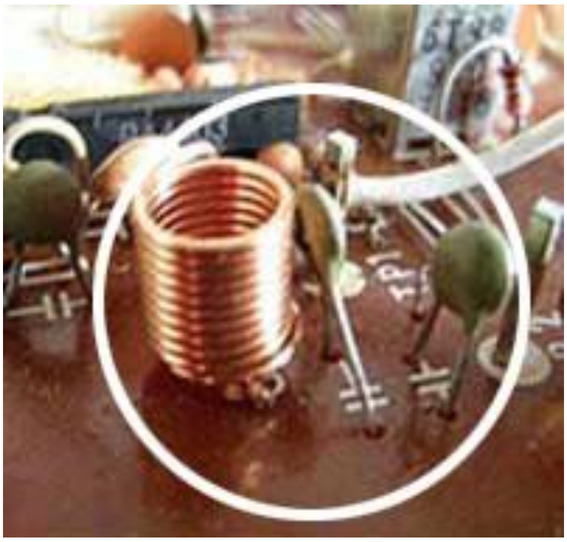
DOING PHYSICS WITH MATLAB

ac CIRCUITS



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Matlab Download Directory

Cac1.m ac voltages and phases

Cac2.m Time evolution and phasor plot for two sinusoidal voltages

Cac3.m Solving textbook style problems on RLC circuits

Cac5.m Modelling a series RLC circuit (time domain)

Cagvoltage.m Animated gif for the rotation of the voltage phasor

simpson1d.m function to evaluate an integral

CN06.m

A collection of Cells makeup the script to solve standard textbook problems on ac circuit theory. Each Cell gives the solution to one problem. To run a cell press <Ctrl><Enter> simultaneously.

COMPLEX POTENTAL DIFFERENCES AND CURRENTS: PHASORS

We will use complex numbers to model ac circuits with passive components of resistors, capacitors and inductors.

The simplest alternating waveforms are sinusoidal waveforms. We will represent all sinusoidal oscillations as cosine functions.

Working with trigonometric functions is extremely cumbersome. A much better approach is to use the voltages and currents expressed as complex functions and using the **Euler Identify**

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad j = \sqrt{-1}$$

The **real** part of the complex function is the actual voltage or current.

Using complex functions means that the analysis of ac circuits is no more difficult conceptually then dc circuits.

NOTE: The lowercase letters v and i are used to represent ac voltages and currents respectively and the uppercase letters V and I used for the peak values.

Potential differences (Voltages)

The sinusoidal voltage (potential difference) $v(t)$ is specified by its amplitude (peak value) V, period T, frequency f, angular frequency ω and phase angle ϕ.

(1) $$v(t) = V\cos\left(\omega t + \phi\right) \quad \omega = 2\pi\, f = \frac{2\pi}{T} \quad f = \frac{1}{T}$$

Equation 1 can be expressed in complex exponential form as

(2) $$v(t) = V\, e^{\,j(\omega t + \phi)}$$

The voltage is then given by the real part of the voltage given in equation 2.

$$\mathrm{Real}\left(V\, e^{\,j(\omega t + \phi)}\right) = V\cos(\omega t + \phi)$$

The term $(\omega t + \phi)$ is called the **phase**. The complex voltage $v(t)$ is called the voltage (potential difference) **phasor**. In the complex plane, the complex voltage is represented by a length V rotating anticlockwise with angular frequency ω. At any instant, the real part of the complex voltage is the actual voltage $V\cos(\omega t+\phi)$ as shown in figure 1.

View animation of voltage phasor

The animation was produced with the script **Cagvoltage.m**

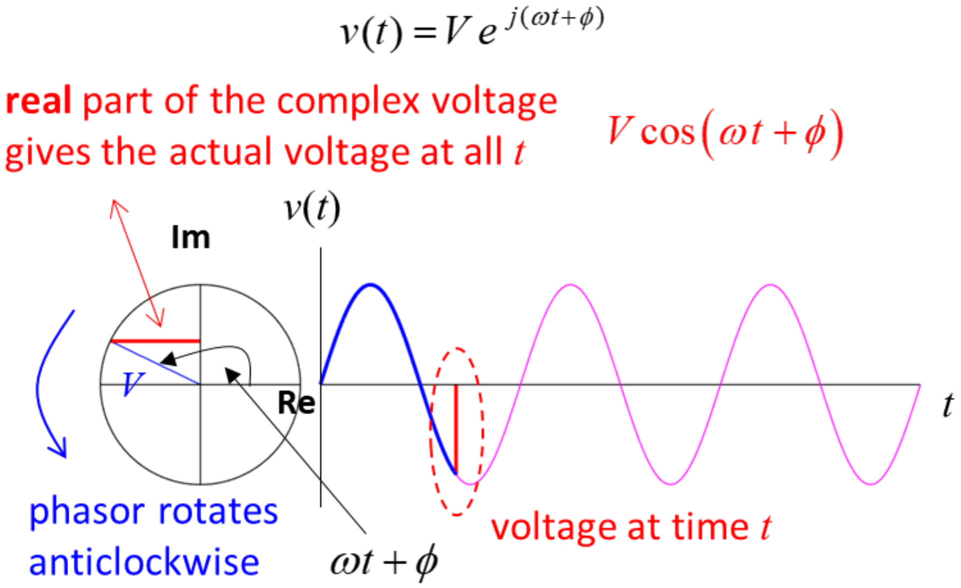


Fig. 1. A voltage can be represented by a complex function. When plotted in the complex plane it is called a phasor. The length of the phasor gives the magnitude of the voltage. The phasor rotates anticlockwise at the angular velocity ω. At any instant, the angle w.r.t the Re axis gives the phase $(\omega t+\phi)$ and the projection onto the Re axis gives the actual voltage.

Currents

The sinusoidal current $i(t)$ is specified by its amplitude (peak) I, period T, frequency f, angular frequency ω and phase angle θ.

(3) $\quad i(t) = I\cos\left(\omega t + \theta\right)$

Equation 3 can be expressed in complex exponential form as

(4) $\quad i(t) = I\,e^{\,j(\omega t + \theta)}$

The current is then given by the real part of the voltage given in equation 4.

$$\mathrm{Real}\left(I\,e^{\,j(\omega t + \phi)}\right) = I\cos(\omega t + \phi)$$

All phases and phase angles are expressed in radians. Any sine function can be represented as a cosine function by subtracting $\left(\pi / 2\right)$ from the argument of the cosine function

$$\sin(\theta) = \cos(\theta - \pi / 2)$$

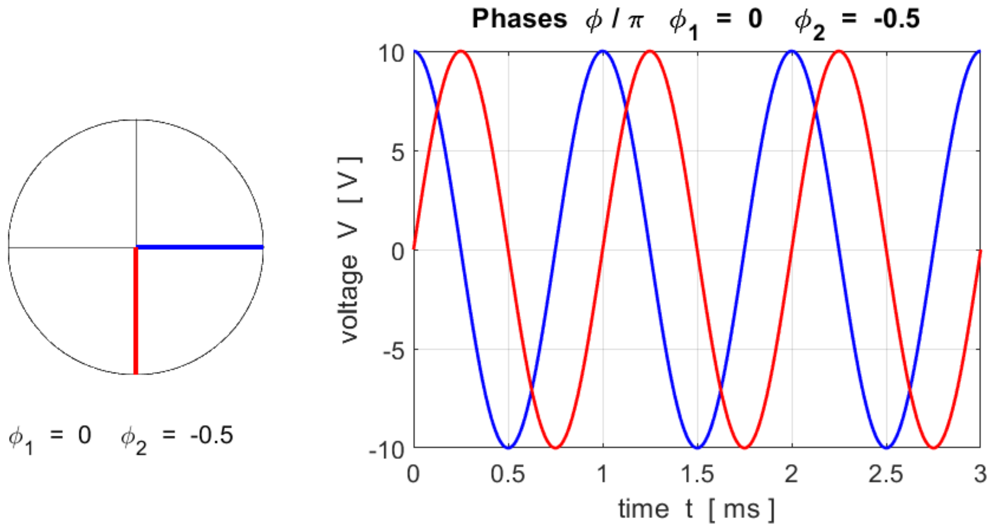


Fig. 2. Blue curve: $v(t) = V\cos(\omega t)$

Red curve: $v(t) = V\cos(\omega t - \pi / 2) = V\sin(\omega t)$

Power P

Power is an important electrical concept. The instantaneous power $p(t)$ absorbed by or transferred to a circuit element is the product of the potential difference $\mathrm{Re}\left[v(t)\right]$ across it and the current into it $\mathrm{Re}\left[i(t)\right]$

$$p(t) = \mathrm{Re}\left[v(t)\right]\,\mathrm{Re}\left[i(t)\right]$$

The average power $\left\langle P \right\rangle$ is called the **root mean square value** for sinusoidal functions is by the integral

$$\left\langle P \right\rangle = \mathrm{P}_{\mathrm{rms}} = \frac{1}{T}\int_0^T p(t)\,dt$$

You can perform an integration of the function $p(t)$ with the script **simpson1d.m**.

An alternative expression for the average power absorbed by the circuit element when $p(t)$ is a sinusoidal function is given by

$$\left\langle P \right\rangle = \tfrac{1}{2}\mathrm{Re}\left(v(t)\; i(t)^*\right)$$

Impedance Z

The ac version of Ohm's Law is

$$Z = \frac{v(t)}{i(t)} \quad v(t) = Z\, i(t) \quad i(t) = \frac{v(t)}{Z}$$

where the complex quantity Z is called the **impedance**. The impedance can be expressed in terms of its real part R and its imaginary part X as

$$Z = R + j\, X$$

where the R is the **resistance** and X is the **reactance**.

VOLTAGES, CURRENTS PHASORS FOR RESISTORS, CAPACITORS AND INDUCTORS

With circuits containing resistors, capacitors and inductors, the voltage across different elements maybe out of phase with each other. Also, for an element, the voltage across it and the current through it may also be out of phase. The script **Cac1.m** can be used to model two ac voltages which have different phases. By carefully examining the following figures 3, 4, and 5 you should be able to gain a better understanding of the difference in the phases of the two signals described by the terms **lead** and **lag**.

