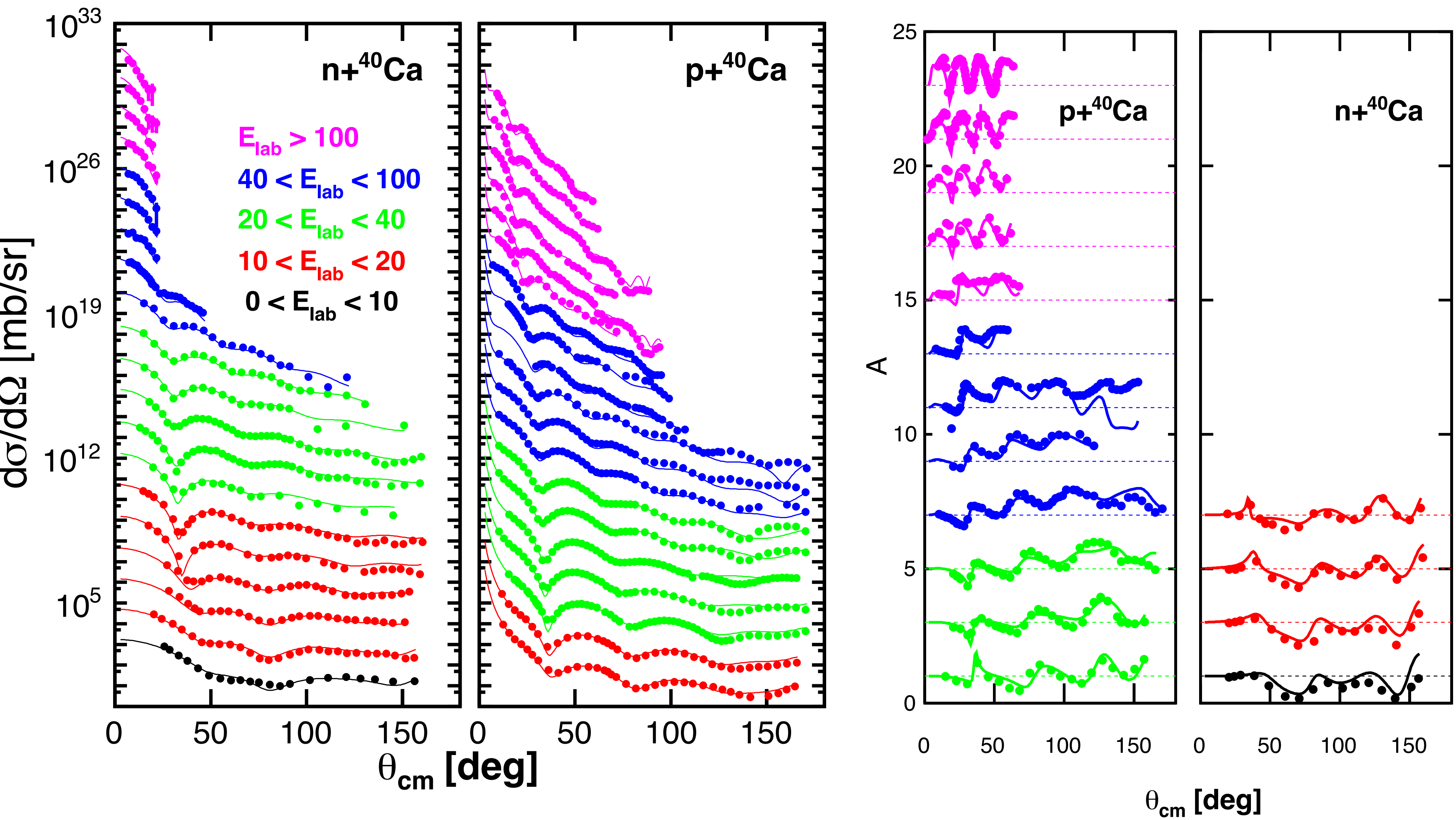
Implications

- Changes the description of hadronic reactions because interior nucleon wave functions depend on non-locality
- Consistency test of the interpretation of (e,e′p) possible
- Independent “experimental” statement on size of three-body contribution to the energy of the ground state--> two-body only:

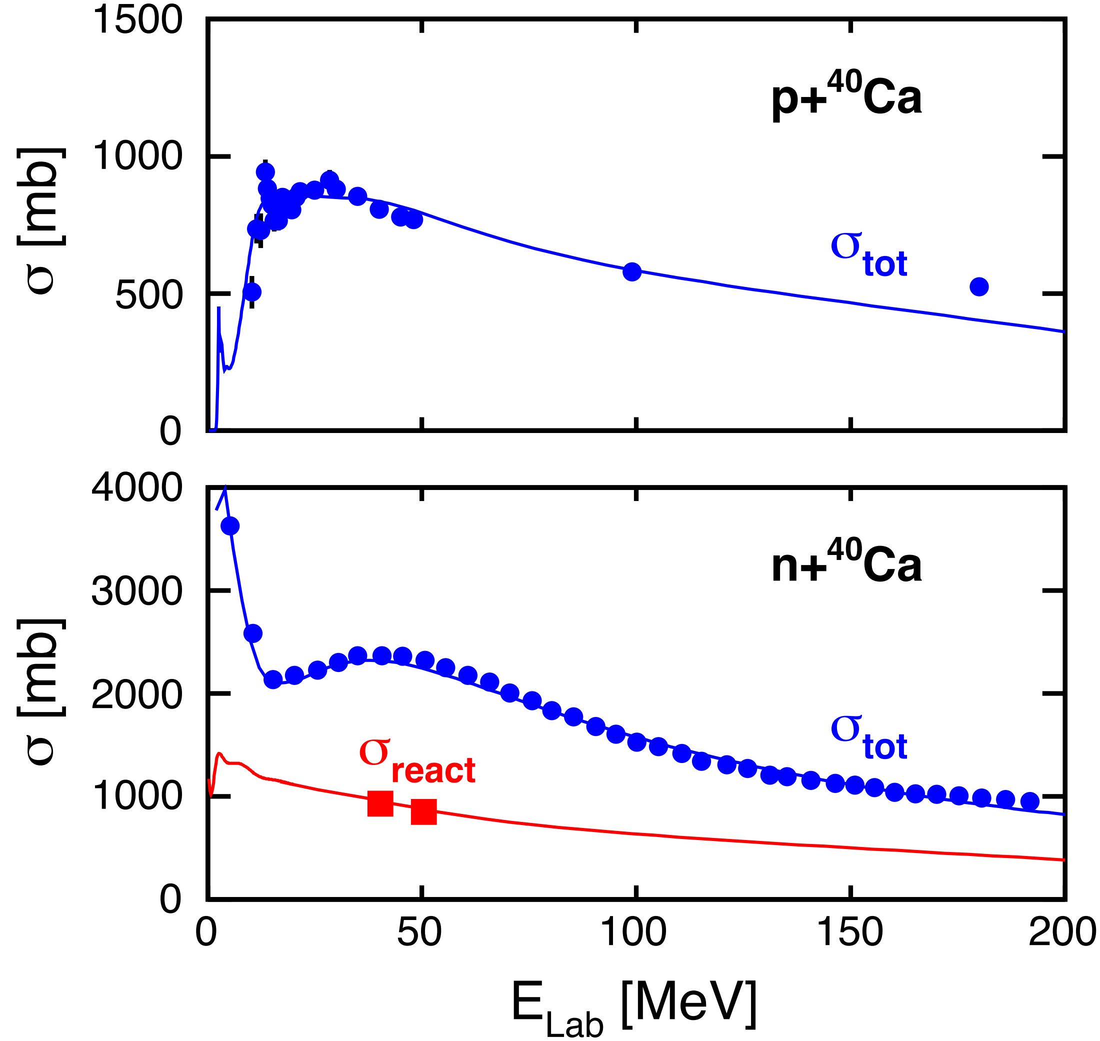
$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ E S_{\ell j}(k;E)$$

reactions and structure

Differential cross sections and analyzing powers



Reaction (p&n) and total (n) cross sections



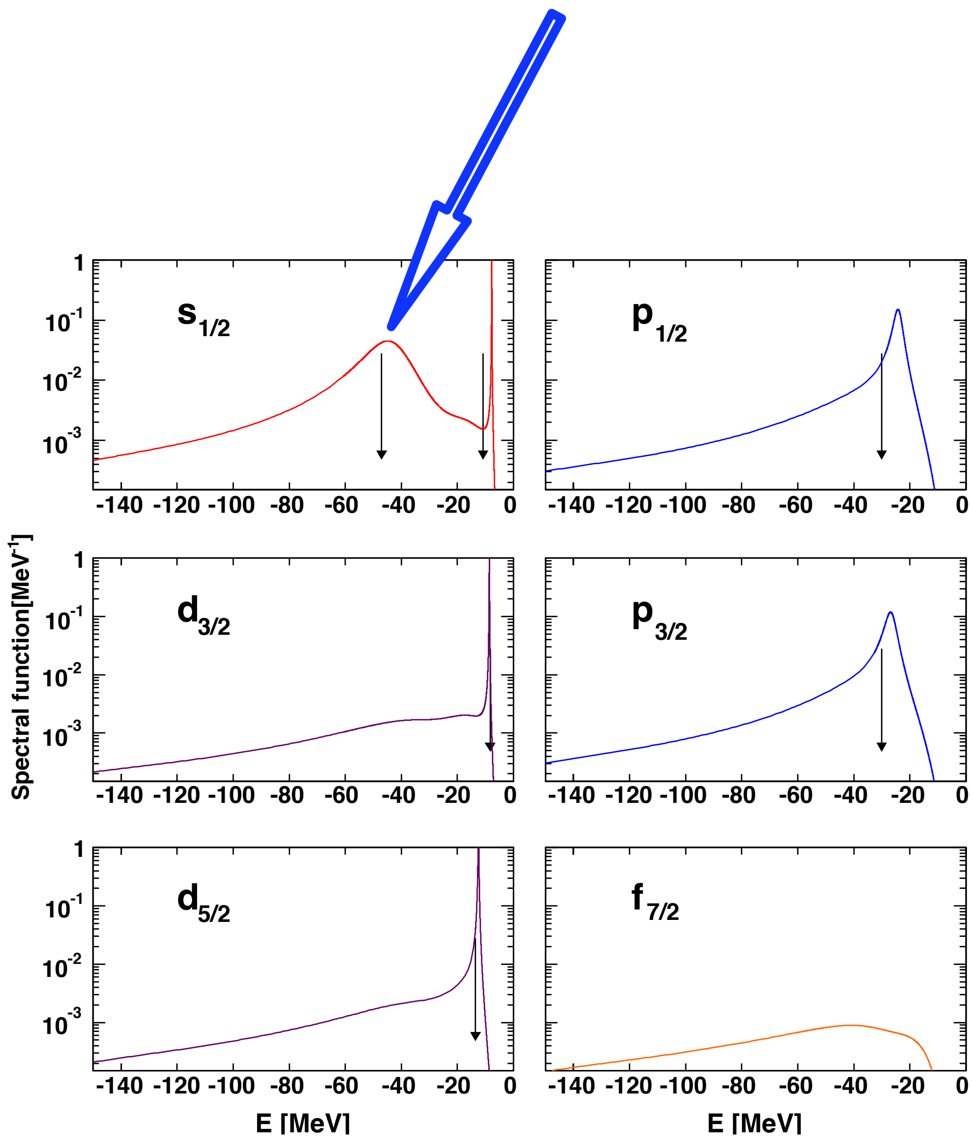
Below ε_F

^{40}Ca spectral function

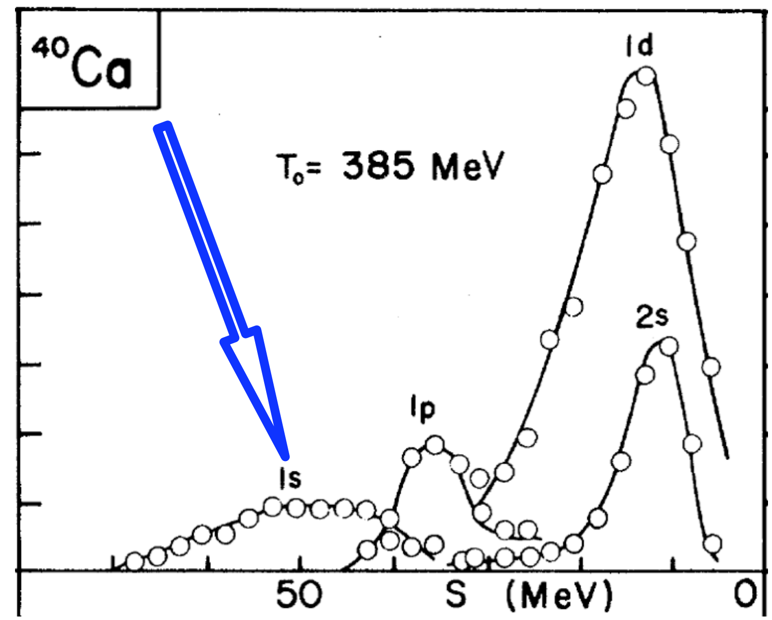
Nonlocal imaginary self-energy:
proton number --> 19.88
neutron number -> 19.79
$S_{0d3/2}$= 0.76
$S_{1s1/2}$ = 0.78
0.15 larger than NIKHEF analysis!

$\ell \leq 5$

Not part of fit!!

