DOINGPHYSICS WITH MATLAB

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A COMPUTATIONAL APPROACH TO ELECTROMAGNETIC THEORY

CHAPTER 1

VECTOR ANALYSIS

The study of electromagnetism requires a considerable understanding of vectors and vector operations. Chapter 1 presents an overview the mathematics of vectors which is essential in learning and using electromagnetic theory. Matlab will be used extensively through the notes as a tool to enhance the appreciation and learning of electromagnetic theory.

DOWNLOAD DIRECTORIES FOR MATLAB SCRIPTS

Google drive

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cemVectorsS.m

Conversion between Cartesian and cylindrical and spherical components

cemVectorsA.m

Inputs: Cartesian components of the vector $\vec{V}$

Outputs: cylindrical and spherical components and [3D] plot of vector

cemVectorsB.m

Inputs: Cartesian components of the vectors A B C

Outputs: dot products, cross products and triple products

cemVectorsC.m

Rotation of XY axes around Z axis to give new of reference X'Y'Z'.

Inputs: rotation angle and vector (Cartesian components) in XYZ frame

Outputs: Cartesian components of vector in X'Y'Z' frame

The mscript can be modified to calculate the rotation matrix for a [3D] rotation and give the Cartesian components of the vector in the X'Y'Z' frame of reference.

DrawArrow.m

You can add an arrow to a Figure Window (plot) by calling the function **DrawArrow.m**

```
DrawArrow(zT,magV,angleV,L,W,LW,col)

% zT       position of vector tail (x,y): complex number zT = x + 1i*y
% magR     magnitude of vector
% angleV   angle of vector  [rad]
% L        length of arrow head
% W        width of arrow head
% LW       vector line width
% col      vector color
```

1.1 VECTOR ALGEBRA

A **scalar** is characterised by a number called its **magnitude**. For example

mass m = 10 kg

A **vector** is a physical quantity that is specified both by its **magnitude** and **direction**. For example

Force $\vec{F}$ Electric field $\vec{E}$ Magnetic field $\vec{B}$

A vector can be specified in terms of its **Cartesian** or **cylindrical** (**polar** in [2D]) or **spherical coordinates**.

XYZ right-handed rectangular Cartesian coordinate system: if we curl our fingers on the right hand so they rotate from the X axis to the Y axis then the Z axis is in the direction of the thumb.

A vector $\vec{V}$ in specified in terms of its X, Y and Z Cartesian components as

$$\vec{V}\left(V_x, V_y, V_z\right) \qquad \vec{V} = V_x\,\hat{i} + V_y\,\hat{j} + V_z\,\hat{k}$$

where $\left(\hat{i}, \hat{j}, \hat{k}\right)$ are **unit vectors** parallel to the X, Y and Z axes respectively.

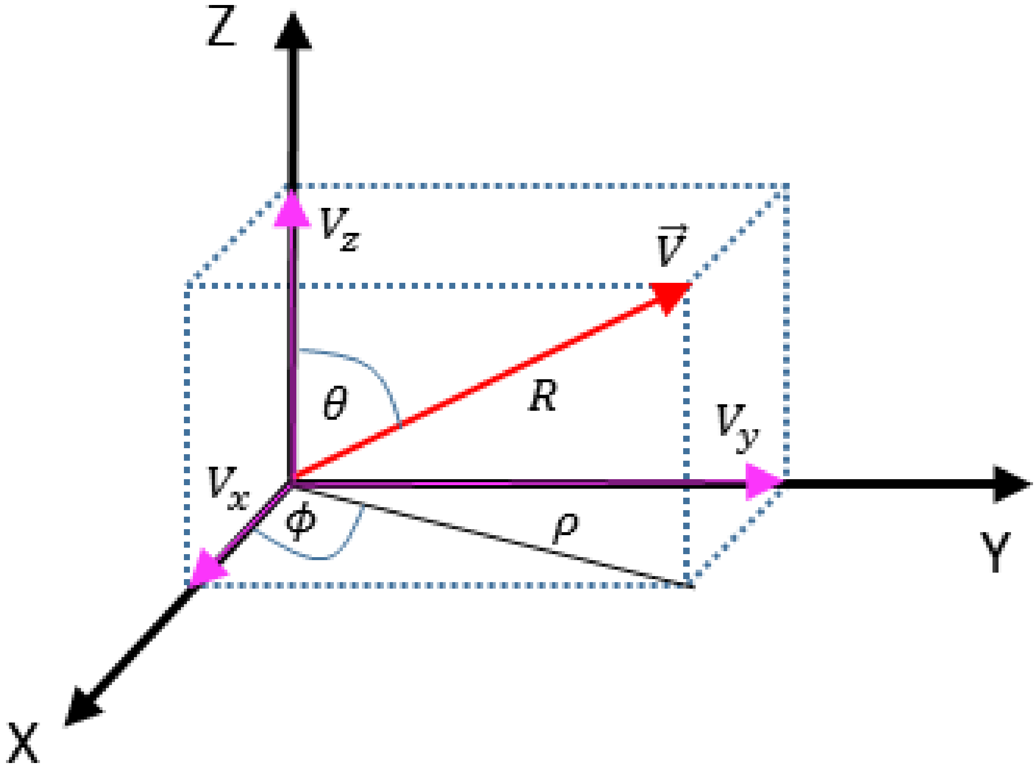


Fig. 1. A vector in an orthogonal Cartesian coordinate system.