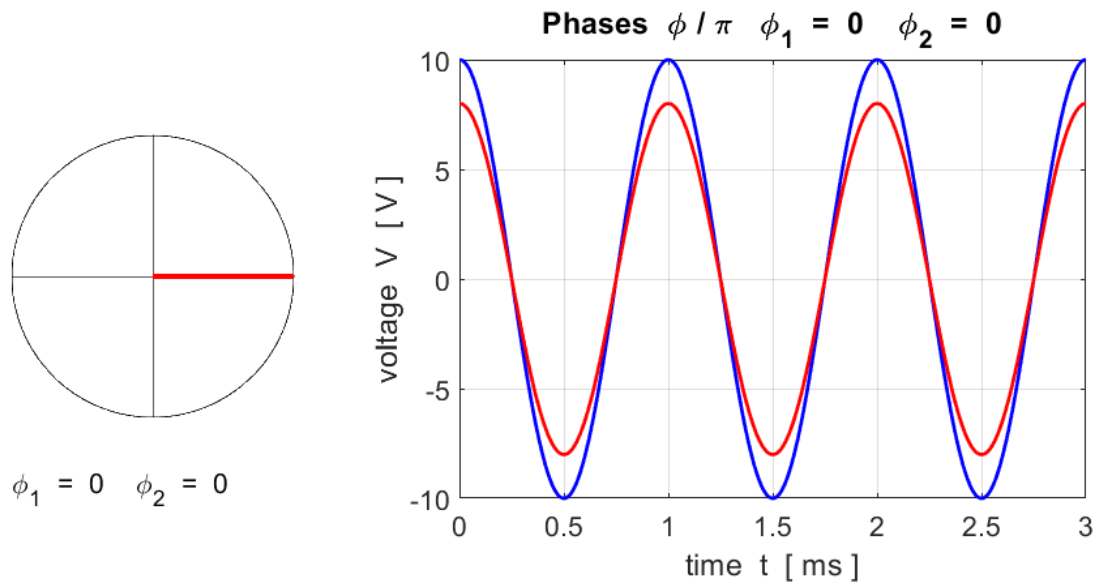


Fig. 3. The two voltages are in phase as they reach their peak values at the same time.

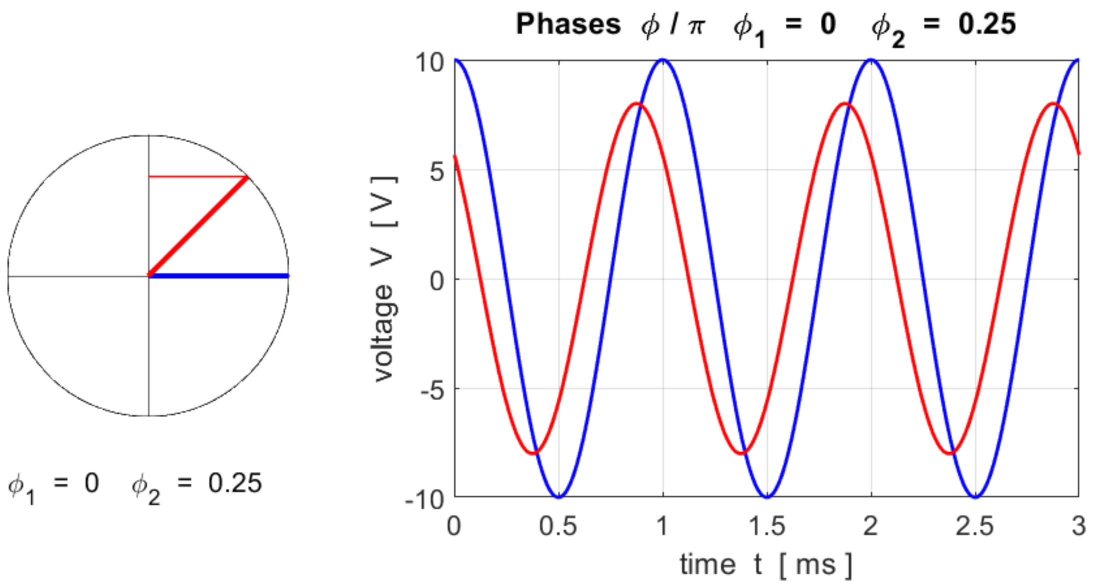


Fig. 4. The two voltages are out of phase as they reach their peak values at different times. Voltage 2 **leads** Voltage 1 by $\pi / 4$ rad. Voltage 1 **lags** Voltage 2 by $\pi / 4$ rad. **Cac1.m**

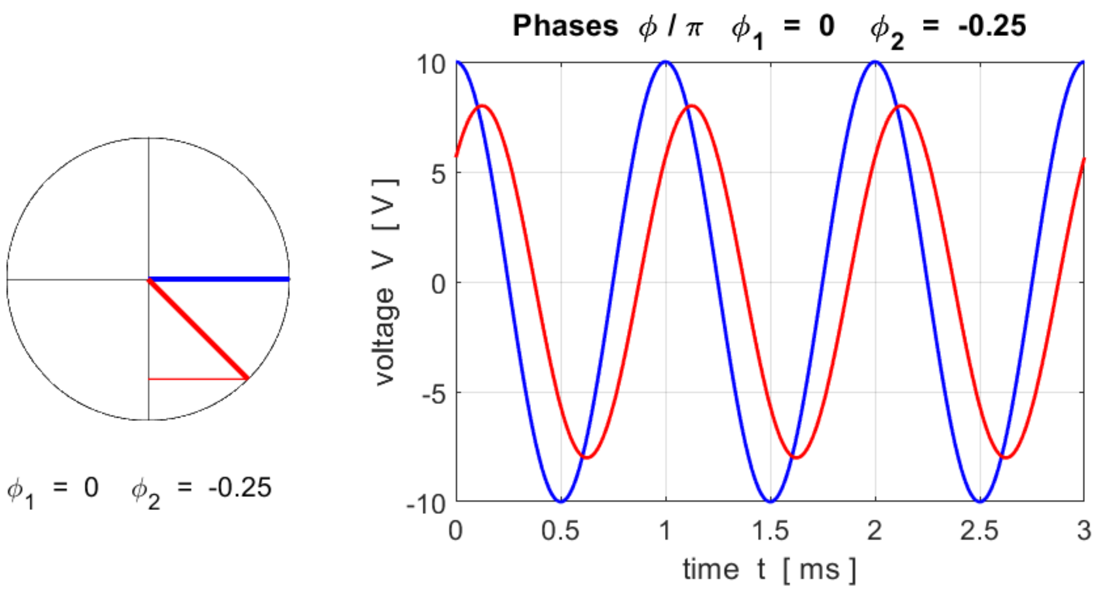


Fig. 5. The two voltages are out of phase as they reach their peak values at different times. Voltage 2 **lags** Voltage 1 by $\pi / 4$ rad. Voltage 1 **leads** Voltage 2 by $\pi / 4$ rad. **Cac1.m**

Resistor R

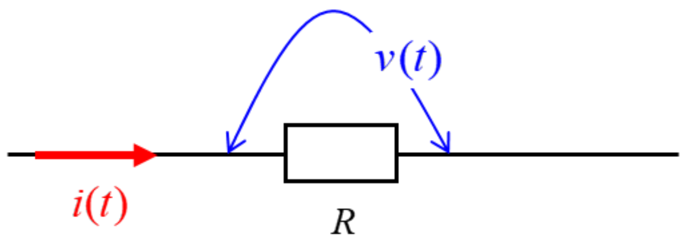
The complex voltage across the resistor is $v(t) = V\,e^{\,j\omega t}$

Ohm's Law $\quad i(t) = \dfrac{v(t)}{R}$

Hence, the complex current is $i(t) = \dfrac{V}{R} e^{\,j\omega t} = I\,e^{\,j\omega t} \qquad I = \dfrac{V}{R}$

The impedance is simply the resistance $Z = R$ (real)

At all instants, the voltage and current are **in phase** (figure 3).



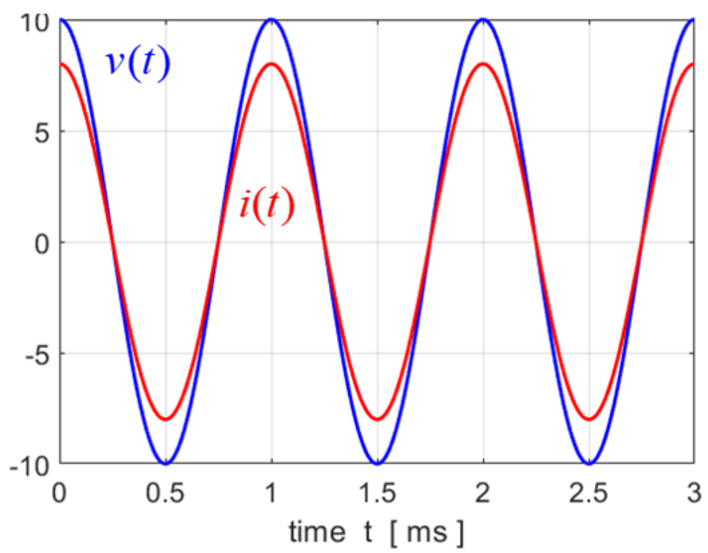


Fig. 6. The real current in a resistor and the real voltage across it reach their peak values simultaneously. The current and voltage are in phase.

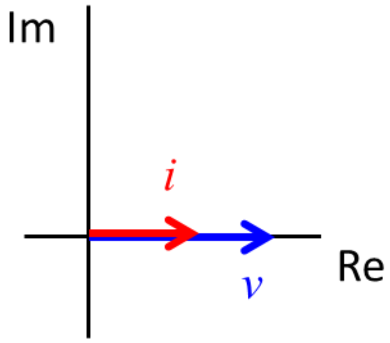


Fig. 7. Phasor diagram of the voltage and current for a resistor at time t = 0. The voltage and current are in phase at all times. The phasors would rotate anticlockwise at the angular frequency ω as time evolves.