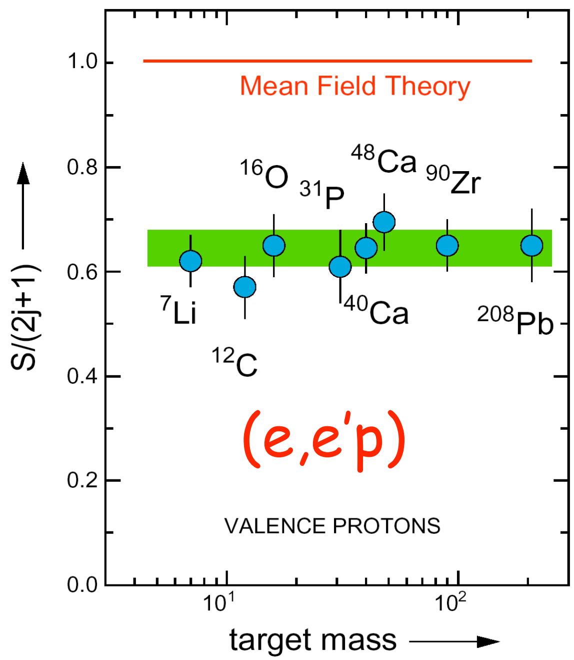


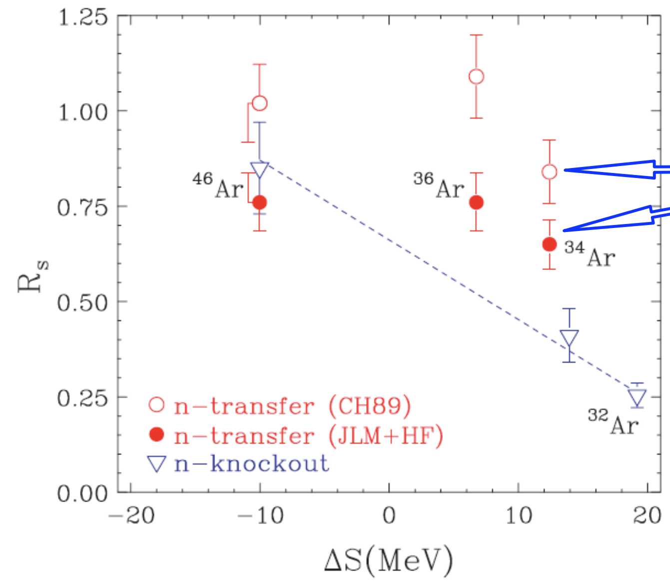
Old (p,2p) data from Liverpool
or (e,e'p) from Saclay



Linking nuclear reactions and nuclear structure —> DOM

Correlations from nuclear reactions



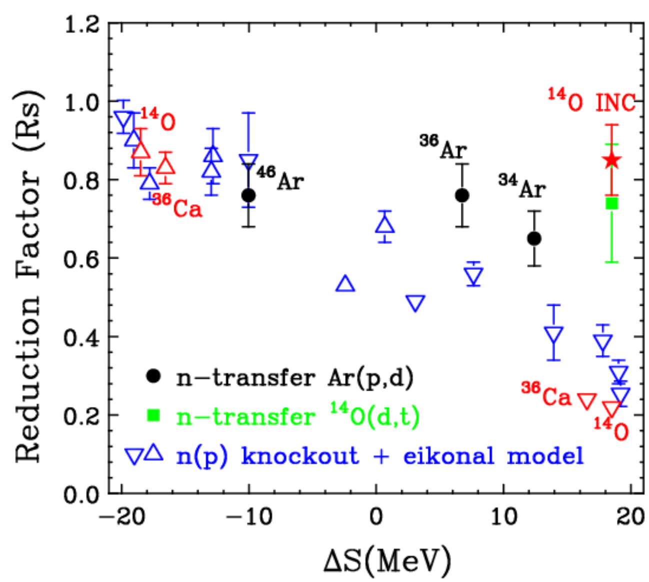


Different optical potentials --> different reduction factors for transfer reactions

Spectroscopic factors > 1 ???

PRL 93, 042501 (2004) HI

PRL 104, 112701 (2010) Transfer



In (e,e'p) proton still has to get out of the nucleus —> optical potential

Nucl. Phys. A553,297c (1993)

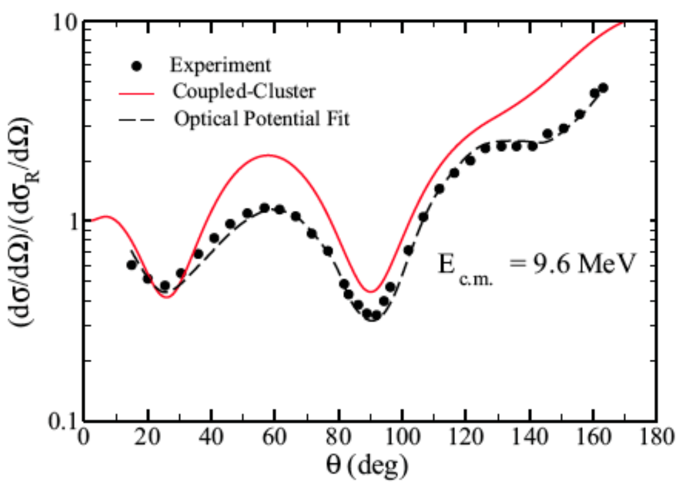
Consistency study in progress

Recent summary —> Jenny Lee

Different reactions different results???

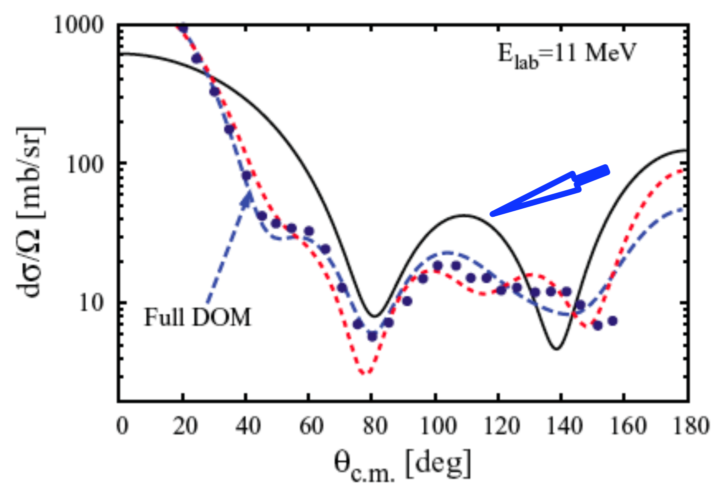
Linking nuclear reactions and nuclear structure

- Extracting information on correlations beyond the independent particle model requires optical potentials in (e,e'p), (d,p),(p,d),(p,pN), etc.
- Quality of ab initio to describe elastic scattering or optical potentials should be improved substantially and urgently



Coupled cluster calculation using overlap functions
PRC86,021602(R)(2012)
Probably limited to low energy

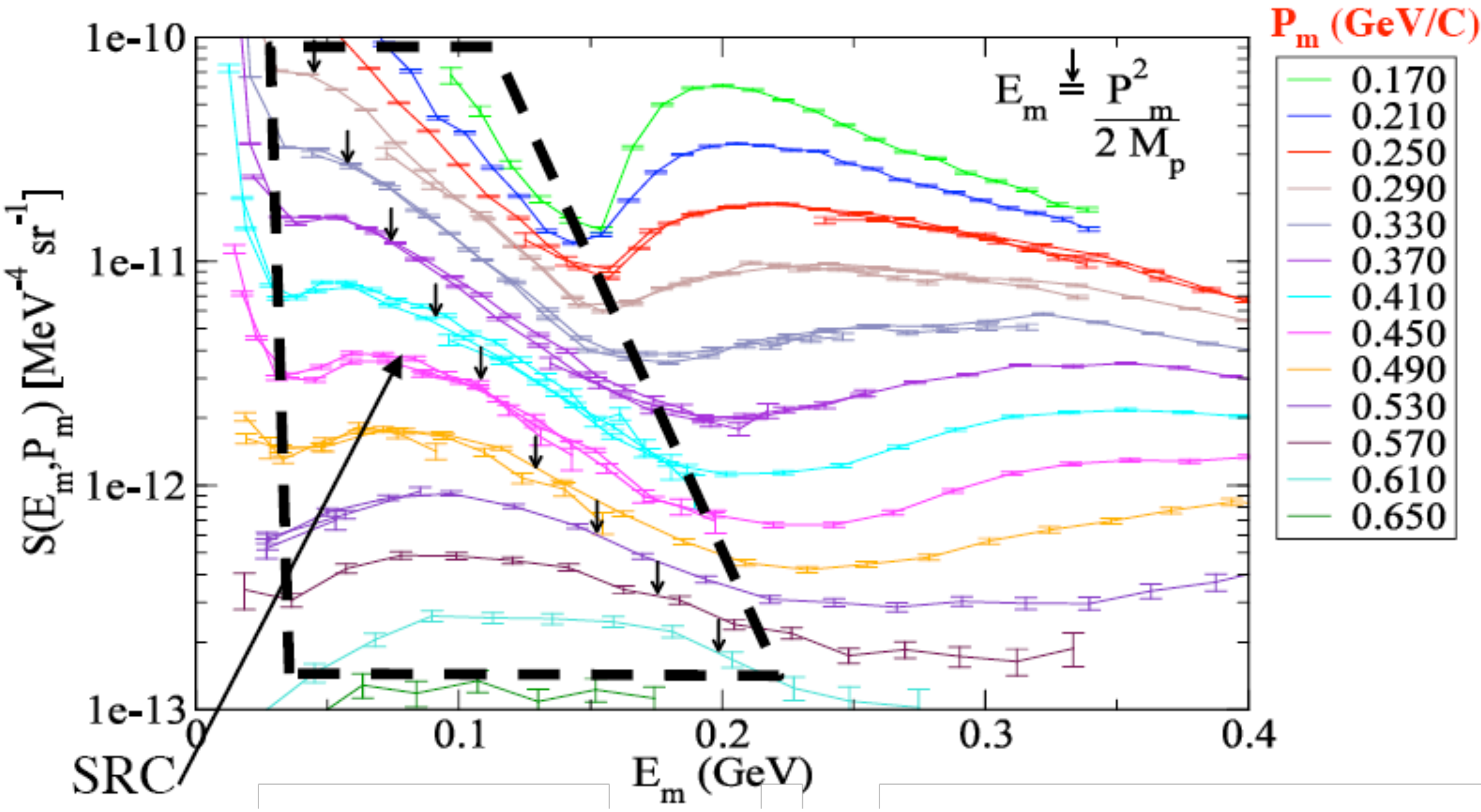
^{40}Ca



Green's function result —> optical potential with emphasis on SRC only
PRC84,044319(2011)

High-momentum protons have been seen in nuclei!

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



^{12}C

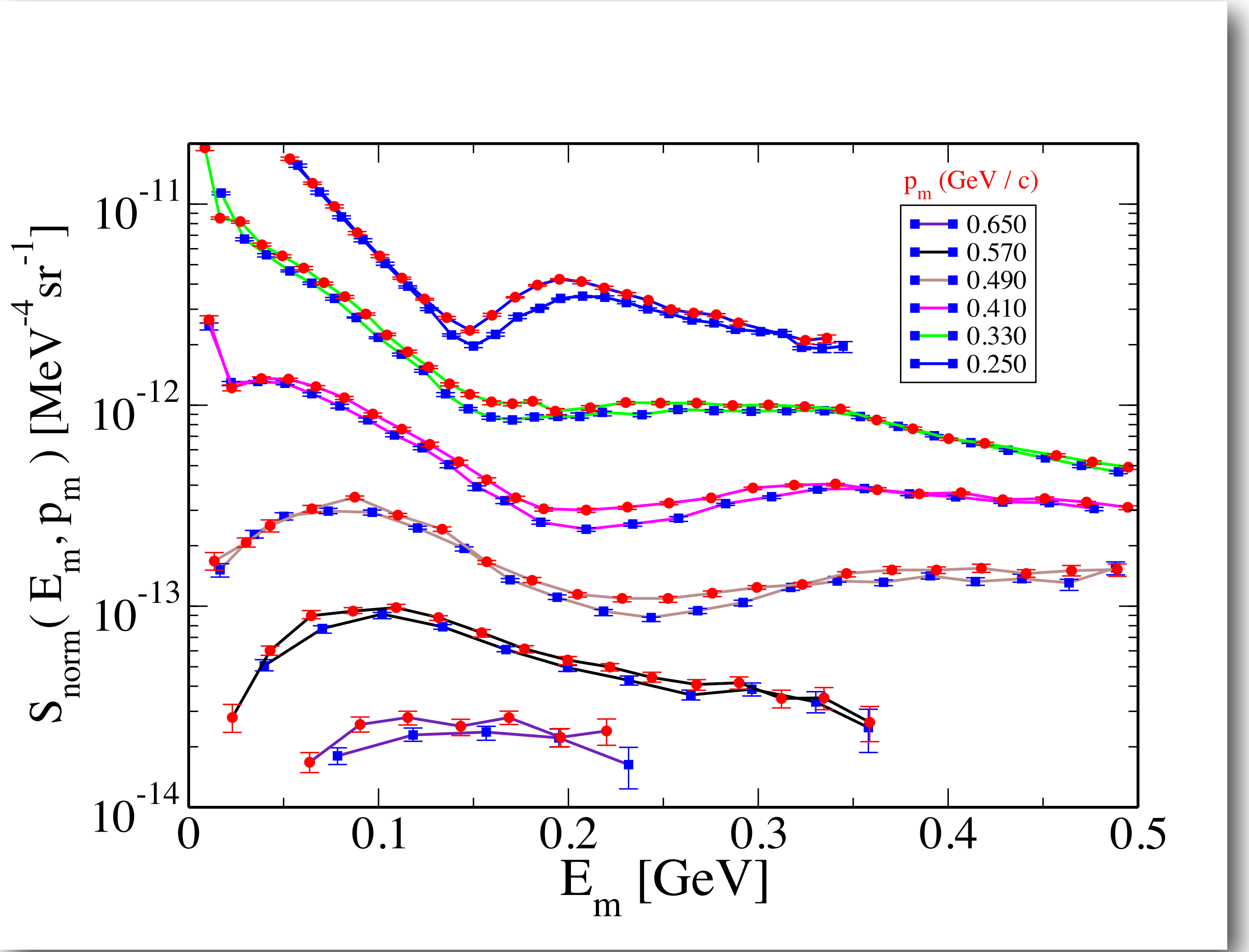
- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)