The **polar angle** θ is the angle down from the Z axis to the vector $\vec{V}$.

Polar angle $0 \leq \theta \leq \pi$

The **azimuthal angle** ϕ is the angle around from the X axis.

Azimuthal angle $0 \leq \phi \leq 2\pi$ or $-\pi \leq \phi \leq +\pi$

Angles can be measured in radians or in degrees where $2\pi \text{ rad} = 360^\circ$

You can use the Matlab functions **rad2deg** and **deg2rad** for the conversions between radians and degrees

deg2rad(30) → $30^o = 0.5236$ rad

rad2deg(pi) → $\pi = 180^o$

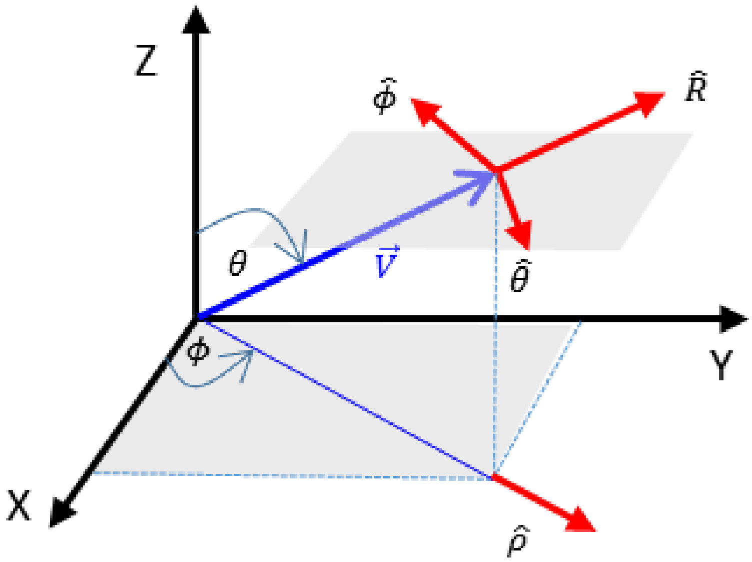


Fig. 2. The unit vectors $\hat{R}, \hat{\theta}, \hat{\phi}, \hat{\rho}$ pointing in the direction of an increase in the corresponding coordinate.

Cartesian components $\vec{V}(V_x, V_y, V_z)$ $\vec{V} = V_x\,\hat{i} + V_y\,\hat{j} + V_z\,\hat{k}$

Cylindrical components $\vec{V}(V_\rho, V_\phi, V_z)$ $\vec{V} = V_\rho\,\hat{\rho} + V_\phi\,\hat{\phi} + V_z\,\hat{k}$

Spherical components $\vec{V}(V_R, V_\theta, V_\phi)$ $\vec{V} = V_R\,\hat{R} + V_\theta\,\hat{\theta} + V_\phi\,\hat{\phi}$

Polar components [2D] $\vec{V}(V_\rho, V_\phi)$ $\vec{V} = V_\rho\,\hat{\rho} + V_\phi\,\hat{\phi}$

Magnitudes

$$\left|\vec{V}\right| \equiv V \equiv R = \sqrt{V_x^{\,2} + V_y^{\,2} + V_z^{\,2}}$$

$$\rho = \sqrt{V_x^{\,2} + V_y^{\,2}}$$

Relationship between coordinates from figure 2

$$V_x = R\sin\theta\cos\phi \qquad V_y = R\sin\theta\sin\phi \qquad V_z = R\cos\theta$$

$$V_x = \rho\cos\phi \qquad V_y = \rho\sin\phi \qquad V_z = V_z$$

$$\tan\phi = \frac{V_y}{V_x} \qquad \tan\theta = \frac{\rho}{V_z} \qquad \cos\theta = \frac{V_z}{R}$$

Spherical coordinates

$$\hat{R} = \sin\theta\,\cos\phi\;\hat{i} + \sin\theta\,\sin\phi\;\hat{j} + \cos\theta\;\hat{k}$$

$$\hat{\theta} = \cos\theta\,\cos\phi\;\hat{i} + \cos\theta\,\sin\phi\;\hat{j} - \sin\theta\;\hat{k}$$