Inductor L

The complex voltage across the inductor is

$$v(t) = V\,e^{\,j\omega t}$$

Current though the inductor is related to the voltage across it

$$v(t) = L\frac{di(t)}{dt}$$

Hence, the complex current is

$$i(t)=\frac{V}{L}\int e^{\,j\omega t}=\frac{V}{j\omega L}e^{\,j\omega t}=\frac{-jV}{\omega L}e^{\,j\omega t}$$

$$e^{\,-j\pi/2}=\cos(-\pi/2)+j\sin(-\pi/2)=-j$$

$$i(t)=\frac{V}{\omega L}e^{\,j\omega t}\,e^{\,-j\pi/2}=\frac{V}{\omega L}e^{\,j(\omega t-\pi/2)}$$

$$i(i)=I\,e^{\,j(\omega t-\pi/2)} \qquad I=\frac{V}{\omega L}$$

Taking the real part of the voltage and current we get the actual voltages and currents

$$v(t)=V\cos(\omega t)$$
$$i(t)=I\cos(\omega t-\pi/2)$$
$$i(t)=I\sin(\omega t)$$

as shown in figure 8.

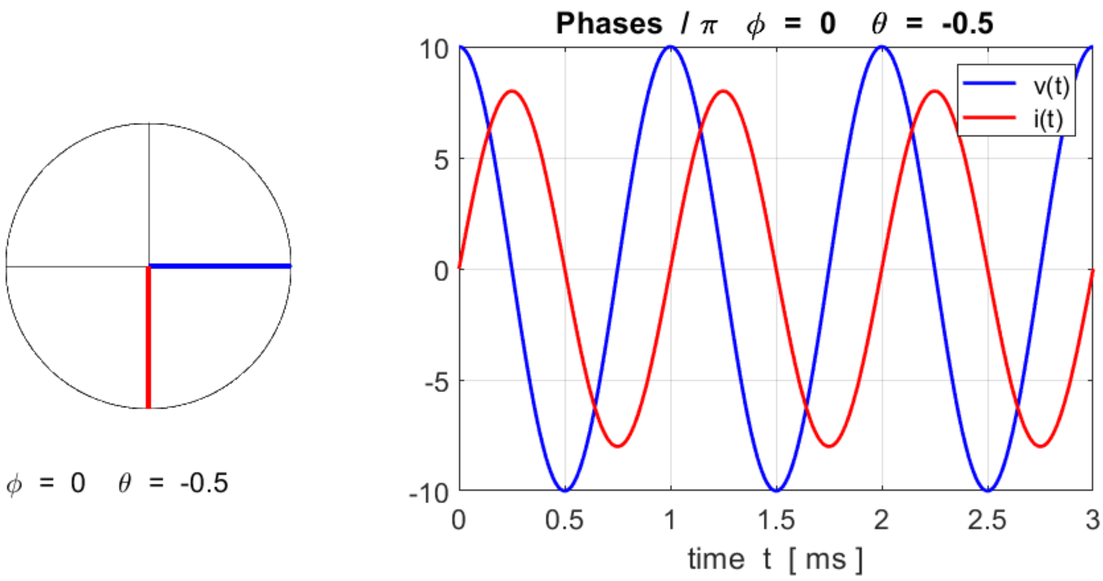


Fig. 8. The voltage and current are out of phase as they reach their peak values at the different time. Voltage **leads** current by $\pi / 2$ rad. Current **lags** Voltage by $\pi / 2$ rad. The voltage reaches its peak before the current.
Cac2.m

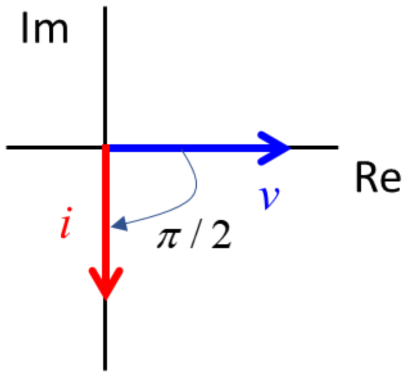


Fig. 9. Phasor diagram of the voltage and current for an inductor at time t = 0. The voltage phasor always leads the current phasor by $\pi / 2$. The phasors would rotate anticlockwise at the angular frequency ω as time evolves.

The impedance of the inductor Z_L is

$$Z_L = \frac{v(t)}{i(t)} = \frac{V\,e^{\,j\omega t}}{I\,e^{\,j(\omega t - \pi/2)}} = \frac{V}{I}\,e^{\,j\pi/2}$$

$$I = \frac{V}{\omega L} \qquad e^{\,j\pi/2} = \cos(\pi/2) + j\sin(\pi/2) = j$$

$$Z_L = j\,\omega L$$

The impedance only has an imaginary part. So, the reactance of the inductor X_L is

$$X_L = \omega L$$

Capacitor C

The complex voltage across the capacitor is

$$v(t) = V\,e^{\,j\omega t}$$

The charge on the capacitor is related to the voltage across it

$$q(t) = C\,v(t)$$

But, the current is related to the charge

$$i(t) = \frac{dq(t)}{dt}$$

So,

$$
\begin{aligned}
i(t) &= \frac{dq(t)}{dt} = C\left(j\omega\right)V\,e^{\,j\omega t} = j\omega\, CV\,e^{\,j\omega t} \\
e^{\,j\pi/2} &= \cos(\pi/2) + j\sin(\pi/2) = j \\
I &= \omega\, CV \\
i(t) &= I\,e^{\,j\omega t}\,e^{\,j\pi/2} \\
i(t) &= I\,e^{\,j\left(\omega t + \pi/2\right)}
\end{aligned}
$$

Taking the real part of the voltage and current we get the actual voltages and currents

$$
\begin{aligned}
v(t) &= V\cos(\omega t) \\
i(t) &= I\cos(\omega t + \pi/2) \\
i(t) &= -I\sin(\omega t)
\end{aligned}
$$

as shown in figure 10.

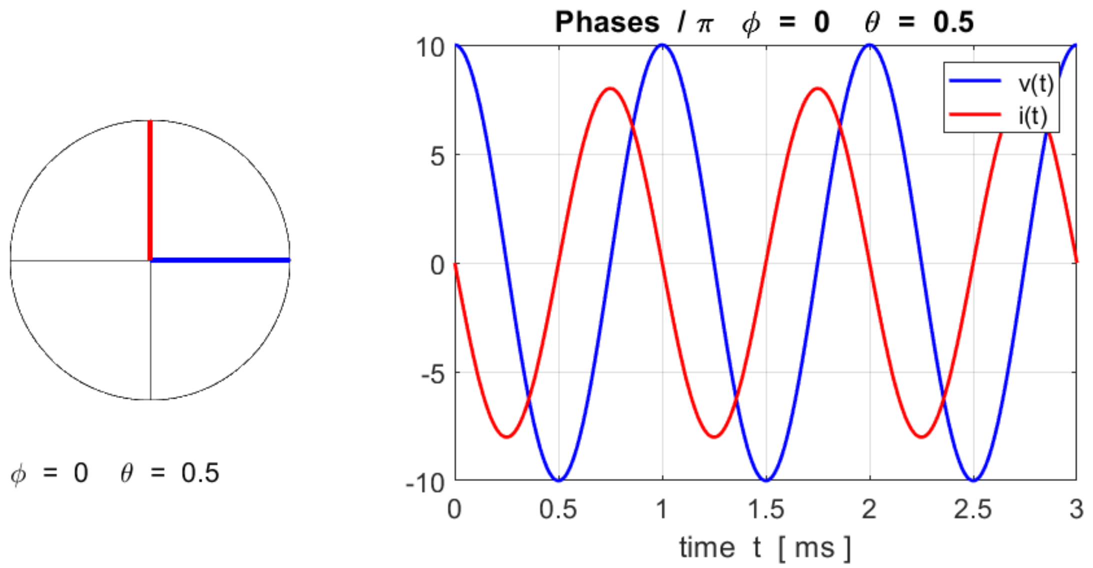


Fig. 10. The voltage and current are out of phase as they reach their peak values at the different time. Voltage **lags** current by $\pi/2$ rad. Current **leads** Voltage by $\pi/2$ rad. The voltage reaches its peak after the current. **Cac2.m**

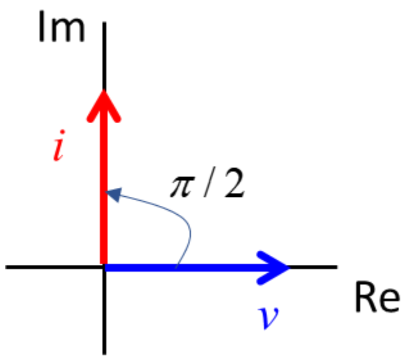


Fig. 11. Phasor diagram of the voltage and current for a capacitor at time t = 0. The voltage phasor always lags the current phasor by $\pi/2$. The phasors would rotate anticlockwise at the angular frequency ω as time evolves.

The impedance of the capacitor Z_C is

$$Z_L = \frac{v(t)}{i(t)} = \frac{V\,e^{\,j\omega t}}{I\,e^{\,j(\omega t+\pi/2)}} = \frac{V}{I}\,e^{-j\pi/2}$$

$$I = \omega\,CV \qquad e^{-j\pi/2} = \cos(\pi/2) - j\sin(\pi/2) = -j$$

$$Z_C = \frac{-j}{\omega\,C}$$

The impedance only has an imaginary part. So, the reactance of the capacitor X_C is

$$X_C = \frac{1}{\omega C}$$

Example 1

We will consider the series RLC circuit shown in figure 12 and use the Matlab script **Cac5.m** to calculate the current, voltages across each element and power absorbed by each element as functions of time.

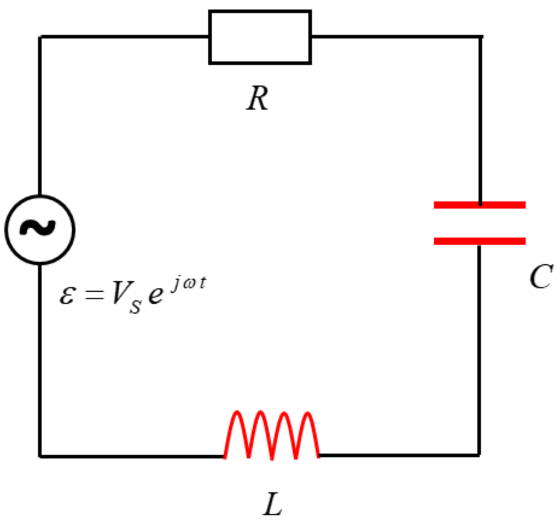


Fig. 12. Series RLC circuit with a sinusoidal source emf.