$\Rightarrow$ ~0.6 protons for ^{12}C $\Rightarrow$ **~10%**

reactions and structure

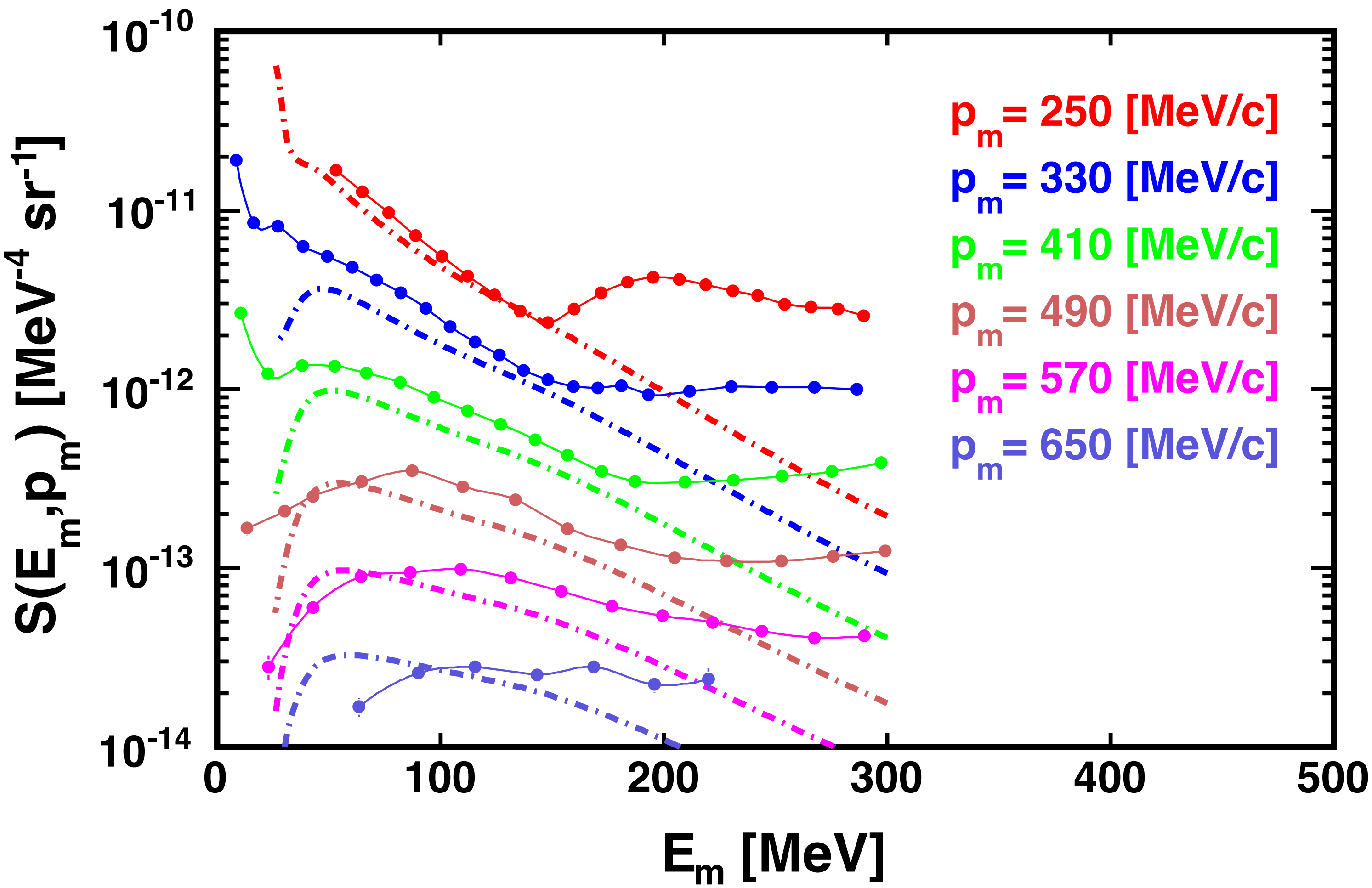
High-momentum components

Rohe, Sick et al. JLab data for Al and Fe (e,e'p) per proton



Jefferson Lab data per proton

- Pion/isobar contributions cannot be described
- Rescattering contributes some cross section (Barbieri, Lapikas)

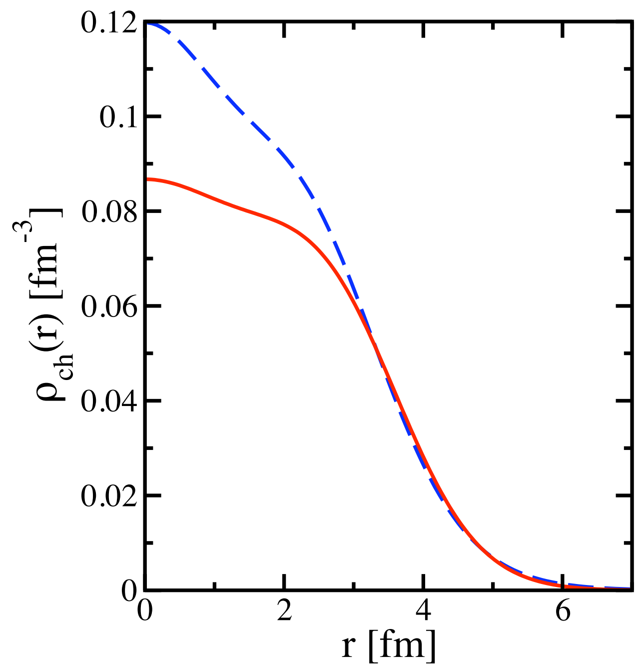


Critical experimental data

Local version

radius correct…

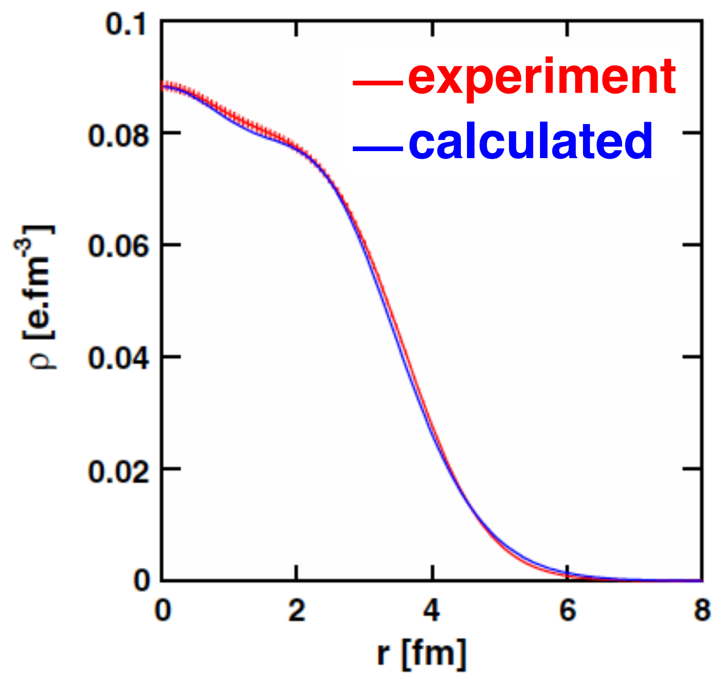
PRC82,054306(2010)



Charge density ^{40}Ca

Non-locality essential

PRL112,162503(2014)



High-momentum nucleons —> JLab can also be described —> E/A

Historical perspective...

- The following authors identify the single-particle propagator (or self-energy) as central quantities in many-body systems

Abrikosov, Gorkov, Dzyaloshinski
(Methods of Quantum Field Theory in Statistical Physics, 1963 Dover Revised edition 1975),

Pines
(The Many-body Problem, 1961 Addison Wesley reissued 1997),

Nozieres
(Theory of Interacting Fermi Systems, 1964 Addison-Wesley reissued 1997),

Thouless
(The Quantum Mechanics of Many-body Systems, 1972 Dover reissue of second edition, 2014),

Anderson
(Concepts in Solids, Benjamin 1963; World Scientific reissued 1998),

Schrieffer
(Theory of Superconductivity, 1964 Benjamin revised 1983),

Migdal
(Theory of Finite Fermi Systems and Applications to Atomic Nuclei (Interscience, 1967),

Fetter and Walecka
(Quantum Theory of Many-particle Systems, 1971 Dover reissued 2003)

- but apart from qualitative features, they don't answer what it looks like for a real system like a nucleus!

Energy of the ground state

- Energy sum rule (Migdal, Galitski & Koltun)

$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ E S_{\ell j}(k;E)$$

- **Not** part of fit because it can only make a statement about the two-body contribution
- Result:
  - DOM ---> 7.91 MeV/A T/A ---> 22.64 MeV/A
  - 10% of particles (momenta > 1.4 fm-1) provide ~⅔ of the binding energy!
  - Exp. 8.55 MeV/A
  - Three-body ---> 0.64 MeV/A "attraction" —> 1.28 MeV/A "repulsion"
  - Argonne GFMC ~ 1.5 MeV/A attraction for three-body <--> Av18

$$E_0^N = \langle \Psi_0^N | \hat{H} | \Psi_0^N \rangle$$

with three-body interaction

$$= \frac{1}{2\pi}\int_{-\infty}^{\varepsilon_F^-} dE \sum_{\alpha,\beta} \{\langle \alpha | T | \beta \rangle + E\ \delta_{\alpha,\beta}\}\ \mathrm{Im}\ G(\beta,\alpha;E) - \frac{1}{2}\langle \Psi_0^N | \hat{W} | \Psi_0^N \rangle$$

reactions and structure

Do elastic scattering data tell us about correlations?

- Scattering T-matrix

$$\Sigma_{\ell j}(k,k';E) = \Sigma^*_{\ell j}(k,k';E) + \int dq q^2 \Sigma^*_{\ell j}(k,q;E) G^{(0)}(q;E) \Sigma_{\ell j}(q,k';E)$$

- Free propagator $G^{(0)}(q;E) = \dfrac{1}{E - \hbar^2 q^2/2m + i\eta}$

- Propagator

$$G_{\ell j}(k,k';E) = \frac{\delta(k-k')}{k^2} G^{(0)}(k;E) + G^{(0)}(k;E)\Sigma_{\ell j}(k,k';E)G^{(0)}(k;E)$$

- Spectral representation

$$G^p_{\ell j}(k,k';E) = \sum_n \frac{\phi^{n+}_{\ell j}(k)\left[\phi^{n+}_{\ell j}(k')\right]^*}{E - E^{*A+1}_n + i\eta} + \sum_c \int_{T_c}^{\infty} dE' \, \frac{\chi^{cE'}_{\ell j}(k)\left[\chi^{cE'}_{\ell j}(k')\right]^*}{E - E' + i\eta}$$

- Spectral density at positive energy

$$S^p_{\ell j}(k,k';E) = \frac{i}{2\pi}\left[G^p_{\ell j}(k,k';E^+) - G^p_{\ell j}(k,k';E^-)\right] = \sum_c \chi^{cE}_{\ell j}(k)\left[\chi^{cE}_{\ell j}(k')\right]^*$$

- Coordinate space $S^p_{\ell j}(r,r';E) = \sum_c \chi^{cE}_{\ell j}(r)\left[\chi^{cE}_{\ell j}(r')\right]^*$

- Elastic scattering explicit

$$\chi^{elE}_{\ell j}(r) = \left[\frac{2mk_0}{\pi\hbar^2}\right]^{1/2}\left\{ j_\ell(k_0 r) + \int dk k^2 j_\ell(kr) G^{(0)}(k;E)\Sigma_{\ell j}(k,k_0;E)\right\}$$

How is it done?

- Solve

$$\Sigma_{\ell j}(k,k';E) = \Sigma^*_{\ell j}(k,k';E) + \int dq q^2 \Sigma^*_{\ell j}(k,q;E) G^{(0)}(q;E) \Sigma_{\ell j}(q,k';E)$$

- with $G^{(0)}(q;E) = \dfrac{1}{E - \hbar^2 q^2/2m + i\eta}$

- See discussion by Arturo Polls
- Note: irreducible self-energy is a complex quantity
- Cross section as shown in first lecture

Elastic nucleon scattering

- Scattering from potential $\langle k_0|\mathcal{S}^\ell(E)|k_0\rangle = \left[1 - 2\pi i \left(\frac{mk_0}{\hbar^2}\right)\langle k_0|\mathcal{T}^\ell(E)|k_0\rangle\right] \equiv e^{2i\delta_\ell}$
- Potential real —> phase shift real
- Scattering amplitude

$$\begin{aligned} f(\theta,\phi) &= \sum_l \frac{2l+1}{k_0}\left\{\frac{-mk_0\pi}{\hbar^2}\right\}\langle k_0|\mathcal{T}^\ell(E)|k_0\rangle P_\ell(\cos\theta) \\ &= \sum_\ell \frac{2\ell+1}{k_0} e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta) \end{aligned}$$

- Elastic nucleon scattering
  - Involves reducible self-energy (see also later)

$$\langle k_0|\,\mathcal{S}_{\ell j}(E)\,|k_0\rangle \equiv e^{2i\delta_{\ell j}} = 1 - 2\pi i\left(\frac{mk_0}{\hbar^2}\right)\langle k_0|\,\Sigma_{\ell j}(E)\,|k_0\rangle$$

  - Scattering amplitude

$$f_{m_s',m_s}(\theta,\phi) = -\frac{4m\pi^2}{\hbar^2}\langle \boldsymbol{k}' m_s'|\Sigma(E)|\boldsymbol{k} m_s\rangle$$

  - Phase shift now includes imaginary part when potential is absorptive

Spin-orbit physics included

- Scattering amplitude $$f_{m_s',m_s}(\theta,\phi) = -\frac{4m\pi^2}{\hbar^2}\langle \boldsymbol{k}' m_s' | \Sigma(E) | \boldsymbol{k} m_s\rangle$$
- Rewrite $$[f(\theta,\phi)] = \mathcal{F}(\theta) I + \boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}\,\mathcal{G}(\theta)$$
  - with $$\hat{\boldsymbol{n}} = \frac{\boldsymbol{k}\times\boldsymbol{k}'}{|\boldsymbol{k}\times\boldsymbol{k}'|} = \frac{\hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}'}{\sin\theta}$$
  - then