$$\hat{\phi} = -\sin\phi\;\hat{i} + \cos\theta\,\sin\phi\;\hat{j}$$

Cylindrical coordinates

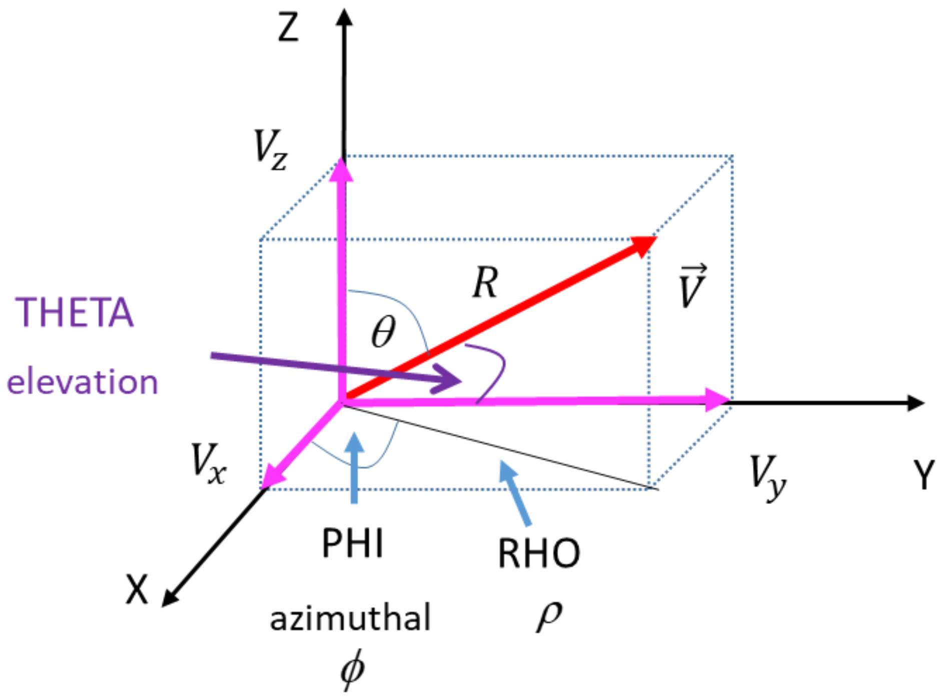
$$\hat{\rho} = \cos\phi\;\hat{i} + \sin\phi\;\hat{j}$$

$$\hat{\phi} = -\sin\phi\;\hat{i} + \cos\theta\,\sin\phi\;\hat{j}$$

$$\hat{z} = \hat{z}$$

Matlab function: Change components

You can also use MATLAB functions to make the conversion between Cartesian, polar, cylindrical, and spherical coordinate systems.



Cartesian components

V_x → X V_y → Y V_z → Z $R = |\vec{V}|$

Polar components

ρ (rho) → RHO ϕ (phi) → PHI

Cylindrical componets

ρ (rho) → RHO ϕ (phi) → PHI V_z → Z

Spherical componets

R (R) → RHO ϕ (phi) → PHI θ (theta) → $\pi/2$ - THETA

THETA + $\theta = \pi/2$

Matlab functions

[PHI, RHO] = cart2pol(X, Y)

[X, Y] = pol2cart(PHI, RHO)

[PHI, RHO, Z] = cart2pol(X, Y, Z)

[X, Y, Z] = pol2cart(PHI, RHO, Z)

[PHI, THETA, R] = cart2sph(X, Y, Z)

[X, Y, Z] = sph2cart(PHI, THETA, R)

Matlab calculations cemVectorsS.m

Vector $\vec{V} = 3\hat{i} + 5\hat{j} - 6\hat{k}$

Azimuthal angle $0 \le \phi \le 2\pi$

Polar angle $0 \le \theta \le \pi$

```
% Cartesian to polar and cylindrical
% >>>>>  Enter the Cartesian components of the vector V
  V = [3; 5; -6];

  X = V(1); Y = V(2); Z = V(3);

  [PHI, RHO] = cart2pol(X, Y);
    disp('Cartesian to polar')
    rho = RHO
    phi = PHI
    phiD = rad2deg(phi)

  [PHI, RHO, Z] = cart2pol(X, Y, Z);
  disp('Cartesian to cylindrical')
```

```
    rho = RHO
    phi = PHI
    phiD = rad2deg(phi)
    Vz = Z

 %  Polar / Cylindrical to Cartesian
% >>>>>  Enter the cylindrical components of the vector V
  V = [1.0304, 5.831, -6];

  PHI = V(1); RHO = V(2); Z = V(3);

  [X, Y, Z] = pol2cart(PHI, RHO, Z);
   disp('cylindrical to Cartesian')
    Vx = X
    Vy = Y
    Vz = Z

% Cartesian to spherical
% >>>>>  Enter the Cartesian components of the vector V
  V = [3; 5; -6];

  X = V(1); Y = V(2); Z = V(3);

  [PHI, THETA, R] = cart2sph(X, Y, Z);
  disp('Cartesian to spherical')
    R
    phi = PHI
    phiD = rad2deg(phi)
    theta = pi/2 - THETA
    thetaD = rad2deg(theta)

 % Spherical to Cartesian
% >>>>>  Enter the spherical components of the vector V
  V = [8.3666, 2.3705, 1.0304];

  R = V(1);  THETA = pi/2 - V(2); PHI = V(3);

  disp('spherical to Cartesian')
  [X, Y, Z] = sph2cart(PHI, THETA, R);
    Vx = X
```

Vy = Y
Vz = Z

Answers
Vx = 3 Vy = 5 Vz = -6

ρ = 5.831 ϕ = 1.0304 rad = 59.0362 deg

R = 8.3666 θ = 2.3705 rad = 135.8186 deg

Figure 3 gives a [3D] plot of a vector plus a summary of the input values for the Cartesian components and the calculated spherical and cylindrical components of the vector using the **mscript cemVectorsA.m**.