The first step is to label the circuit to identify the elements, currents and voltages as shown in figure 13.

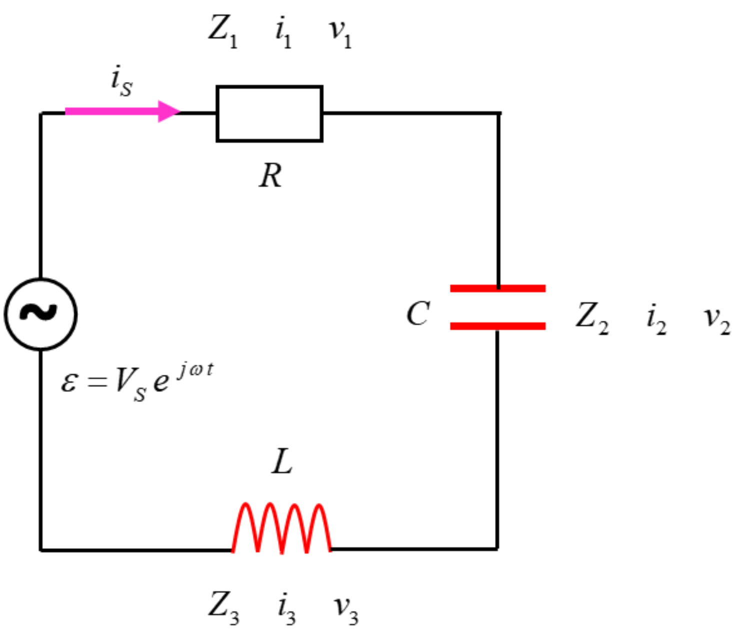


Fig. 13. Labelled series RLC circuit with a sinusoidal source emf.

Script Cac5.m

Specify the input parameters

```
% source emf (peak or amplidue value) [10 10e3]
   VS = 10;
   f = 10e3;
% resistance Z1     [1e3]
   R = 1e3;
% capacitance Z2 [1.0e-8 F]
   C = 1.0e-8;
% inductance Z1 [10e-3 H]
   L = 10e-3;
```

Compute:

Angular frequency and period of the source emf.

The grid for time (number of grid points must be odd).

The source emf as a function of time.

The resonance frequency for the LC combination.

```
w = 2*pi*f;                % angular frequency
T = 1/f;                   % period
t = linspace(0,3*T,5001);
     % time  N must be an odd number for Simpson's Rule
vS = VS .* exp(1i*w*t);   % emf as a function of time
f0 = 1/(2*pi*sqrt(L*C)); % Resonance frequency
```

Compute: Impedances and reactances. Z4 is the total circuit impedance.

```
Z1 = R;                   % resistance
 X2 = 1/(w*C);            % capacitive reactance
 Z2 = -1i * X2;           % capacitive impedance
 X3 = w*L;                % inductive reactance
 Z3 = 1i * X3;            % inductive impedance
 Z4 = Z1 + Z2 + Z3;       % total circuit impedance
```

Compute: Functions of time – currents, voltages and phases (lowercase letters); peak values (uppercase letters).

```
iS = vS ./ Z4;                    % source current
i1 = iS; i2 = iS; i3 = iS;         % element currents
theta = angle(iS);                 % current phase
IS = max(abs(iS));                 % peak current

v1 = i1 .* Z1;                     % element voltages
v2 = i2 .* Z2;
v3 = i3 .* Z3;

V1 = max(abs(v1));                 % peak voltages
V2 = max(abs(v2));
V3 = max(abs(v3));

phi1 = angle(v1);                  % voltage phases
phi2 = angle(v2);
phi3 = angle(v3);
phiS = 0;
```

Compute: Powers as function of time (lowercase letters) and rms values (uppercase letters).

```
pS = real(vS) .* real(iS);    % powers
p1 = real(v1) .* real(i1);
p2 = real(v2) .* real(i2);
p3 = real(v3) .* real(i3);

Prms = 0.5*real((vS(1) .* conj(iS(1))));
Prms_N = simpson1d(pS,0,t(end)/t(end));
P1 = simpson1d(p1,0,t(end)/t(end));
P2 = simpson1d(p2,0,t(end)/t(end));
P3 = simpson1d(p3,0,t(end)/t(end));
```

A summary of the parameters for the modelling of the series RLC circuit are displayed in the Command Window

```
Inputs
 Source peak voltage VS =  10.00  V
 Source frequency f  =  1.00e+04 Hz
  R =  1000.00 ohms
  C =  1.00e-08 F
  L =  1.00e-02 H

Outputs
 Resonance freq  f0 =  15915.49  Hz
  XC =  1591.55  ohms
  XL =  628.32  ohms
  peak current  IS =  7.20
Peak Values
  IS =  7.20 mA
  emf  VS =  10.00 V
  VR =  7.20 V
  VC =  11.46 V
  VL =  4.53 V
Phases
  phi_S =  0.00 pi rad
  phi_R =  0.24 pi rad
  phi_C =  -0.26 pi rad
  phi_L =  0.74 pi rad
Power  rms values
  Prms =  25.94 pi mW
  Prms (Simpsons Rule) Prms =  25.94 mW
  Simpsons Rule PR =  25.94 mW
  Simpsons Rule PC =  0.00 mW
  Simpsons Rule PL =  -0.00 mW
```

Results are displayed in a series of Figure Windows.

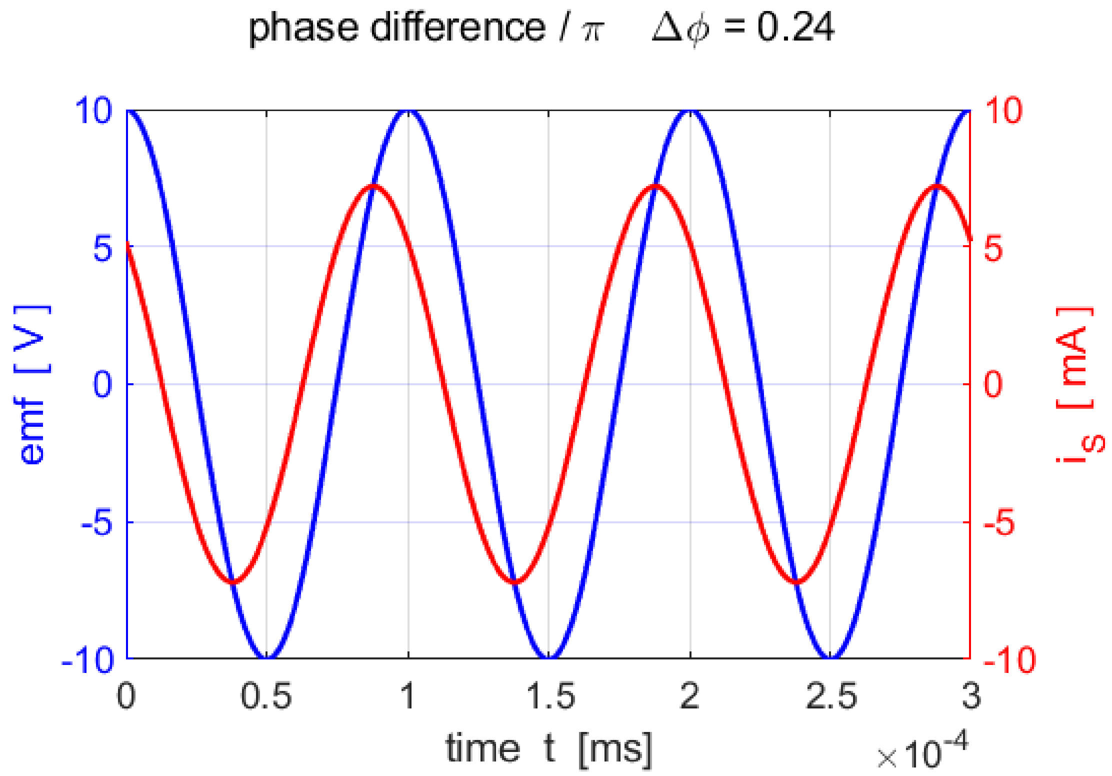
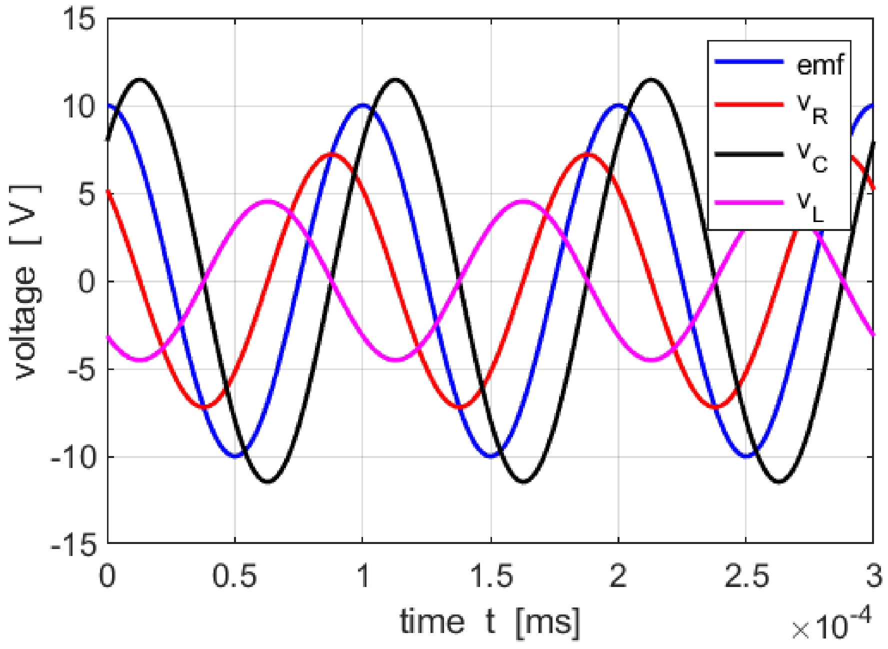


Fig. 14. The time variation of the source emf and the current in the circuit. The same current that is drawn from the source passes through each element since it is a series circuit. The current leads the emf by 0.24 π rad.