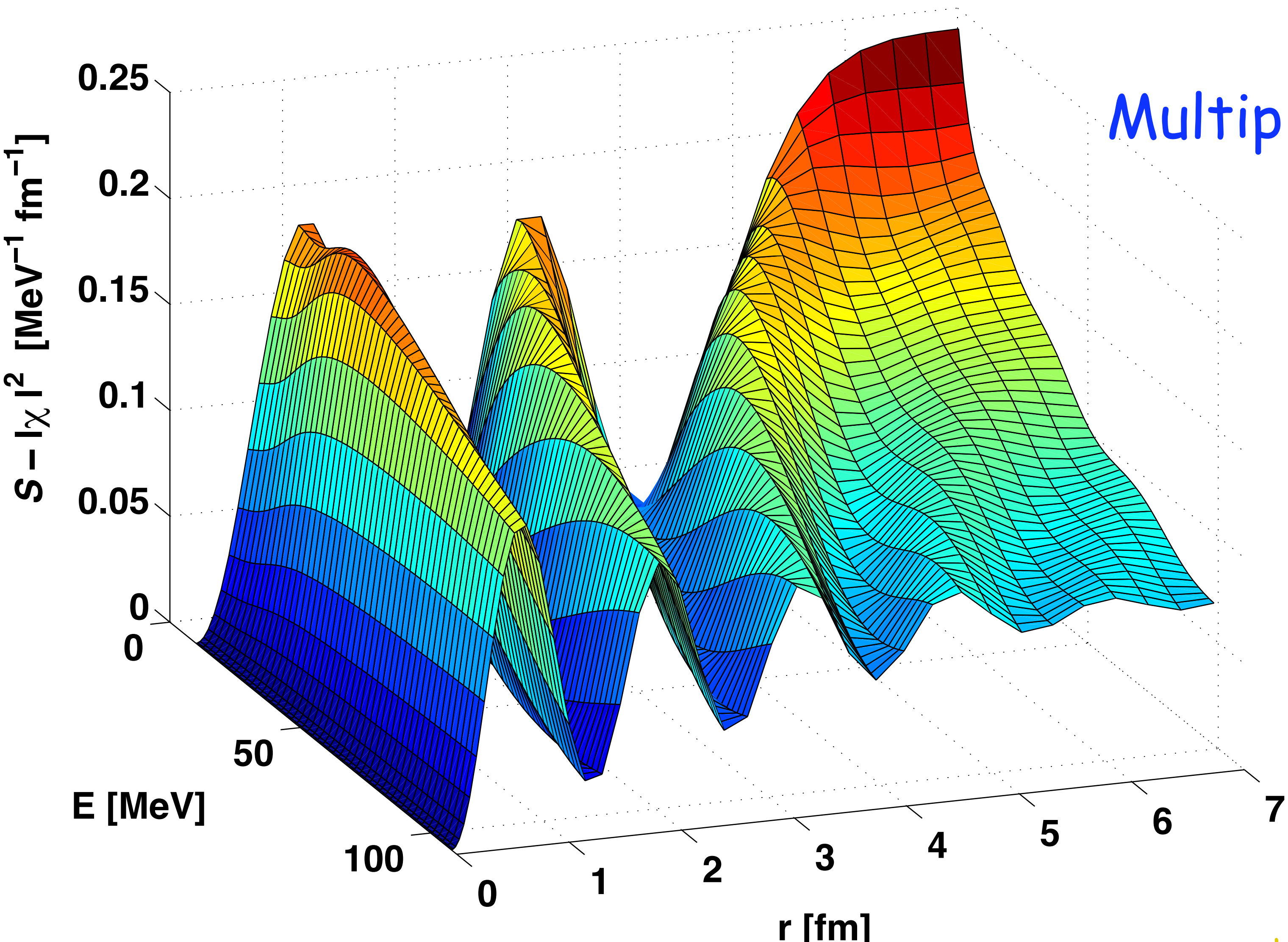
$$\mathcal{F}(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}\left[(\ell+1)\left\{e^{2i\delta_{\ell+}}-1\right\} + \ell\left\{e^{2i\delta_{\ell-}}-1\right\}\right] P_\ell(\cos\theta)$$

$$\mathcal{G}(\theta) = \frac{\sin\theta}{2k}\sum_{\ell=1}^{\infty}\left[e^{2i\delta_{\ell+}} - e^{2i\delta_{\ell-}}\right] P_\ell'(\cos\theta)$$

  - Unpolarized differential cross section $$\left(\frac{d\sigma}{d\Omega}\right)_{unpol} = |\mathcal{F}|^2 + |\mathcal{G}|^2$$

Adding an $s_{1/2}$ neutron to ^{40}Ca

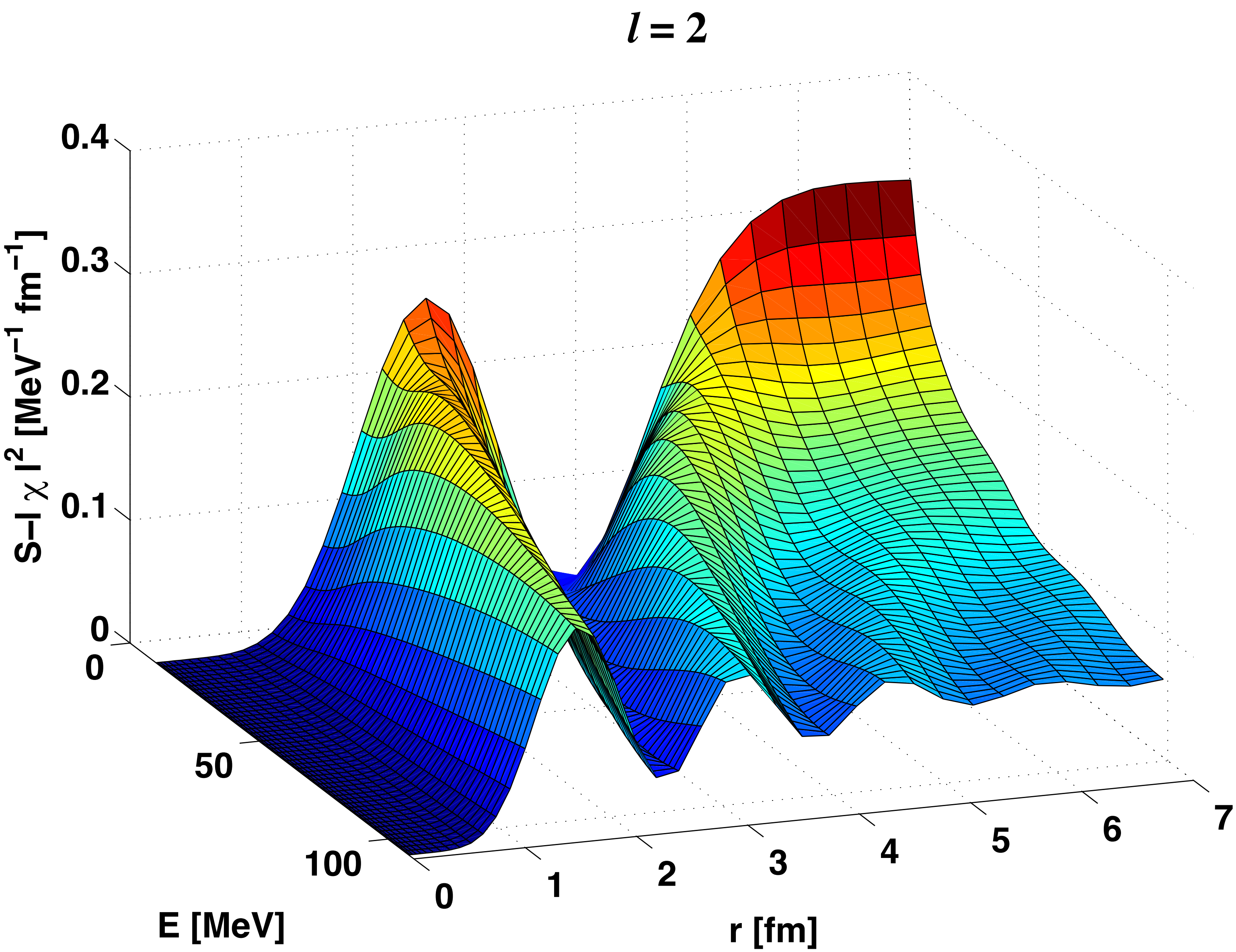
- Inelastically!
- Zero when there is no absorption!



Multiplied by r^2

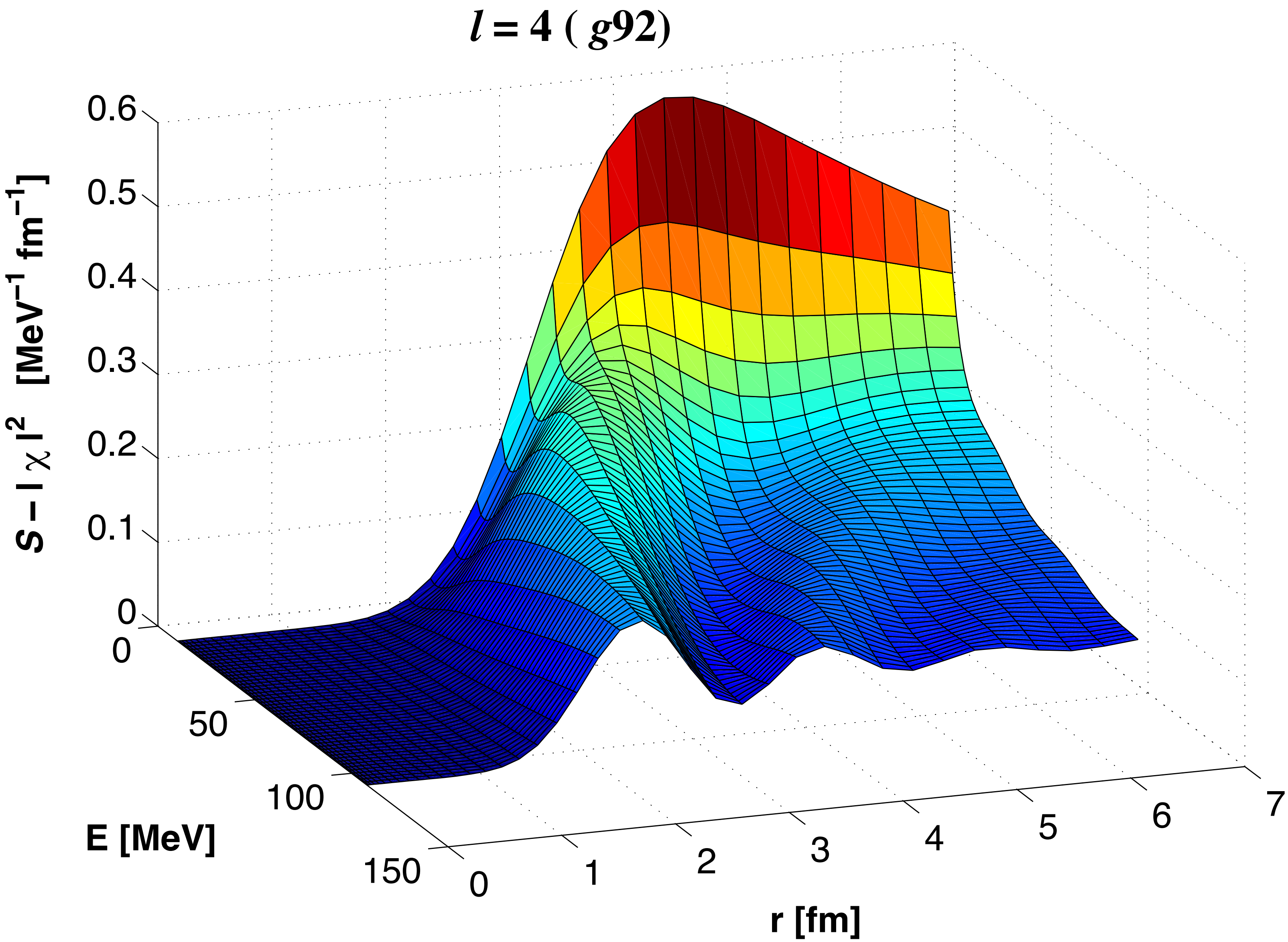
$d_{3/2}$

- One node now



No nodes

- Asymptotically determined by inelasticity



Determine location of bound-state strength

- Fold spectral function with bound state wave function

$$S_{\ell j}^{n+}(E) = \int dr\ r^2 \int dr'\ r'^2 \phi_{\ell j}^{n-}(r) S_{\ell j}^{p}(r, r'; E) \phi_{\ell j}^{n-}(r')$$

- —> Addition probability of bound orbit
- Also removal probability

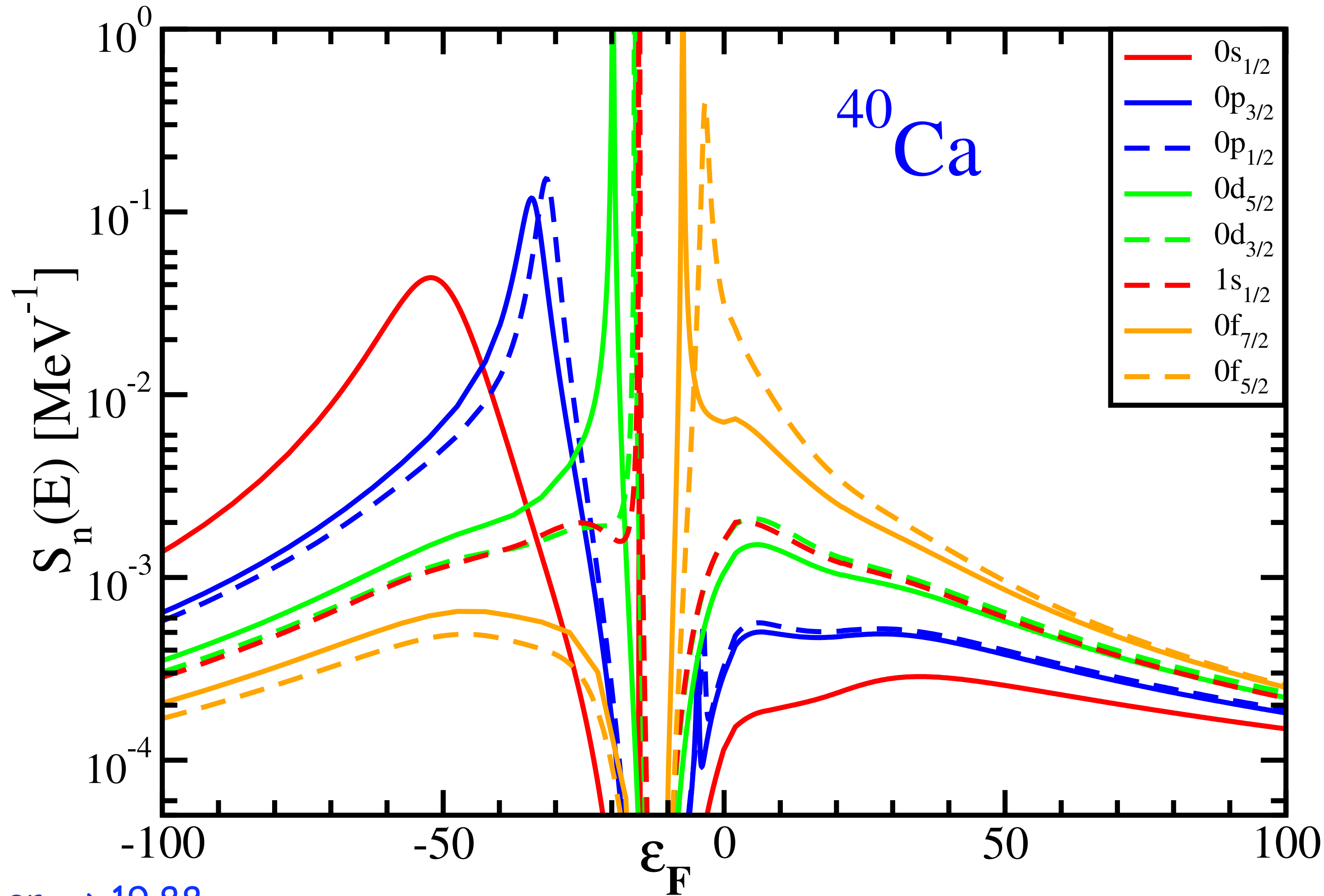
$$S_{\ell j}^{n-}(E) = \int dr r^2 \int dr' r'^2 \phi_{\ell j}^{n-}(r) S_{\ell j}^{h}(r, r'; E) \phi_{\ell j}^{n-}(r')$$