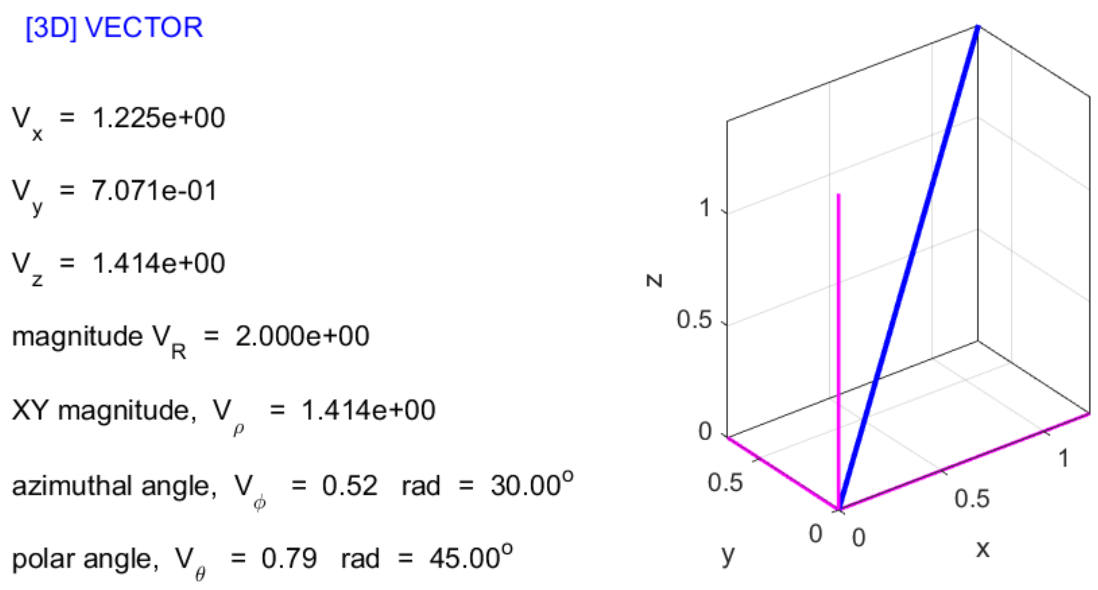


Fig. 3. Figure Window for a vector with inputs as the Cartesian components. **cemVectorsA.m**

1.2 VECTOR ALGEBRA

Addition / Subtraction / Scalar multiplication

To add or subtract vectors, you add or subtract the components. For multiplication of a vector by a scalar, simply multiply each component by the number for the scalar.

For example: consider the vectors in Cartesian coordinates

$$\hat{A}(1, 2, 3) \quad \hat{B}(-1, -3, -5) \quad \hat{C}(2, 4, -3)$$

$$\vec{V} = 3\vec{A} + \vec{B} - 2\vec{C}$$

$$\vec{V} = (3A_x + B_x - 2C_x)\hat{\imath} + (3A_y + B_y - 2C_y)\hat{\jmath} - (3A_z + B_z - 2C_z)\hat{k}$$

$$\vec{V} = (3 - 1 - 4)\hat{\imath} + (6 - 3 - 8)\hat{\jmath} - (9 - 5 + 6)\hat{k}$$

$$\vec{V} = -2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$$

$$\vec{V}(-2, -5, 10) \quad V_x = -2 \quad V_y = -5 \quad V_z = 10$$

Matlab Command Window

```
A = [1 2 3]
B = [-1 -3 -5]
C = [2 4 -3]
V = 3*A+B-2*C    →    V = [ -2   -5   10]
```

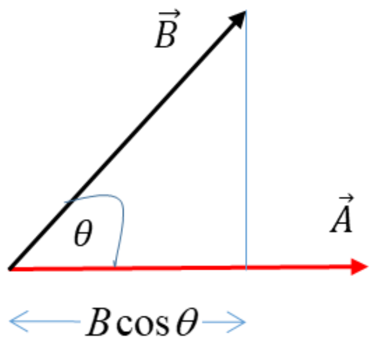
Dot product (scalar product) of two vectors

$$\vec{A}\bullet\vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

where θ is the angle between the two vectors when they are placed tail to tail

$\vec{A}\bullet\vec{B}$	scalar
$\vec{A}\bullet\vec{B} = \vec{B}\bullet\vec{A}$	commutative
$\vec{A}\bullet\left(\vec{B}+\vec{C}\right) = \vec{A}\bullet\vec{B}+\vec{A}\bullet\vec{C}$	distributive