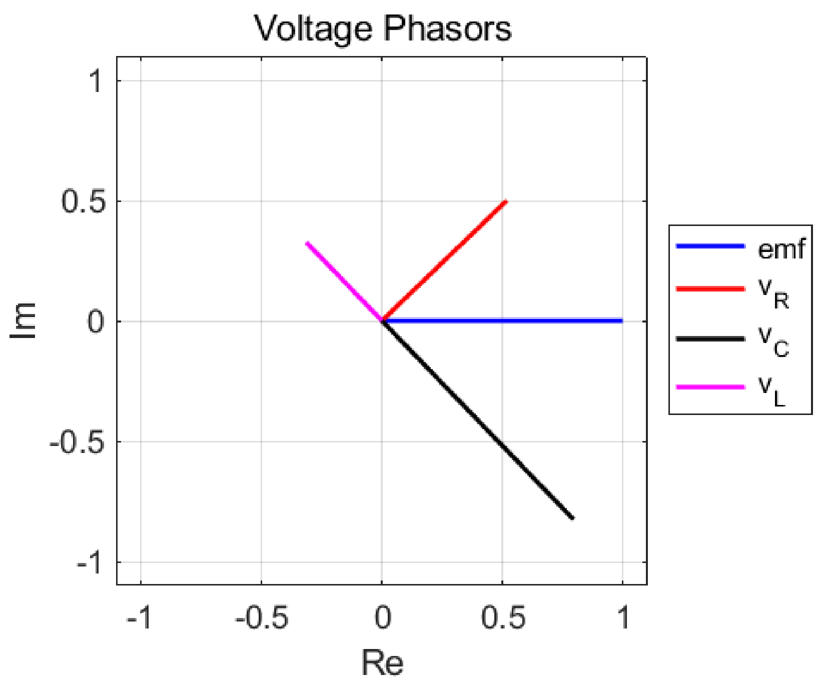


Fig. 15. The time variation of the voltages for the source, and across each element and a phasor diagram in the complex plane (t = 0). The voltage across the capacitor is greater than the emf. This is OK because voltages add like vectors since you need to consider the magnitude and phase of each voltage when they are added together. You can see this from both the above plots that $emf(t) = v_R(t) + v_C(t) + v_L(t)$.

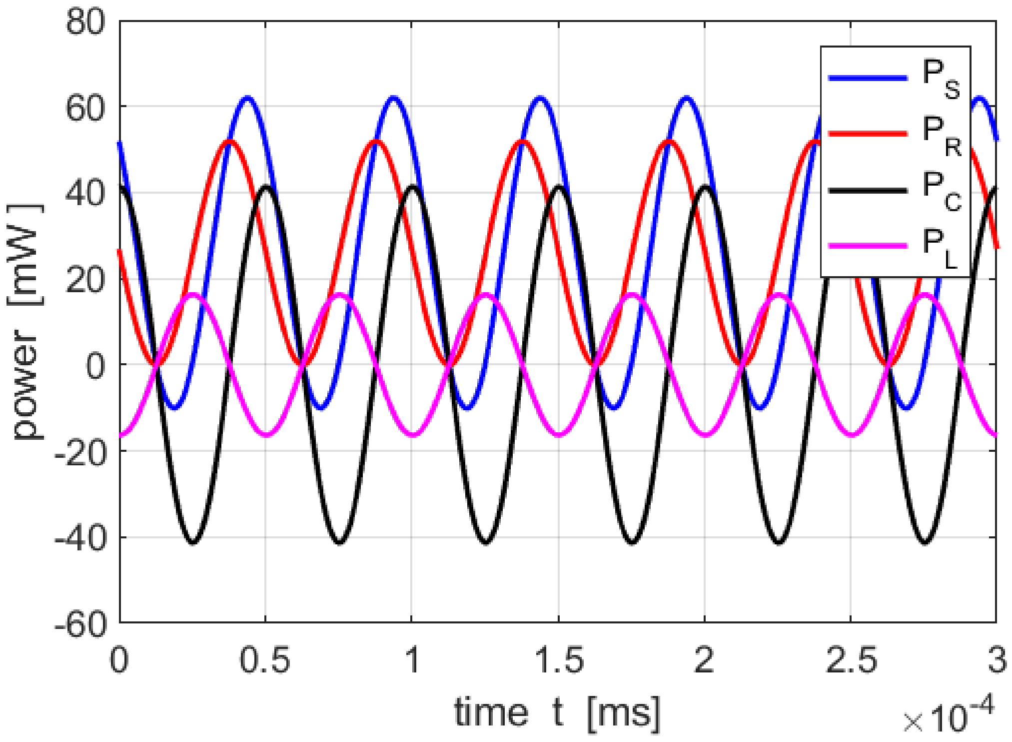


Fig. 16. This is a very interesting plot and reveals lots about the behaviour of resistor, capacitors and inductors in ac circuits. The power supplied from the source is equal to the total power absorbed by the elements (conservation of energy): resistor, capacitor and inductor

$$p(t) = p_R(t) + p_C(t) + p_L(t).$$

The average power absorbed by the capacitor or inductor is zero (the curves are symmetrical about 0 power), but at any instance the power may not be zero. The power is always dissipated by the resistor $\left(P_R \geq 0\right)$.

However, things are very different for the capacitor and inductor.

When $P_C > 0$, the capacitor stores energy in the electric field between the capacitor plates and energy is absorbed from the circuit. When, $P_C < 0$, energy is returned to the circuit. The net effect is that the time average power transferred to or from the circuit is zero.

When $P_L > 0$, the inductor stores energy in the magnetic field surrounding the coil and energy is absorbed from the circuit. When, $P_L < 0$, energy is returned to the circuit. The net effect is that the time average power transferred to or from the circuit is zero.

At Resonance

The series resonant frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

We can run the script Cac5.m with the input frequency set at the resonant frequency f_0 = 15915.49 Hz.

At the resonant frequency, the reactances of the capacitor and inductor are equal X_C = X_L = 1000 Ω and they are π rad out of phase. So, there effects cancel each other – the capacitor and inductor together are like a short circuit (R = 0 Ω). The circuit is purely resistive with the source emf and current in phase.

Figures 17, 18 and 19 shows the graphical output of the script **Cac5.m** when the frequency is set to the resonant frequency.

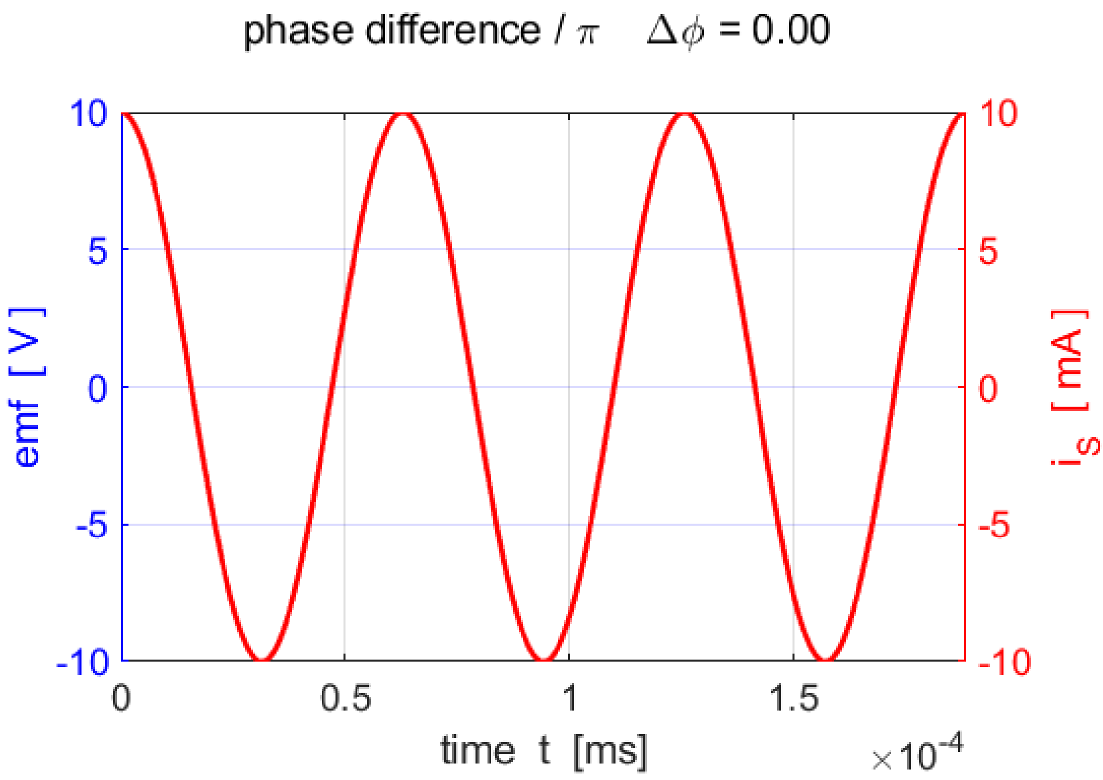


Fig. 17. The source emf and current are in phase (the blue and red curves are superimposed on each other).

$$\left| emf \right| = V_S = 10.0 \text{ V} \quad R = 1000\ \Omega \ \Rightarrow \ I_S = 10.0 \text{ mA}$$