- Overlap function $\sqrt{S_{\ell j}^{n}}\phi_{\ell j}^{n-}(r) = \langle \Psi_n^{A-1} | \, a_{r\ell j} \, | \Psi_0^A \rangle$

- Sum rule $1 = n_{n\ell j} + d_{n\ell j} = \int_{-\infty}^{\varepsilon_F} dE\ S_{\ell j}^{n-}(E) + \int_{\varepsilon_F}^{\infty} dE\ S_{\ell j}^{n-}(E)$

Spectral function for bound states

- [0,200] MeV —> constrained by elastic scattering data



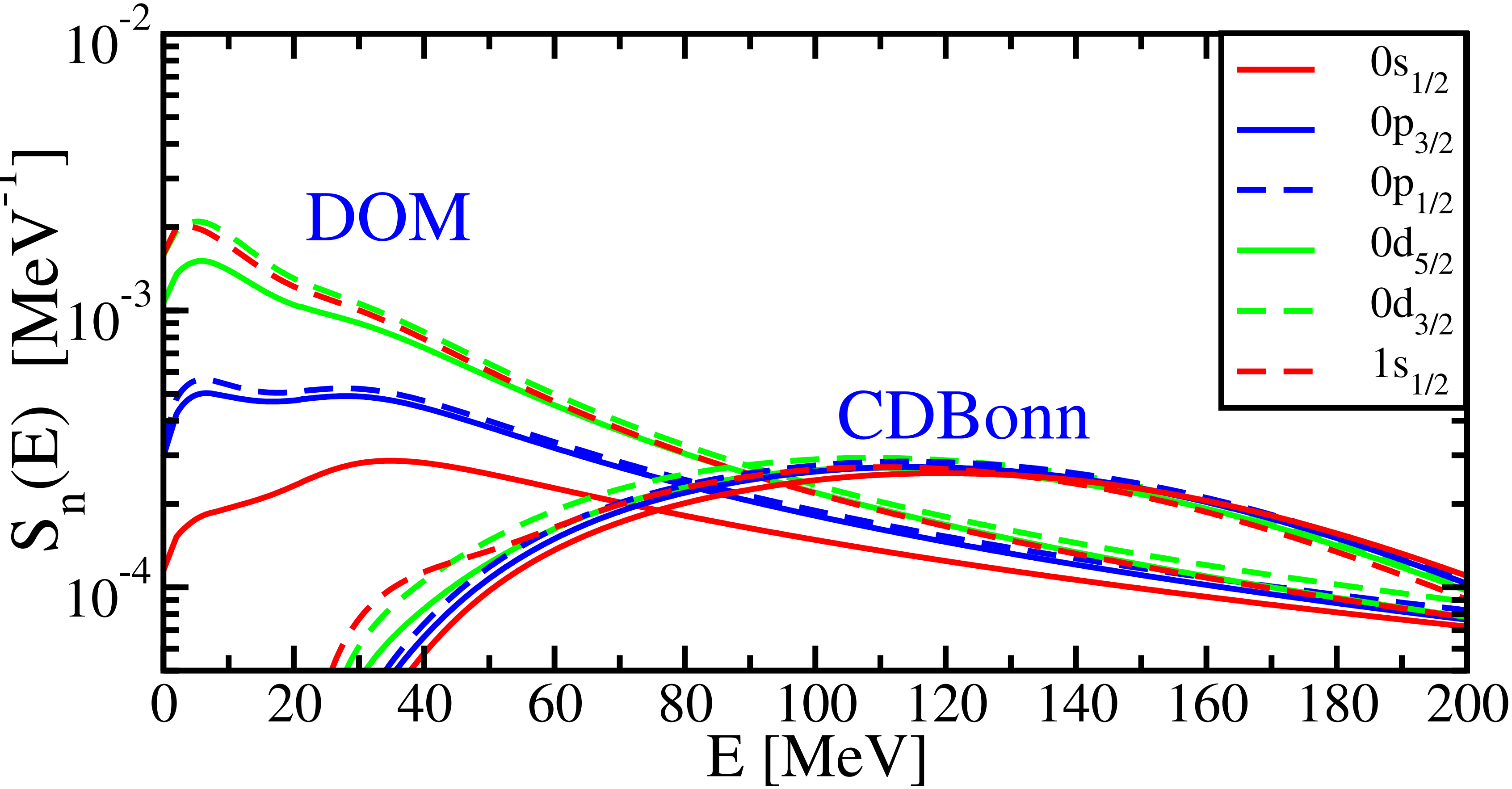
proton number --> 19.88
neutron number -> 19.79
$S_{0d3/2}$= 0.76
$S_{1s1/2}$ = 0.78
0.15 larger than NIKHEF analysis!

PRC90, 061603(R) (2014)

reactions and structure

Compared with ab initio —> SRC only

- CDBonn probably too soft
- SRC relevant at higher energy



PRC90, 061603(R) (2014)

Quantitatively

- Orbit closer to the continuum —> more strength in the continuum
- Note "particle" orbits
- Drip-line nuclei have valence orbits very near the continuum

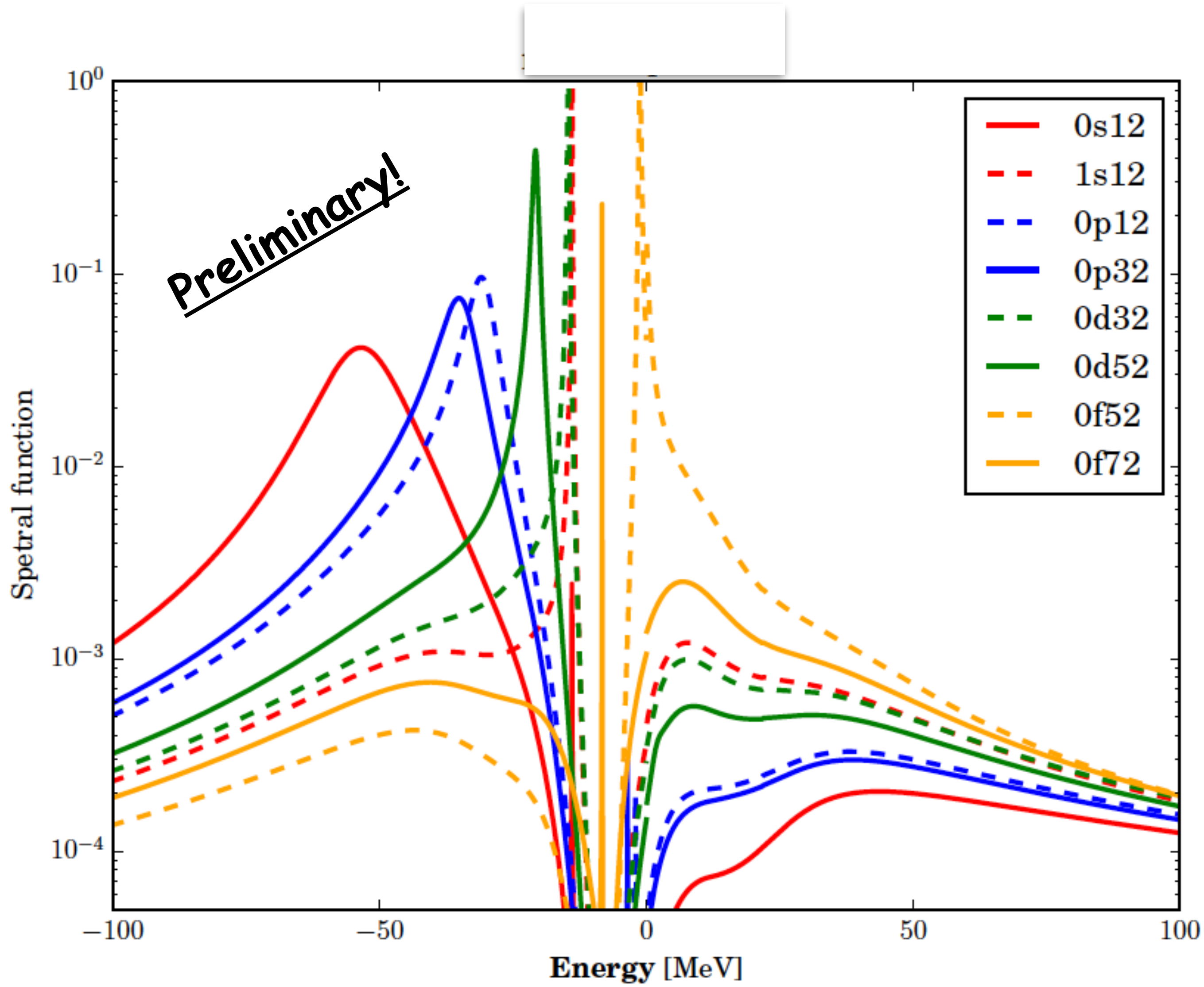
Table 1: Occupation and depletion numbers for bound orbits in ^{40}Ca. $d_{nlj}[0,200]$ depletion numbers have been integrated from 0 to 200 MeV. The fraction of the sum rule that is exhausted, is illustrated by $n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$. Last column $d_{nlj}[0,200]$ depletion numbers for the CDBonn calculation.

orbit	$n_{n\ell j}$ DOM	$d_{n\ell j}[0,200]$ DOM	$n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$ DOM	$d_{n\ell j}[0,200]$ CDBonn
$0s_{1/2}$	0.926	0.032	0.958	0.035
$0p_{3/2}$	0.914	0.047	0.961	0.036
$1p_{1/2}$	0.906	0.051	0.957	0.038
$0d_{5/2}$	0.883	0.081	0.964	0.040
$1s_{1/2}$	0.871	0.091	0.962	0.038
$0d_{3/2}$	0.859	0.097	0.966	0.041
$0f_{7/2}$	0.046	0.202	0.970	0.034
$0f_{5/2}$	0.036	0.320	0.947	0.036

PRC90, 061603(R) (2014)

Neutron spectral function in ^{48}Ca

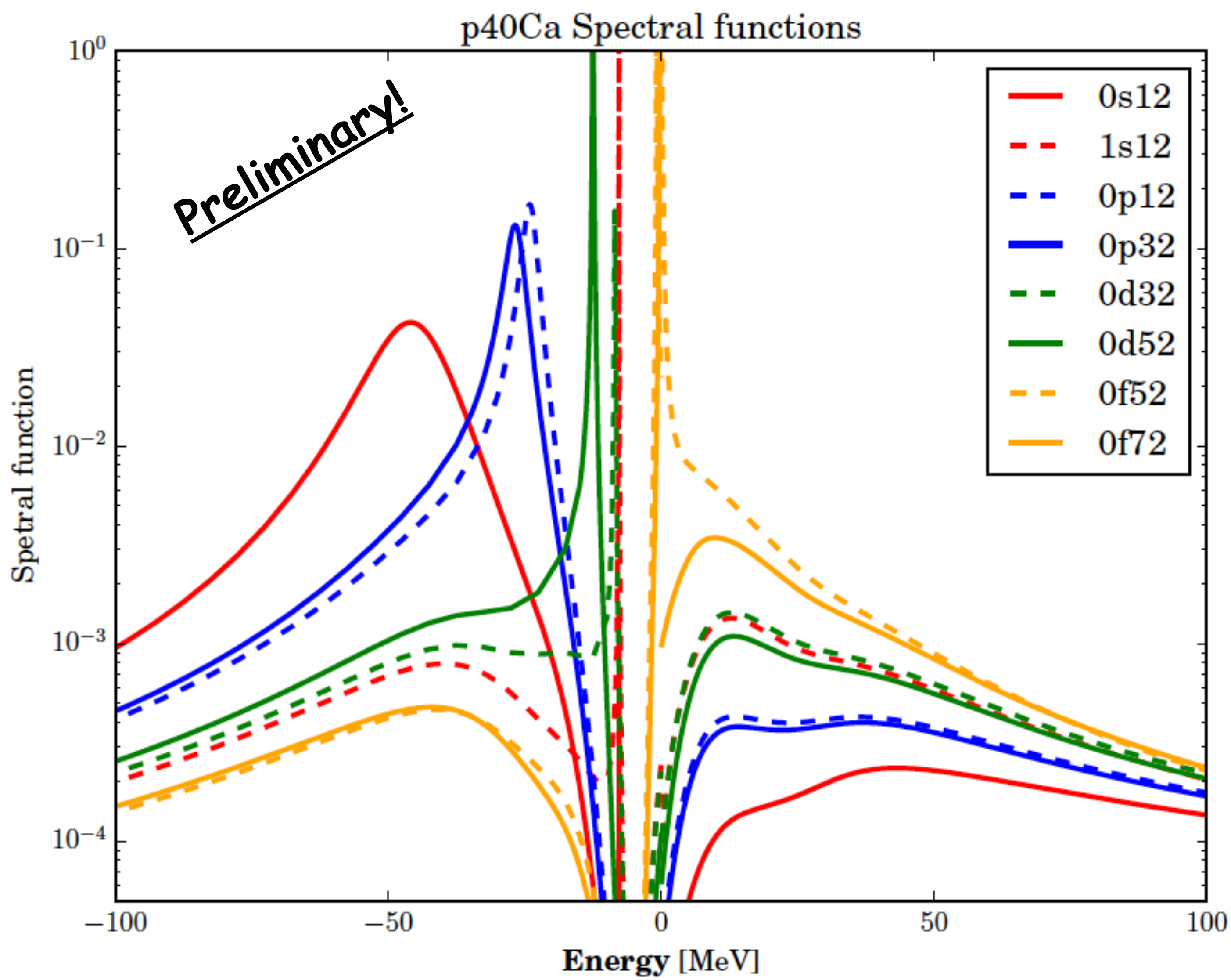
- Neutrons in ^{48}Ca less correlated <-> ^{40}Ca but qualitatively similar