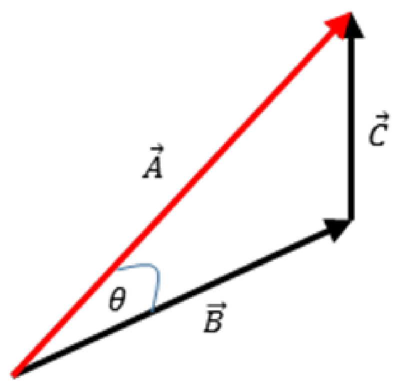
Geometrically $\vec{A}\bullet\vec{B}$ is the product of $\vec{A}$ times the projection of $\vec{B}$ along $\vec{A}$ or the product of $\vec{B}$ times the projection of $\vec{A}$ along $\vec{B}$



Angle between the two vectors

$$\cos\theta = \frac{\vec{A}\bullet\vec{B}}{AB} = \frac{A_x B_x + A_y B_y + A_z B_z}{AB}$$

Law of cosines



$$\vec{A} = \vec{B} + \vec{C} \qquad \vec{C} = \vec{A} - \vec{B}$$

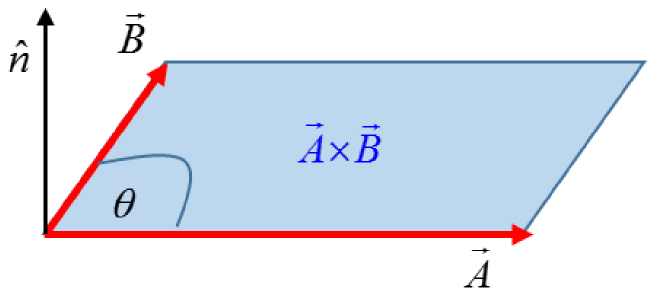
$$\vec{C} \bullet \vec{C} = \left(\vec{A} - \vec{B}\right) \bullet \left(\vec{A} - \vec{B}\right) = \vec{A} \bullet \vec{A} + \vec{B} \bullet \vec{B} - 2\vec{A} \bullet \vec{B}$$

$$C^2 = A^2 + B^2 - 2AB\cos\theta$$

Matlab function dot product is **dot(A,B)** where A and B are row vectors or column vectors of the same length.

Cross product or vector product of two vectors

$$\vec{A} \times \vec{B} = AB\sin\theta\ \hat{n}$$



where θ is the angle between the two vectors placed tail to tail and $\hat{n}$ is a unit vector that is normal to the plane defined by the two vectors and whose direction is determined by the right-hand rule (fingers curl from $\vec{A}$ to $\vec{B}$ then extended thumb points in direction of $\hat{n}$).

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$ non-commutative

$$\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B} + \vec{A}\times\hat{C}$$ distributive

$$\vec{A}\,||\,\vec{B} \quad \Rightarrow \quad \theta = 0 \quad \vec{A}\times\vec{B} = 0$$

$$\vec{A}\times\vec{A} = 0$$

$$\vec{A}\perp\vec{B} \quad \Rightarrow \quad \theta = \pi / 2\ rad \quad \left|\vec{A}\times\vec{B}\right| = A\,B$$

The cross product is the vector area of the parallelogram having $\vec{A}$ and $\vec{B}$ on adjacent sides.

Determinant form

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \left(A_y\,B_z - A_z B_y\right)\hat{i} + \left(A_z\,B_x - A_x B_z\right)\hat{j} + \left(A_x\,B_y - A_y B_x\right)\hat{k}$$

Matlab function **cross(A,B)** returns the cross product of the vectors A and B. A and B must be 3 element vectors.

Triple Products

Examples of triple products of three vectors

$$\vec{A}\bullet\left(\vec{B}\times\vec{C}\right) = \vec{B}\bullet\left(\vec{C}\times\vec{A}\right) = \vec{C}\bullet\left(\vec{A}\times\vec{B}\right)$$

$$\vec{A}\bullet\left(\vec{C}\times\vec{B}\right) = \vec{B}\bullet\left(\vec{A}\times\vec{C}\right) = \vec{C}\bullet\left(\vec{B}\times\vec{A}\right)$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}\bullet\left(\vec{A}\bullet\vec{C}\right) - \vec{C}\bullet\left(\vec{A}\bullet\vec{B}\right)$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) \neq \left(\vec{A}\times\vec{B}\right)\times\vec{C}$$

Calculations of triple products are given in the following examples. You should compare the numerical outputs with the above relationships.

Results of running the mscript cemVectorsB.m

Inputs vectors

A = [1 0 1]   B = [0 1 1]   C = [2 4 5]

Outputs

Magnitudes of vectors

```
Amag = norm(A)
```

Amag = 1.4142   Bmag = 1.4142   Cmag = 6.7082

Dot products

```
AdotB = dot(A,B)
```

AdotA = 2   AdotB = 1   BdotA = 1   BdotC = 9   CdotA = 7

Cross products

```
AB = cross(A,B)
AA = 0
```

AB = [-1 -1 1]   BA = [1 1 -1]   BC = [1 2 -2]   CA = [4 3 -4]

Cross products: magnitudes

```
ABmag = norm(AB)
```

ABmag = 1.7321   BAmag = 1.7321   BCmag = 3   CAmag = 6.4031

Angles between vectors

```
ABangle = asin(norm(AB) /(Amag * Bmag))
ABangle_deg = rad2deg(ABangle)
```