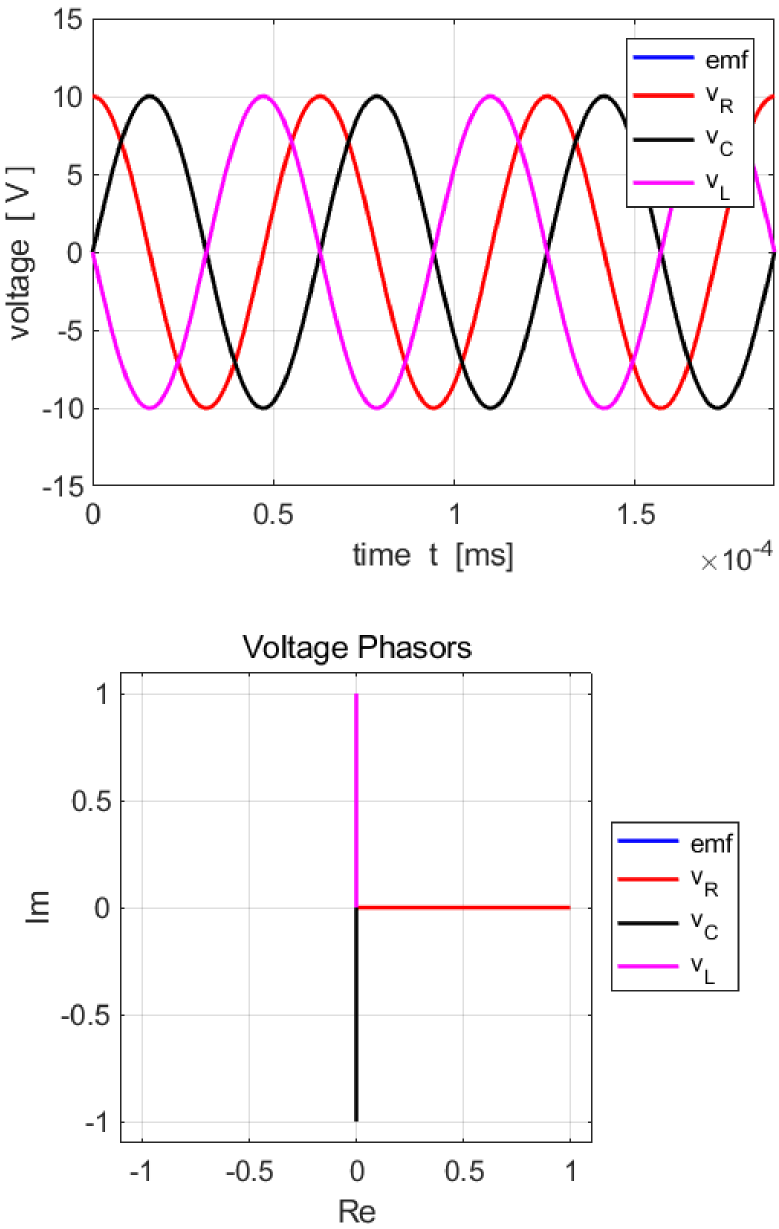


Fig. 18. The time variation of the voltage across the capacitor and inductor are identical except they are out of phase by π rad. Hence, and the effects of the capacitative and inductive reactances exactly cancel each other, with the result that the current is in phase with the source emf. (The voltage across the resistor is identical to the source emf, so the red is superimposed over the blue curve).

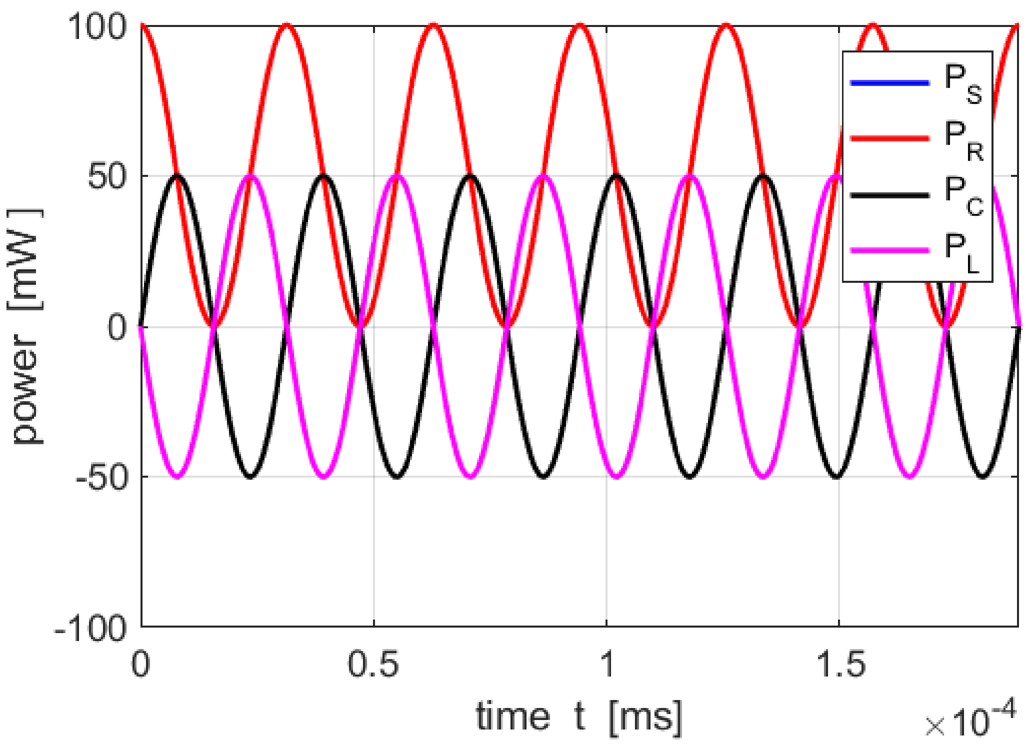


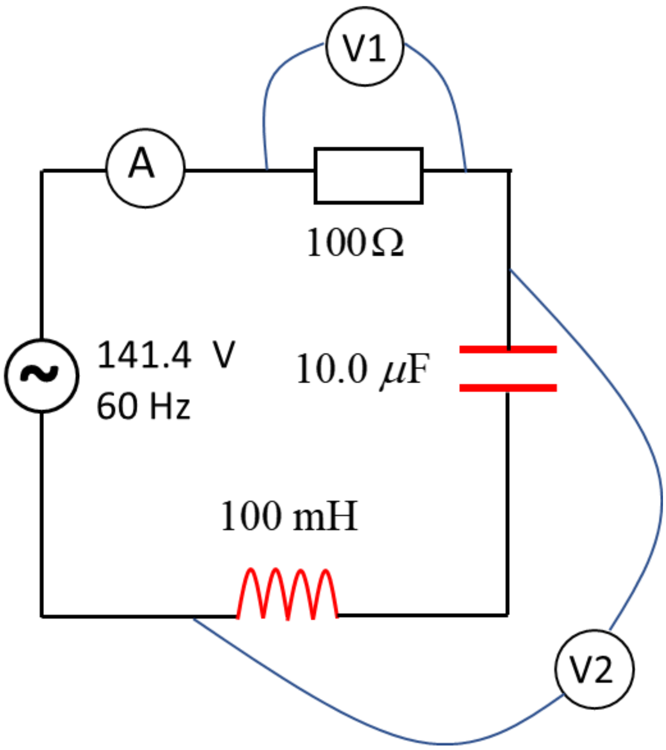
Fig. 19. The time variation of the power transfer to or from the circuit by the capacitor and inductor are identical except they are out of phase by π rad. Hence, at each instant $p_C(t) + p_L(t) = 0$. So, all the power supply by the energy source is dissipated by the current through the resistor.

Textbook style problems

It is almost a trivial task to do traditional textbook problems on ac circuits using Matlab complex functions. The code can be done in small scripts or even by entering text into the command Window.

Textbook Example 1

In the circuit shown, the applied ac emf has a frequency of 60 Hz and the peak voltage is 141.4 V. Compute the readings of the voltmeters and the ammeter. Give the expression for the instantaneous current in the circuit. Assume the impedance of the generator is small. Note: the meters record the rms values.



Solution Cac3.m (run Cell #1)

Script

```
%% CELL #1  Textbook Example 1
clear all
close all
clc

% INPUTS [SI Units] -------------------------------------
---------------
R = 100;
C = 10e-6;
L = 100e-3;
f = 60;
VS = 141.4;

% CALCULATIONS  [SI Units] ------------------------------
--------------
w = 2*pi*f
XC = 1/(w*C)
XL = w*L
Z1 = R;
Z2 = -1i*XC
Z3 = 1i*XL
Z = Z1+Z2+Z3
IS = VS/Z
Ipeak = abs(IS)
Irms = abs(IS)/sqrt(2)
theta = angle(IS)
V1 = IS * Z1
V2 = IS * Z2
V3 = IS * Z3
V23 = V2+V3
V123 = V1+V2+V3
V1rms = abs(V1)/sqrt(2)
V23rms = abs(V23)/sqrt(2)
V123rms = abs(V123)/sqrt(2)
```

Command Window

```
w = 376.9911
XC = 265.2582
XL = 37.6991
Z2 = 0.0000e+00 - 2.6526e+02i
Z3 = 0.0000 +37.6991i
Z = 1.0000e+02 - 2.2756e+02i
IS = 0.2289 + 0.5208i
Ipeak = 0.5689
Irms = 0.4023
theta = 1.1568
```

```
V1 = 22.8865 +52.0803i
V2 = 1.3815e+02 - 6.0708e+01i
V3 =  -19.6338 + 8.6280i
V23 = 1.1851e+02 - 5.2080e+01i
V123 =  1.4140e+02 + 1.2434e-14i
V1rms = 40.2253
V23rms = 91.5364
V123rms =  99.9849
```

The ammeter reading is 0.40 A

The voltmeter readings are

Across the resistor = 40.2 V

Across the capacitor and inductor = 91.5 V

The voltage drop across resistor, capacitor and inductor = 100 V which is the equal to the source emf of 100 V_{rms} (141.4 V_{peak})

The emf is

complex $$emf = V_{peak}\, e^{j\omega t}$$

real $$emf = V_{peak} \cos\left(\omega t\right) = 141.4\cos(377t) \text{ V}$$

The current is

complex

$$i_S = I_{peak} \cos\left(\omega t\right) = 0.5689\cos(377t + 1.1568) \text{ A}$$

real

$$i_S = I_{peak} \cos\left(\omega t + \theta\right) = 0.5689\cos(377t + 1.1568) \text{ A}$$

Textbook Example 2

A circuit has a 1.00 kΩ resistor, a 10 μF capacitor and a 100 mH inductor connected in series to a 110 V, 60 Hz voltage source. Calculate: The reactances, the current and the voltage across each element (magnitude and phase) and the total voltage drop across the resistor, capacitor and inductor.