reactions and structure

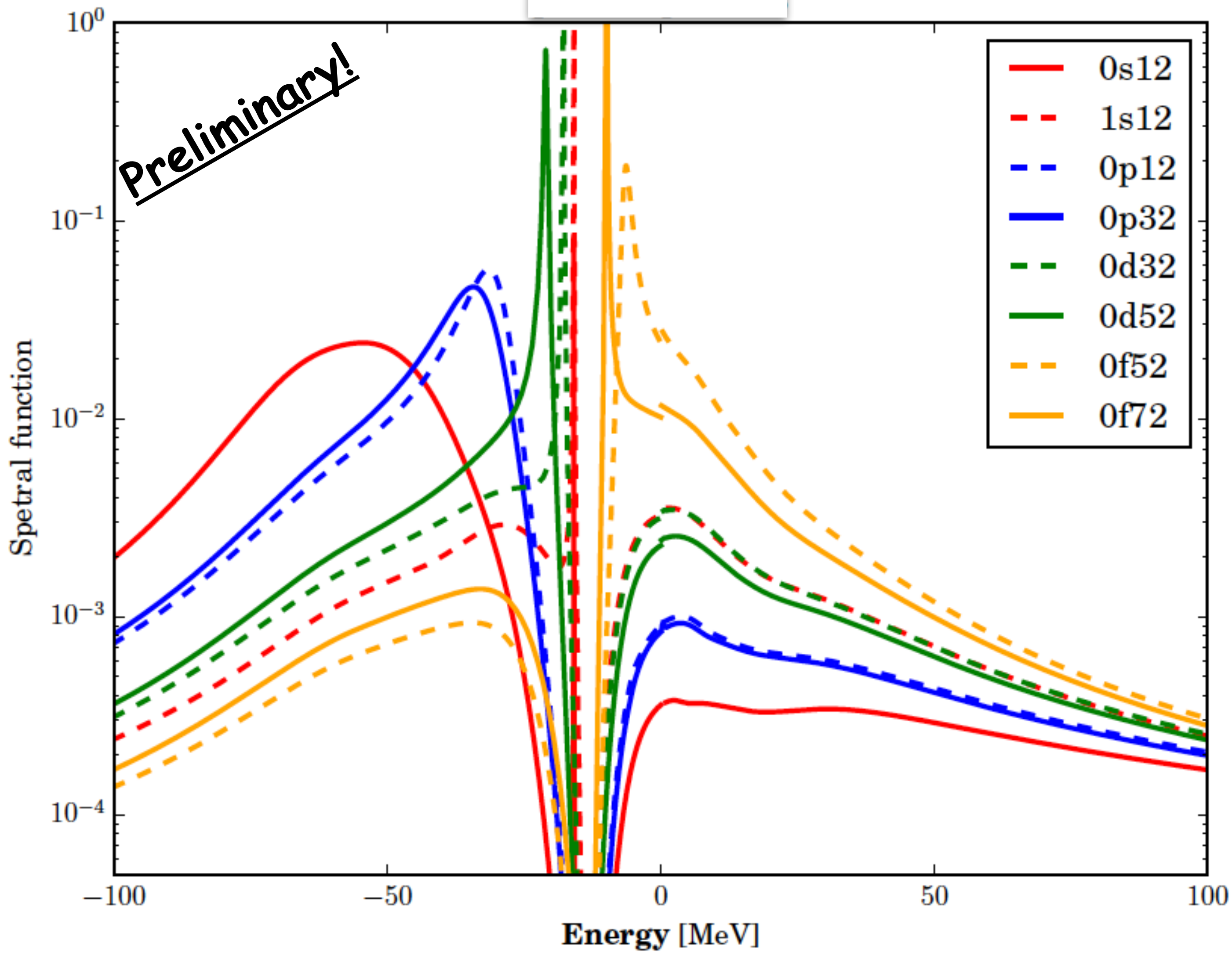
Proton spectral function in ^{40}Ca

- Learning how to deal with Coulomb in momentum space



Protons in ^{48}Ca

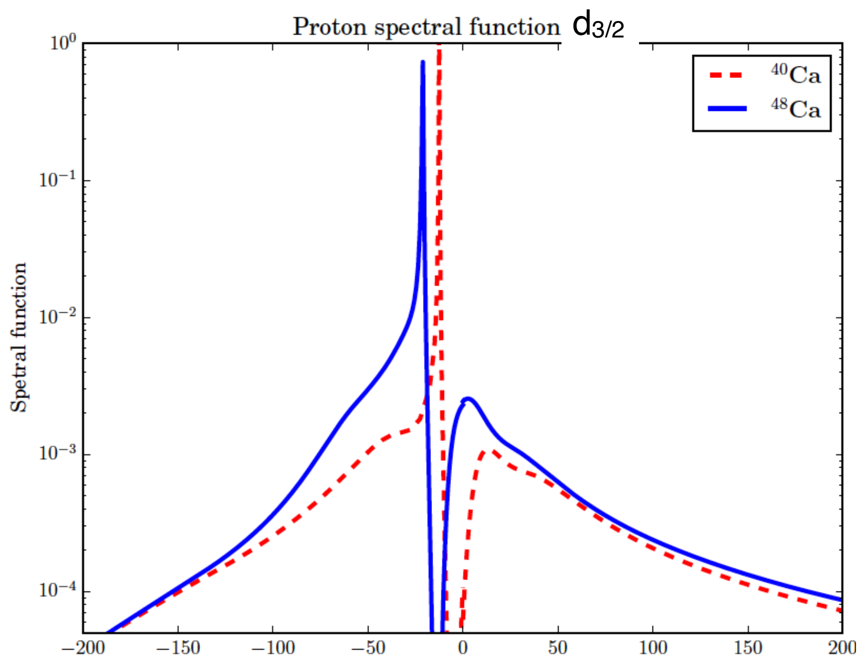
- Protons in ^{48}Ca more correlated than in ^{40}Ca

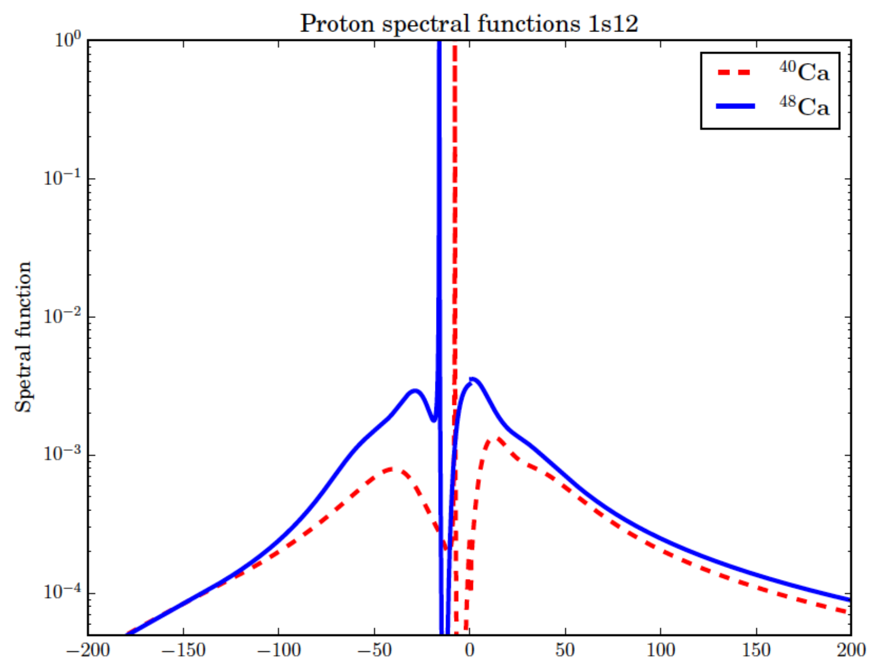


Quantitative comparison of ^{40}Ca and ^{48}Ca

Spectroscopic factors	^{40}Ca	p ^{48}Ca	n ^{48}Ca
$0d_{3/2}$	0.76	0.65 ↓	0.80 ↑
$1s_{1/2}$	0.78	0.71 ↓	0.83 ↑
$0f_{7/2}$	0.73	0.59 ↓	0.84 ↑

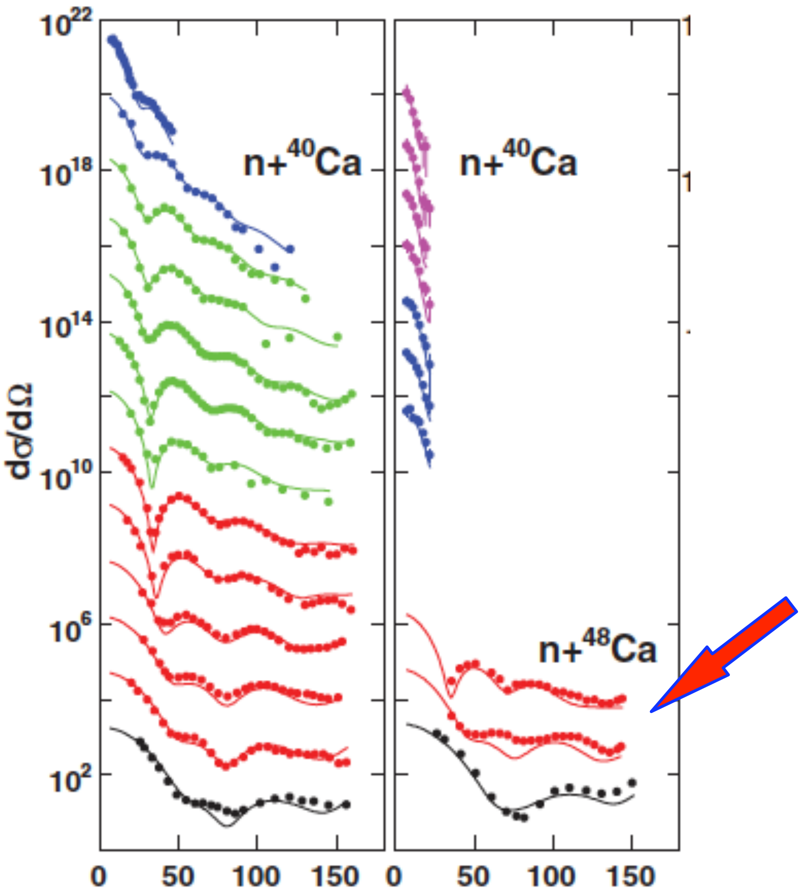
Comparison for $d_{3/2}$ and $s_{1/2}$ protons

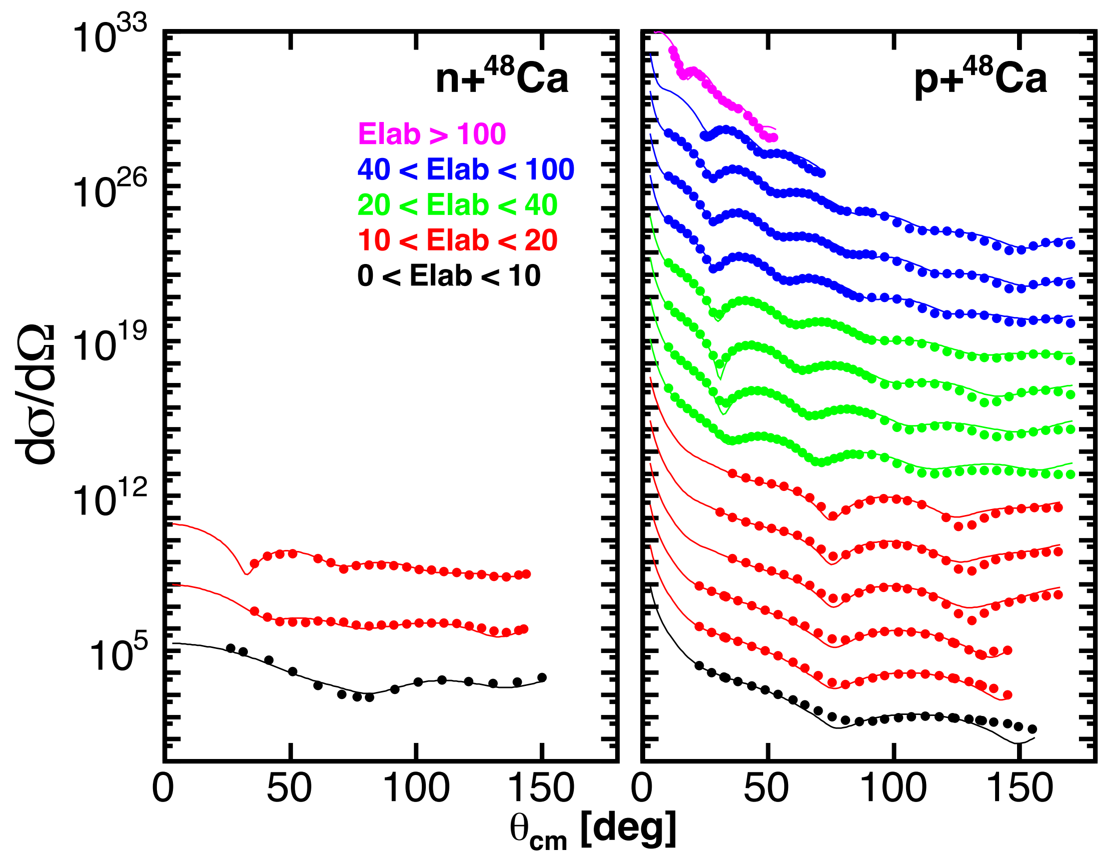




In progress

- ^{48}Ca —> charge density has been measured
- Recent neutron elastic scattering **data** —> PRC83,064605(2011)
- Local DOM **OLD** Nonlocal DOM **NEW**