ABangle = 1.0472 rad = 60.0000 deg

BAangle = 1.0472 rad = 60.0000 deg

BCangle = 0.3218 rad = 18.4349 deg

CAangle = 0.7409 rad = 42.4502 deg

Triple products (cross product of vectors A and B written as AB)

```
AdotBC = dot(A,cross(B,C))
```

AdotBC = -1

BdotCA = -1

CdotAB = -1

AdotCB = 1

BdotAC = 1

CdotBA = 1

$\vec{A}\times\left(\vec{B}\times\vec{C}\right)$ `cross(A,cross(B,C))` → [-2 3 2]

$\vec{B}\bullet\left(\vec{A}\bullet\vec{C}\right)-\vec{C}\bullet\left(\vec{A}\bullet\vec{B}\right)$

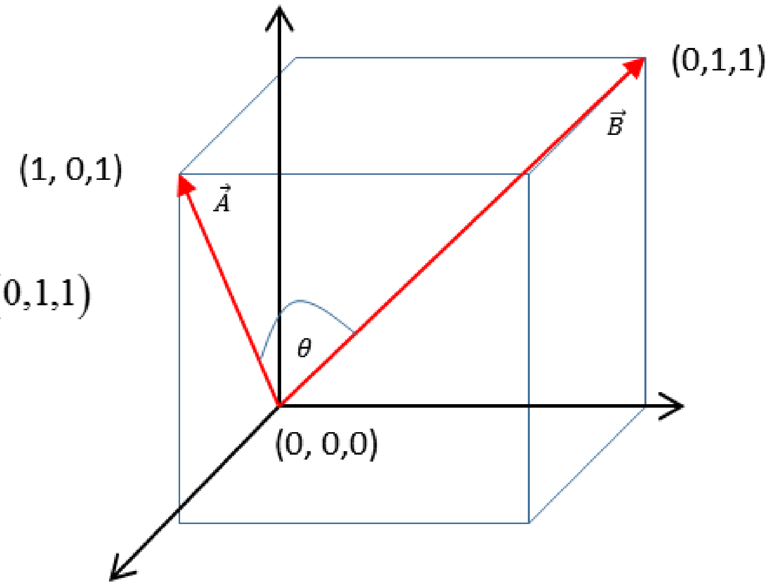
`B .* dot(A,C) - C .* dot(A,B)` → [-2 3 2]

$\left(\vec{A}\times\vec{B}\right)\times\vec{C}$

`ABcrossC = cross(cross(A,B),C)` → [-9 7 -2]

Example Find the angle between the face diagonals of a cube

$\vec{A}(1,0,1)$ $\vec{B}(0,1,1)$



⇒

The angle between the two vectors can be found from the cross product of the two vectors

$$\vec{A}\times\vec{B}=AB\sin\theta\ \hat{n}$$

$$\sin\theta=\frac{\left|\vec{A}\times\vec{B}\right|}{\left|\vec{A}\right|\left|\vec{B}\right|}$$

Run the mscript **cemVectorsB.m**

A = [1 0 1] B = [0 1 1]

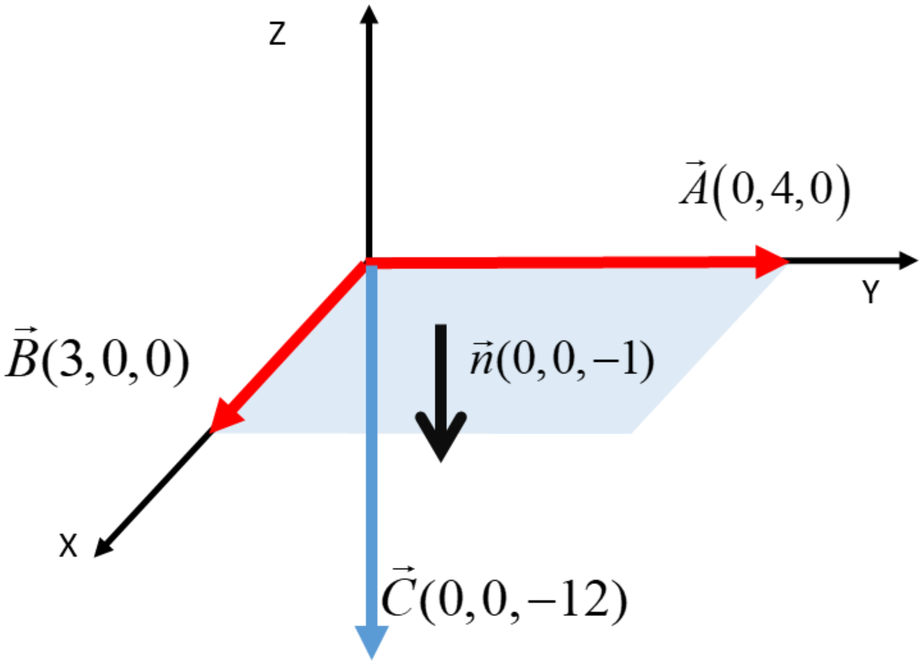
angle is θ = 1.0472 rad = 60.0000 deg

⇐

Example Find the components of the unit vector $\hat{n}$ perpendicular to the shaded regions formed by the vectors $\vec{A}$ and $\vec{B}$

$$\vec{C} = \vec{A} \times \vec{B} = -12\hat{k} = \left|\vec{C}\right|\hat{n} \quad n = \frac{\vec{C}}{\left|\vec{C}\right|} = \frac{-12\hat{k}}{12} = -\hat{k}$$