For a capacitor / inductor combination the resonance frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Repeat the calculations at the resonance frequency. What is the significance of the calculations at the resonance frequency?

If you did this calculation the traditional way using lots of algebra, it would take you a long time and it is tedious work. However, doing it in Matlab is almost a trivial task.

Solution Cac3.m (run Cell #2)

Script

```
%%   CELL #1
   clear all
   close all
   clc
% INPUTS              SI units
   R = 1000;
   C = 10e-6;
   L = 100e-3;
   f = 159.15;
   Vin = 110;

% CALCULATIONS    SI units
   w = 2*pi*f;
   ZR = R;
   XC = 1/(w*C);
   XL = w*L;
   ZC = -1j * XC;
   ZL =  1j * XL;
   Z = ZR + ZC +ZL;
   Iin = Vin / Z;
   VR = Iin * ZR;
   VC = Iin * ZC;
   VL = Iin * ZL;
   phiC = angle(VC)/pi;
   phiL = angle(VL)/pi;
   f0 = 1/(2*pi*sqrt(L*C));

 % DISPLAY RESULTS  actual (real) values
   disp('Inputs  [SI Units]  ');
     fprintf('  R =  %3.2f  \n',R);
     fprintf('  C =  %3.2e  \n',C);
     fprintf('  L =  %3.2e  \n',L);
     fprintf('  f =  %3.2f  \n',f);
     fprintf('  Vin =  %3.2f  \n',Vin);
   disp('Calculations  [SI UNITS  ')
     fprintf('  XC =  %3.2f  \n',XC);
     fprintf('  XL =  %3.2f  \n',XL);
     fprintf('  Iin =  %3.2f  \n',abs(Iin));
     fprintf('  VR =  %3.2f  \n',abs(VR));
     fprintf('  VC =  %3.2f  \n',abs(VC));
     fprintf('  phiC/pi =  %3.2f  \n',phiC)
     fprintf('  VL =  %3.2f  \n',abs(VL));
```

```
      fprintf('  phiL/pi =  %3.2f  \n',phiL);
      fprintf(  'VR + VC + VL =  %3.2f
\n',abs(VR+VC+VL));
    disp('  ');
      fprintf('  resonance frequency f0 =  %3.2f  \n',f0);
```

Command Window

```
Inputs  [SI Units]
  R =  1000.00
  C =  1.00e-05
  L =  1.00e-01
  f =  60.00
  Vin =  110.00

Calculations  [SI UNITS]
  XC =  265.26
  XL =  37.70
  Iin =  0.11
  VR =  107.26
  VC =  28.45
  phiC/pi =  -0.43
  VL =  4.04
  phiL/pi =  0.57
  VR + VC + VL =  110.00

  resonance frequency f0 =  159.15
```

Note:

- The voltage across the capacitor or inductor may be larger than the source voltage. This is because you need to consider the phase of the voltage as well as its magnitude.
- You can not simply add the magnitudes of the voltages across each element. The voltages add like vectors, you need to consider the phase and magnitude of each voltage added. Examine the script to see how the voltages are added.

At the resonance frequency

```
Inputs  [SI Units]
  R =  1000.00
  C =  1.00e-05
  L =  1.00e-01
  f =  159.15
  Vin =  110.00
Calculations  [SI UNITS]
  XC =  100.00
  XL =  100.00
  Iin =  0.11
  VR =  110.00
  VC =  11.00
  phiC/pi =  -0.50
  VL =  11.00
  phiL/pi =  0.50
  VR + VC + VL =  110.00

  resonance frequency f0 =  159.15
```

At the resonance frequency, the reactance of the capacitor is equal to the reactance of the inductor. The voltages across the capacitor and inductor are equal in magnitude but are π rad out of phase. So, when added the two voltages cancel each other and the total impedance of the circuit is purely resistive and the current is simply

$$I_{in} = V_{in} / R = 110 / 1000 \text{ A} = 0.11 \text{ A}$$

Textbook Example 3

A circuit has a 1.00 kΩ resistor, a 10 μF capacitor and a 100 mH inductor. The source voltage (100 V peak, 50 Hz) and the resistor is connected in series with the capacitor and inductor connected in parallel. Find the current through the resistor, capacitor and inductor. Give the expressions for the instantaneous currents through each element.

Solution Cac3.m (run Cell #3)

Script

```
%%  CELL #3
 clear all
 close all
 clc
 format shorte
 % INPUTS
   R = 1000;
   C = 10e-6;
   L = 100e-3;
   f = 50;
  Vin = 100;

% CALCULATIONS
   w = 2*pi*f;
   ZR = R;
   ZC = -1j / (w*C);
   ZL = 1j * w*L;
   Zp = 1/(1/ZC + 1/ZL);
   Z = ZR + Zp;
   Iin = Vin / Z;
   IR = Iin;
   VR = IR * ZR;
   Vp = Vin - VR;
   IC = Vp / ZC;
```

```
   IL = Vp / ZL;
   f0 = 1/sqrt(L*C);

   thetaR = angle(IR);
   thetaC = angle(IC);
   thetaL = angle(IL);

   % Display results  actual values (real)
    disp('Inputs  ');
      fprintf('R =  %3.2f  ohms  \n',R);
      fprintf('C =  %3.2e  F  \n',C);
      fprintf('L =  %3.2e  H  \n',L);
      fprintf('f =  %3.2f  Hz  \n',f);
      fprintf('Vin =  %3.2f  V  \n',Vin);
    disp('Calculations  ');
      fprintf('Vin =  %3.2f  V  \n',Vin);
      fprintf('VR =  %3.2f   V \n',abs(VR));
      fprintf('Vp =  %3.2f  V  \n',abs(Vp));
      fprintf('VR + Vp =  %3.2f  V  \n',abs(VR + Vp));
      disp('  ');
      fprintf('IR =  %3.2f  mA  \n',1e3*abs(IR));
      fprintf('IC =  %3.2f  mA  \n',1e3*abs(IC));
      fprintf('IL  =  %3.2f  mA  \n',1e3*abs(IL));
      fprintf('IC + IL =  %3.2f  \n',1e3*abs(IC+IL));
      fprintf('thetaR / pi =  %3.2f  \n',thetaR/pi);
      fprintf('thetaC / pi=  %3.2f  \n',thetaC/pi);
      fprintf('thetaL / pi  =  %3.2f  \n',thetaL/pi);
      fprintf('resonance frequency f0 =  %3.2f Hz
\n',f0);
```

Output in Command Window

```
Inputs
R =  1000.00  ohms
C =  1.00e-05  F
L =  1.00e-01  H
f =  50.00  Hz
Vin =  100.00  V
Calculations
Vin =  100.00  V
VR =  99.94   V
Vp =  3.48  V
VR + Vp =  100.00  V

IR =  99.94  mA
IC =  10.94  mA
```

```
IL  =  110.88  mA
IC + IL =  99.94
thetaR / pi =  -0.01
thetaC / pi=  0.99
thetaL / pi  =  -0.01
resonance frequency f0 =  1000.00 Hz
```

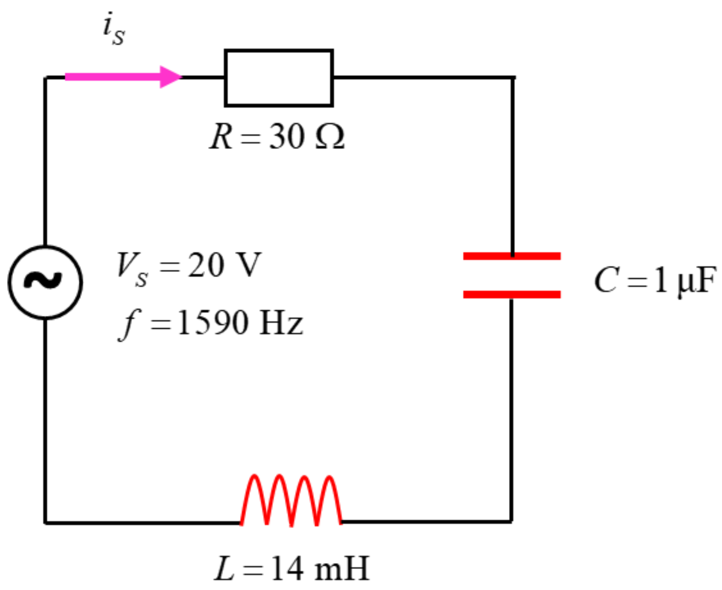
- The phase difference between the currents in the capacitor branch and the inductor branch is π rad.
- The numerical results show that Kirchhoff's Voltage and Current Laws are satisfied. You can not simply add voltages or currents. You must take into account the magnitude and phase of each voltage or current. Voltages and currents add like vectors.

Instantaneous emf and currents;

$$emf = 100\cos(314\,t)\ \mathrm{V}$$
$$i_R = 99.94\cos(314t - 0.01)\ \mathrm{mA}$$
$$i_C = 10.94\cos(314t + 0.99)\ \mathrm{mA}$$
$$i_L = 110.88\cos(314t - 0.01)\ \mathrm{mA}$$

Problem 1 (Cell 1)

Find the magnitude of the current and its phase for the circuit shown in the following figure.




```
%%  CELL 1
clear all
close all
clc

  R = 30
  L = 14e-3
  C = 1e-6
  VS = 20
  f = 1590
```

```
w = 2*pi*f
ZR = R
ZC = -1j/(w*C)
ZL = 1j*w*L
Z = ZR + ZC + ZL

iS = VS/Z
IS = abs(iS)
theta = rad2deg(angle(iS))
```

All the values are displayed in the Command Window.

The magnitude of the current is 0.0415 A and the current lags the source emf by 53^{o}.