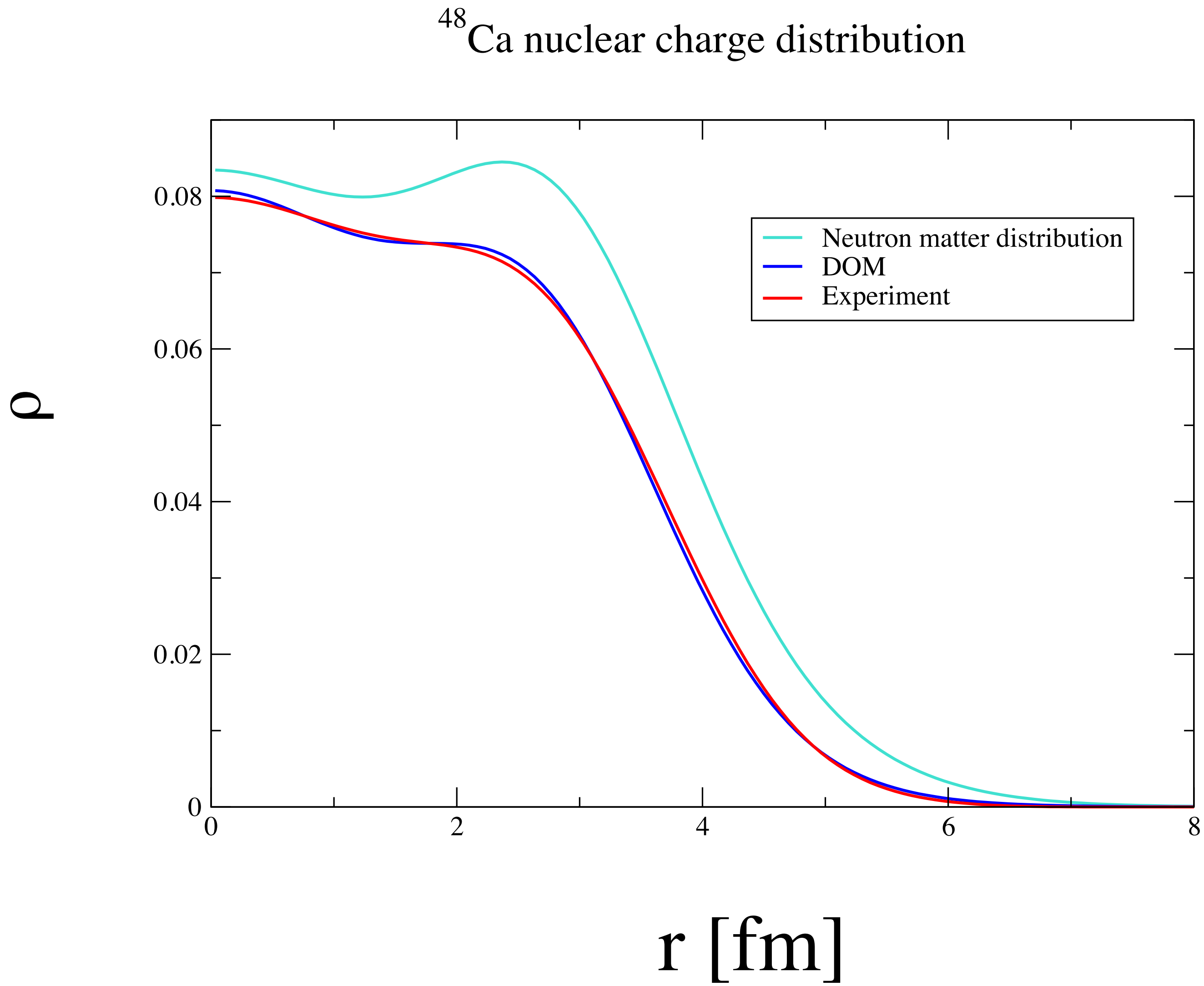




reactions and structure

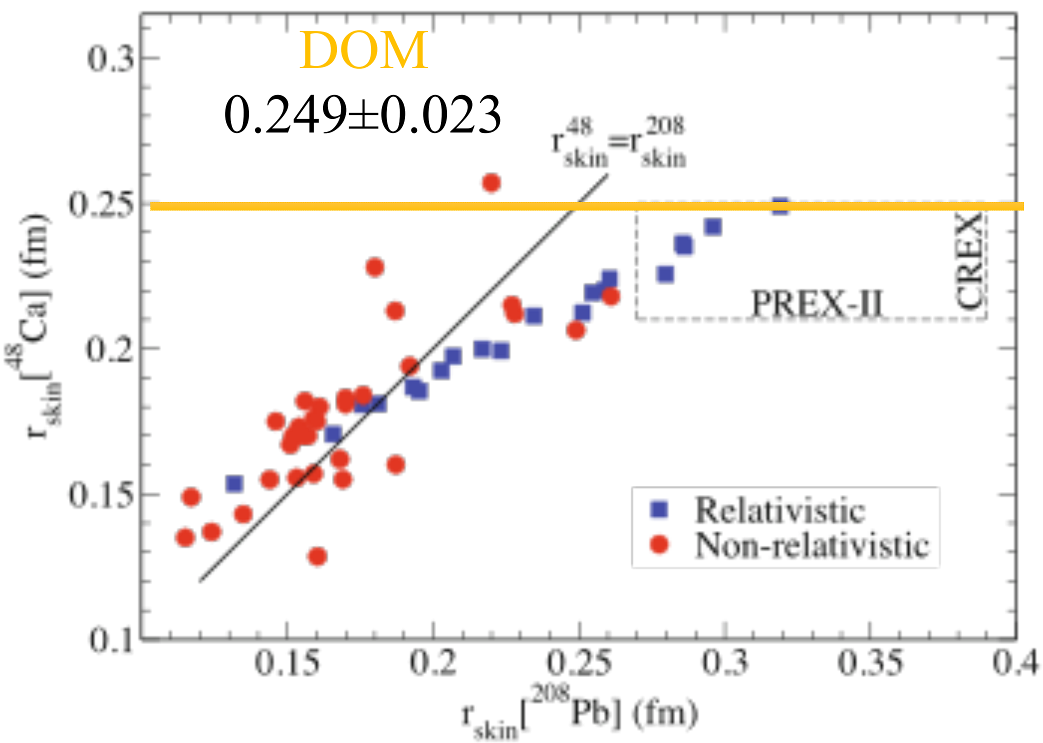
Results for ^{48}Ca

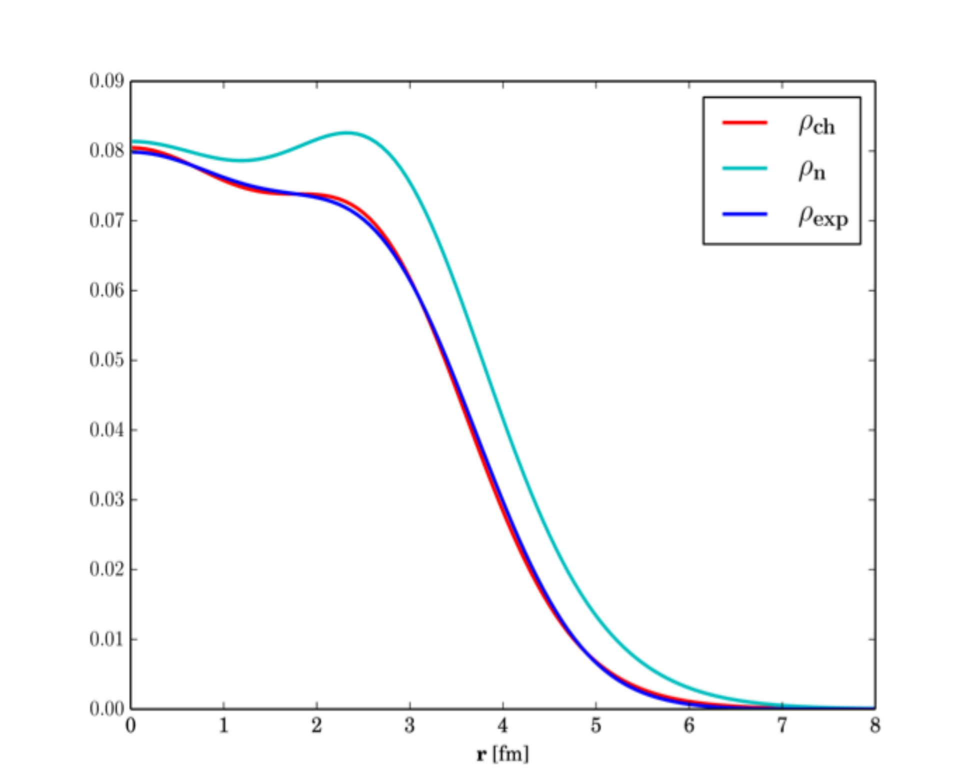
- Density distributions
- DOM —> neutron distribution —> R_n-R_p



reactions and structure

^{48}Ca Densities





Eur. Phys. J. A (2014)
C.J. Horowitz, K.S. Kumar, and R. Michaels

R_n-R_p for ^{48}Ca

- Charge density for ^{40}Ca ✔
- Charge density for ^{48}Ca ✔
- Neutrons in ^{40}Ca well constrained ✔
- 8 extra neutrons in ^{48}Ca constrained by new elastic scattering data at low energy and total cross sections up to 200 MeV, level structure, and particle number ✔
- neutron skin “large”
- neutron distribution smooth like the charge density ✔

Question

- How important is the “straightjacket effect” for the relation between the slope of the symmetry energy and R_n-R_p?

PRL **106,** 252501 (2011) PHYSICAL REVIEW LETTERS week ending 24 JUNE 2011

Neutron Skin of ^{208}Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment

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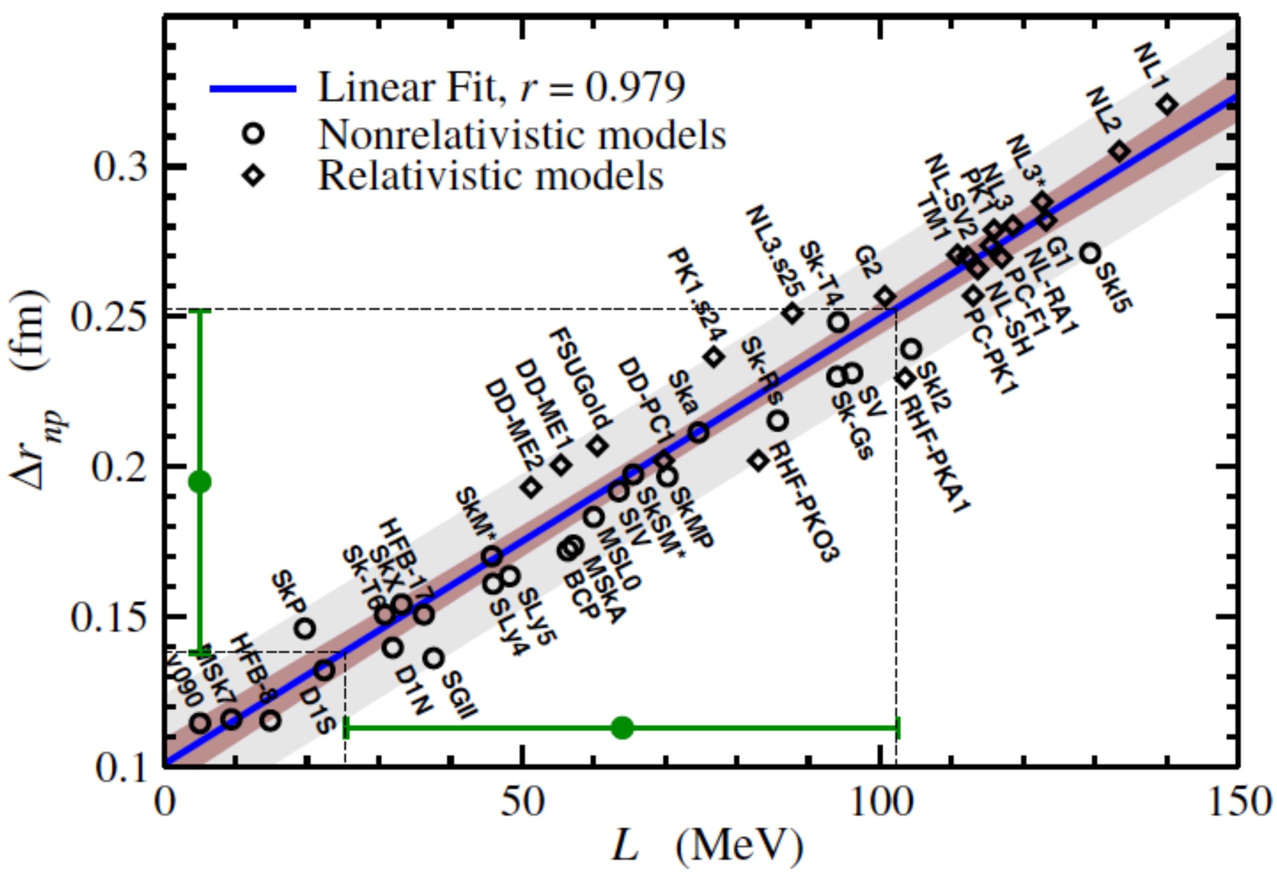
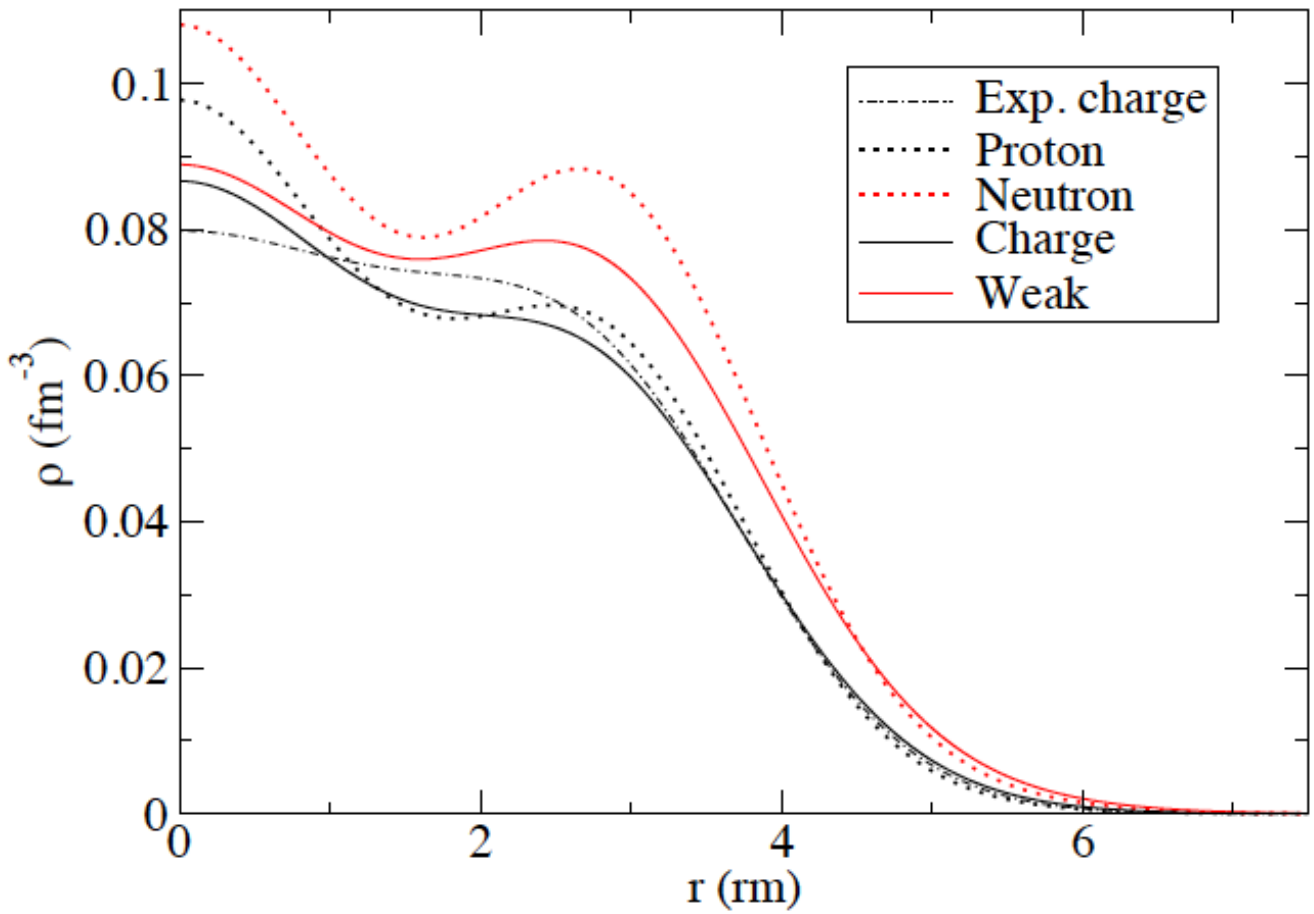
(Received 7 March 2011; published 21 June 2011)



reactions and structure

Hagen et al. based on PRL109,032502(2012)