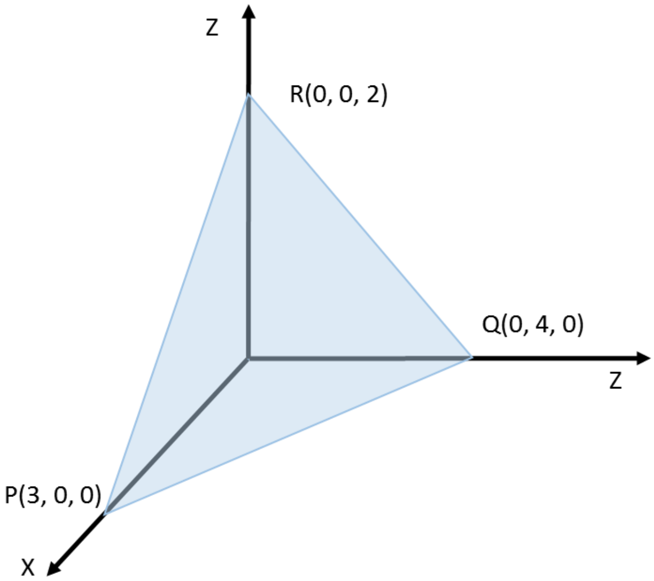
$$n_x = 0 \quad n_y = 0 \quad n_z = -1$$



A = [0 4 0]; B = [3 0 0];

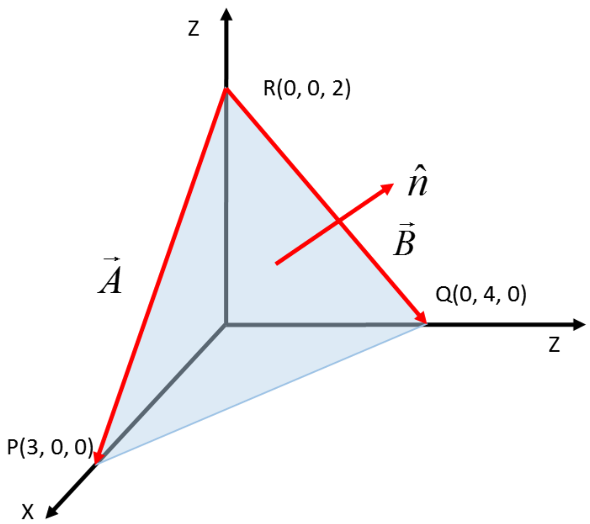
C = cross(A,B) → C = [0 0 -12]

Example Find the components of the unit vector $\hat{n}$ perpendicular to the shaded regions formed by the points R, P, Q.



Let $\vec{A}$ be the vector pointing from R to P and $\vec{B}$ be the vector pointing from R to Q. Then the vectors are $\vec{A}(3,0,-2)$ and $\vec{B}(0,4,-2)$.

$$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$$



Matlab Command Window

A = [3 0 -2] B = [0 4 -2]

C = cross(A,B)

C = [8 6 12]

Cmag = norm(C)

Cmag = 15.6205

n = C./Cmag

n = [0.5121 0.3841 0.7682] The Cartesian components of the vector $\hat{n}$ are (0.5121, 0.3841, 0.7682)

1.3 TRANSFORMATION OF COORDINATES DUE TO ROTATION

What is the change in the components of a vector due to a rotation of the coordinate system from X Y Z to X 'Y 'Z'?

The transformation matrix $\mathbf{R}$ due to a rotation uses the following notation