Problem 2 (Cell 2)

For the RL circuit shown, find the magnitude (rms values) and phases with respect to the emf of each of the following

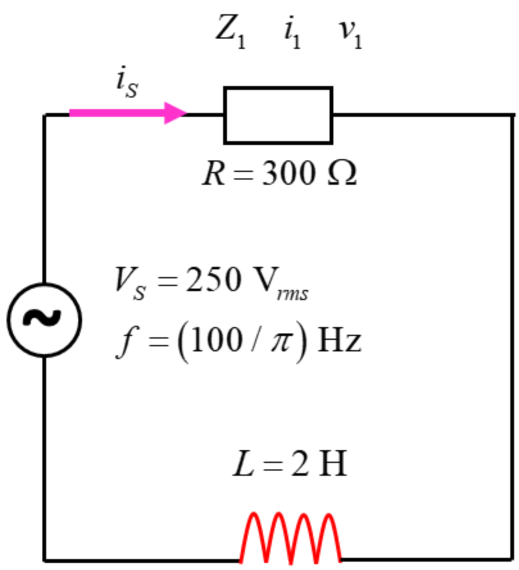
Current

Potential difference across resistor

Potential difference across inductor

Calculate the average power supplied by the source.

Draw a phasor diagram ("vector diagram") showing the source emf, voltage across the resistor and voltage across the inductor.



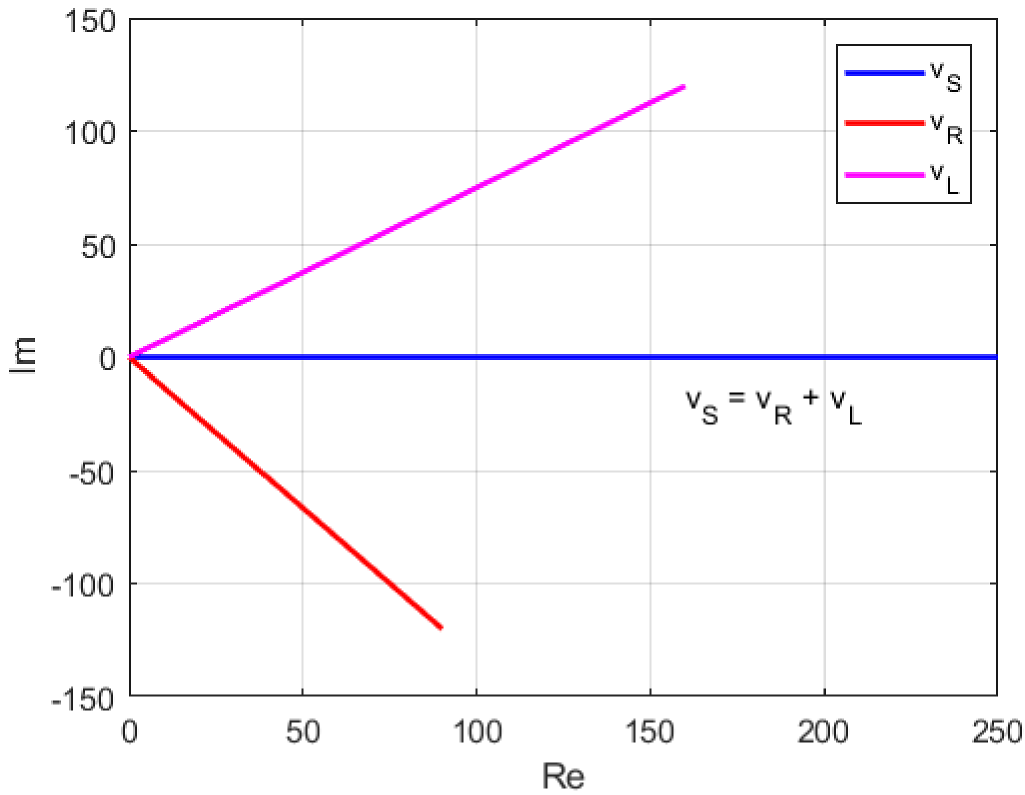

```
%% CELL 2
close all
clear all
clc

  R = 300
  L = 2
```

```
VS = 250
f = 100/pi

w = 2*pi*f
ZR = R
ZL = 1j*w*L
Z = ZR + ZL
iS = VS/Z
vR = iS * ZR
vL = iS * ZL

IS = abs(iS)
 theta = rad2deg(angle(iS))

VR = abs(vR)
phiR = rad2deg(angle(vR))

VL = abs(vL)
phiL = rad2deg(angle(vL))

P = real(iS) * VS
```

All the values are displayed in the Command Window (S.I. units and rms values) .

```
R = 300

L = 2

VS = 250

f = 31.8310

w = 200
```

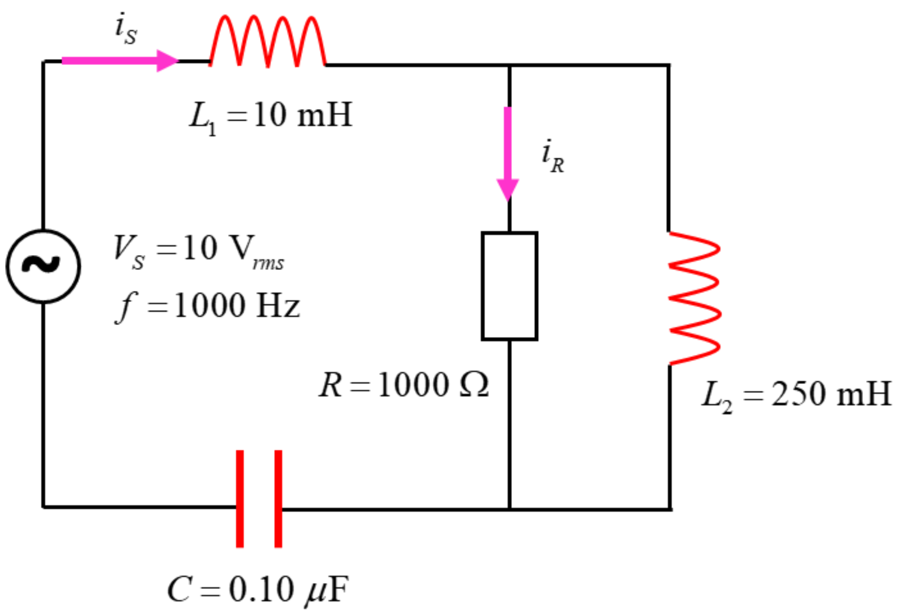
```
ZR = 300

ZL = 0.0000e+00 + 4.0000e+02i

Z = 3.0000e+02 + 4.0000e+02i

iS = 0.3000 - 0.4000i

vR = 9.0000e+01 - 1.2000e+02i

vL = 1.6000e+02 + 1.2000e+02i

IS = 0.5000

theta =  -53.1301

VR = 150

phiR = -53.1301

VL = 200

phiL = 36.8699

P = 75
```


150
100
50
0
-50
-100
-150
Im
0
50
100
150
200
250
Re
v_S
v_R
v_L
v_S = v_R + v_L

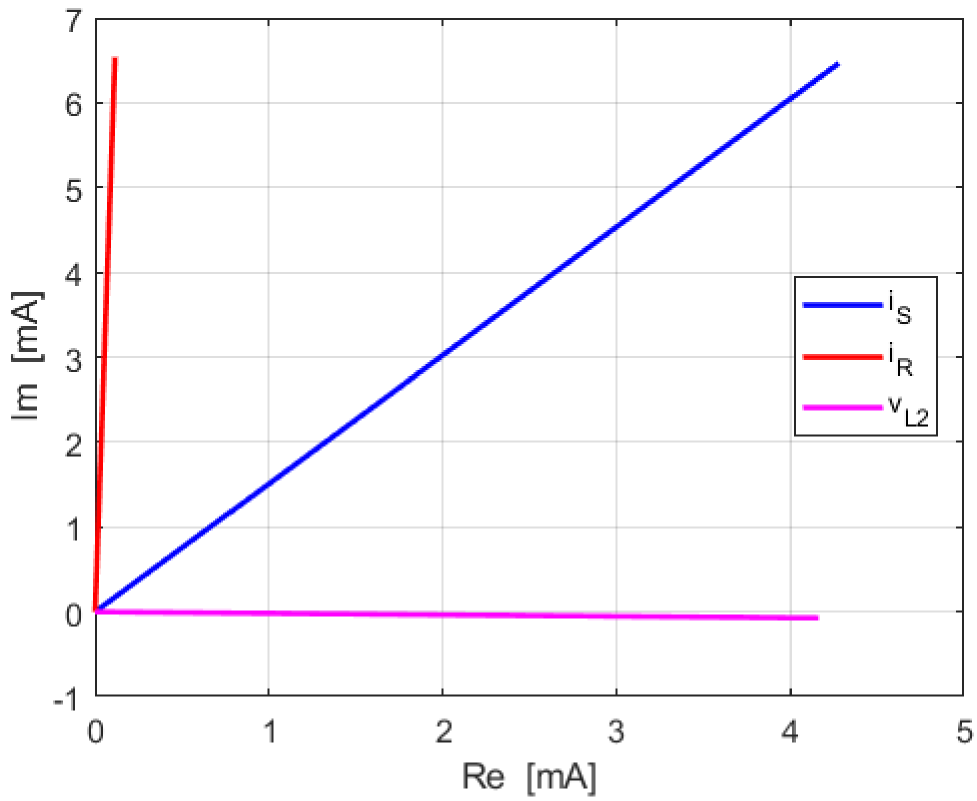
Problem 3 (Cell 3)

For the circuit shown in the diagram, calculate the magnitude and phases for the source current, the currents through the resistor and inductor L_2.

Draw a phasor diagram showing the source current, resistor current and inductor L_2 current.



Run the script to see the results in the Command Window.



Problem 4 (Cell 4)

Three impedances of $(2+j4)\,\Omega\,(2+j4)\,\Omega$, $(2+j1)\,\Omega$ and $(1-j3)\,\Omega$ are all connected in parallel. The parallel combination is connected in series with a coil of 3 Ω and reactance 2 Ω and to a 200 V ac source emf.

Find:

The total circuit impedance

The source current

The power factor

Run Cell 4 and view answers in Command Window