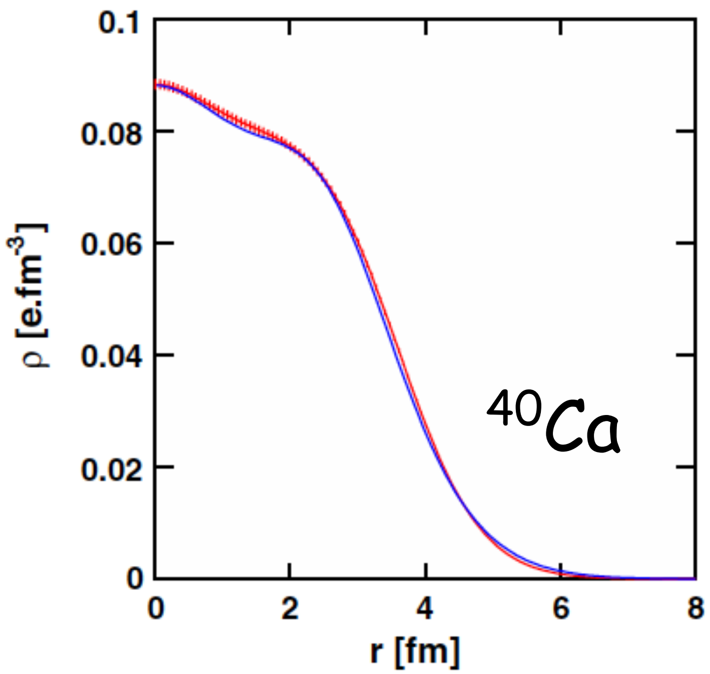
- Coupled-cluster ab initio
- Chiral forces have limitations
- Coupled-cluster method also
- ^{48}Ca

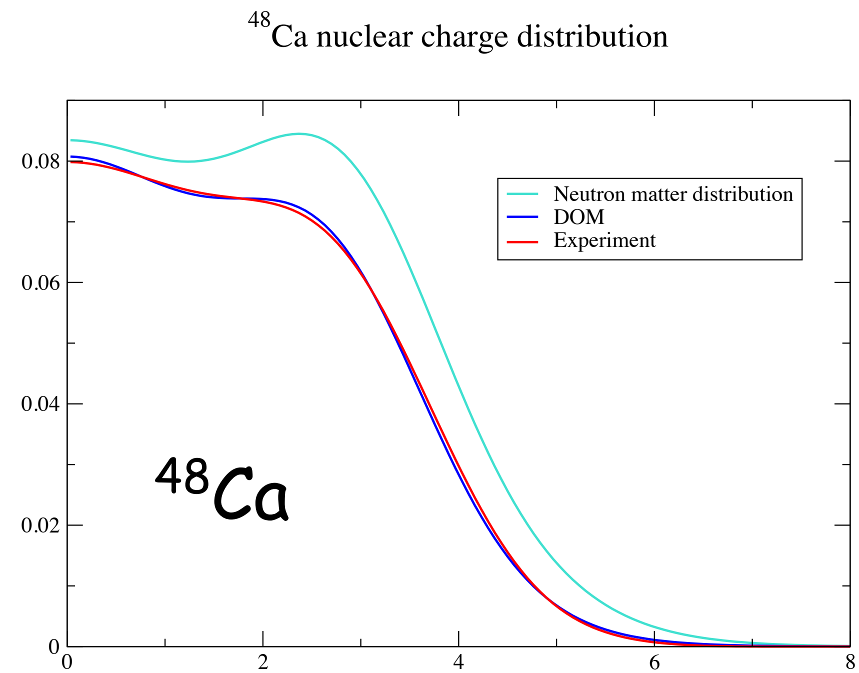


R_p	3.438
R_n	3.594
$R_n - R_p$	0.156
R_W	3.697
R_{ch}	3.526
R_{ch} (exp)	3.48

Can we get *L* from the DOM?

- Perhaps…

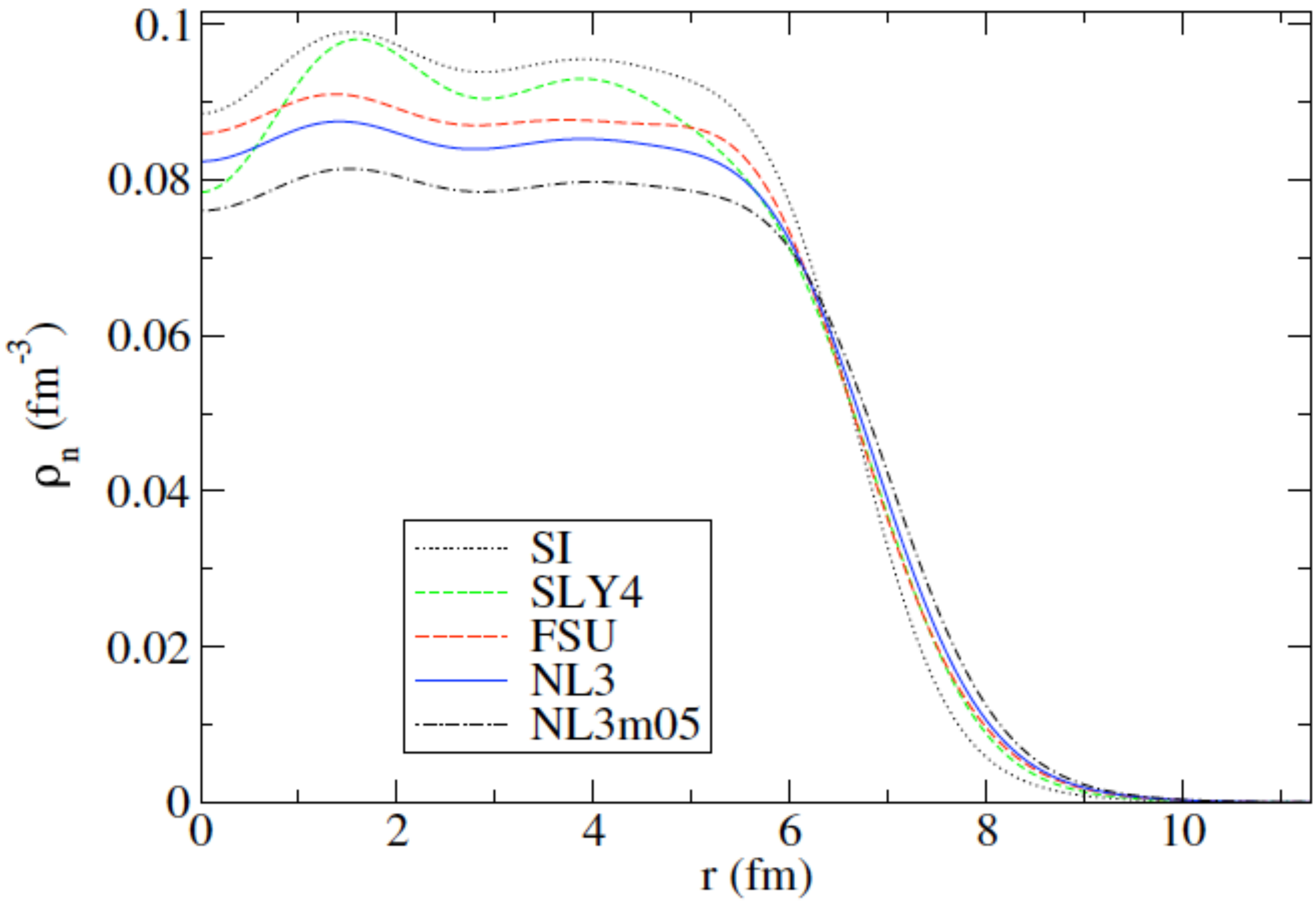




- We could calculate energy density as a function of r for both nuclei…
- Identify the normal density part from the interior…

Mean-field for ^{208}Pb neutrons

B. Shufang, C J Horowitz and R Michaels J. Phys. G: Nucl. Part. Phys. 39 (2012) 015104



Conclusions

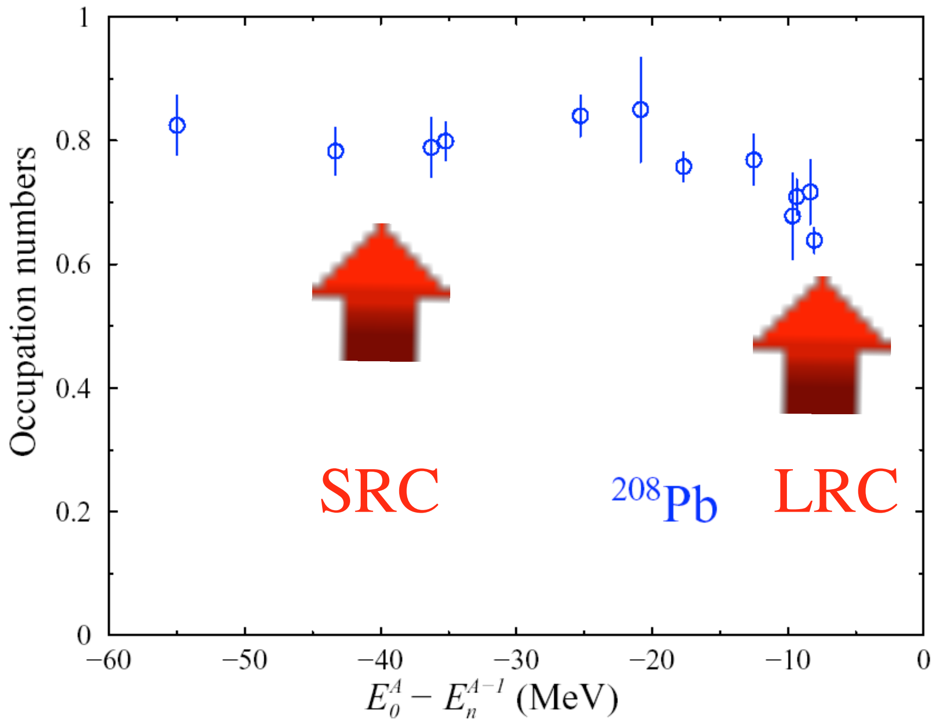
- It is possible to link nuclear reactions and nuclear structure
- Vehicle: nonlocal version of **Dispersive Optical Model** (Green's function method) pioneered by Mahaux —> DSM
- Can be used as input for analyzing nuclear reactions
- Can predict properties of exotic nuclei
- "Benchmark" for ab initio calculations: e.g. V_{NNN} —> binding
- Can describe ground-state properties
  - charge density & momentum distribution
  - spectral properties including high-momentum Jefferson Lab data
- **Elastic scattering determines depletion of bound orbitals**
- Outlook: reanalyze many reactions with nonlocal potentials...
- For N ≳ Z exhibits sensitivity to properties of neutrons —> weak charge —> neutron skin and perhaps more

reactions and structure

M. van Batenburg & L. Lapikás from ^{208}Pb (e,e´p) ^{207}Tl

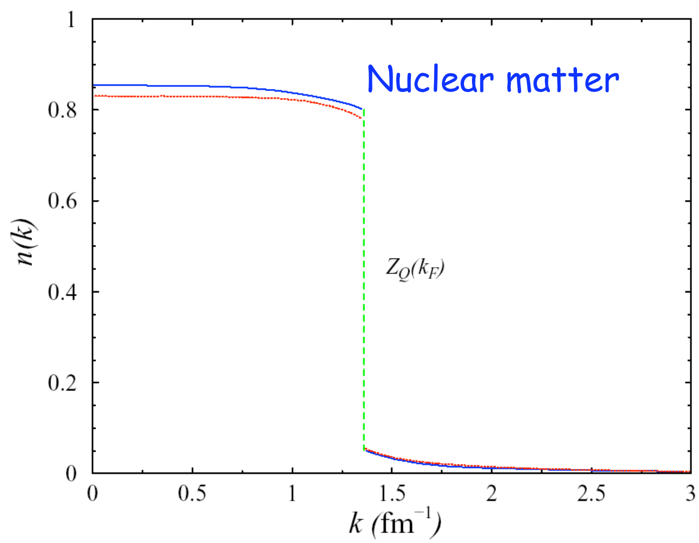
NIKHEF 2001 data (one of the last experiments)

Occupation of deeply-bound proton levels from **EXPERIMENT**



Up to 100 MeV missing energy and 270 MeV/c missing momentum

Covers the **whole** mean-field domain!!



Confirms predictions for depletion

n(0) ⇒ 0.85 Reid

0.87 Argonne V18

0.89 CDBonn/N3LO