$$X \,\&\, X' \to 1 \quad Y \,\&\, Y' \to 2 \quad Z \,\&\, Z' \to 3$$

θ_{11} is the angle between axes X' and X

θ_{32} is the angle between axes Z' and Y

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \quad \text{where}$$

$$R_{nm} = \cos\left(\theta_{mn}\right) \qquad m = 1,2,3 \quad n = 1,2,3$$

$\vec{V}\left(V_x, V_y, V_z\right)$ in the XYZ coordinate system

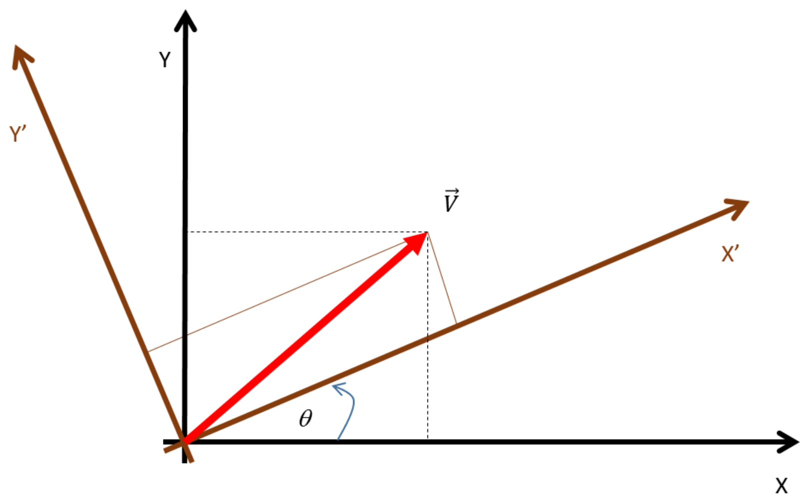
$\vec{V}'\left(V'_x, V'_y, V'_z\right)$ in the X'Y'Z' coordinate system

$$\vec{V}' = \mathbf{R}\vec{V}^T \quad \text{where} \quad \vec{V}^T = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

The mscript **cemVectorsC.m** can be modified to calculate the rotation matrix and the components of the vector in the X'Y'Z' frame of reference.

Rotation of the XY axes around the Z axis

Consider the vector $\vec{V} = V_x\,\hat{i} + V_y\,\hat{j} + V_z\,\hat{k}$ in the unprimed frame of reference. What will be its components in the primed frame of reference that is rotated by an angle θ in an anticlockwise direction in the XY plane?



Angles between the axes XYZ and X'Y'Z'

$$\theta_{11} = \theta \qquad \theta_{12} = 90^o - \theta \qquad \theta_{13} = 90^o$$

$$\theta_{21} = \theta + 90^o \qquad \theta_{22} = \theta \qquad \theta_{23} = 90^o$$

$$\theta_{31} = 90^o \qquad \theta_{32} = 90^o \qquad \theta_{33} = 0^o$$

Transformation of vector components

$$\left(V'_x, V'_y, V'_z\right) = \begin{pmatrix} \cos(\theta) & \cos(90^o - \theta) & \cos(90^o) \\ \cos(\theta + 90^o) & \cos(\theta) & \cos(90^o) \\ \cos(90^o) & \cos(90^o) & \cos(0^o) \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$\left(V'_x, V'_y, V'_z\right) = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$V'_x = \cos(\theta)V_x + \sin(\theta)V_y$$
$$V'_y = -\sin(\theta)V_x + \cos(\theta)V_y$$
$$V'_z = V_z$$

Example

V(2, 3, 0) in XYZ frame of reference

XYZ rotated by 30° anticlockwise in XY plane to give the X'Y'Z' frame

What are the components of V in the X'Y'Z' frame?

⇒

Run the mscript **cemVectorsC.m**

Inputs: V = [2 3 0] theta = 30

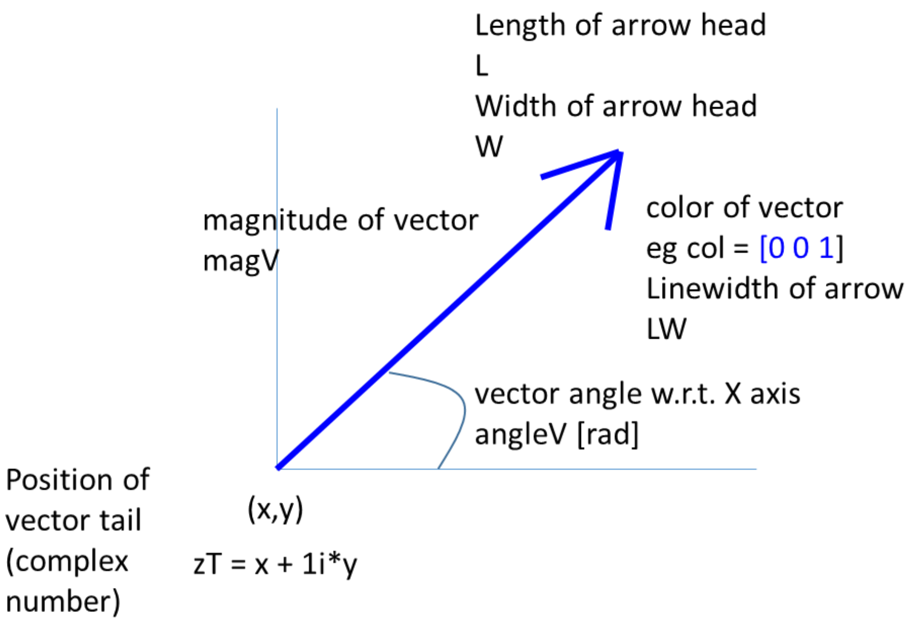
Output displayed in Command Window: Vdash = [3.2321 1.5981 0]

⇐

1.4 ADDING ARROWS TO A PLOT

Often you want to add arrows to a plot. This can be done using the Matlab Script **DrawArrow.m**

DrawArrow(zT,magV,angleV,L,W,LW,col)



DrawArrow(1+1i,2,1,0.51,0.1,2,'b')

zT = 1+1i   magV = 2   angleV = 1   L = 0.51   W = 0.1   LW = 2   col = ‘b’