DOING PHYSICS WITH MATLAB

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A COMPUTATIONAL APPROACH TO ELECTROMAGNETIC THEORY

CHAPTER 4

VECTOR ANALYSIS

PROBLEMS AND SOLUTION USING MATLAB

DOWNLOAD DIRECTORIES FOR MATLAB SCRIPTS

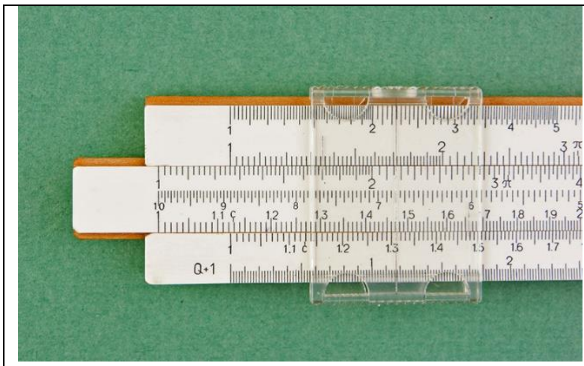
Google drive

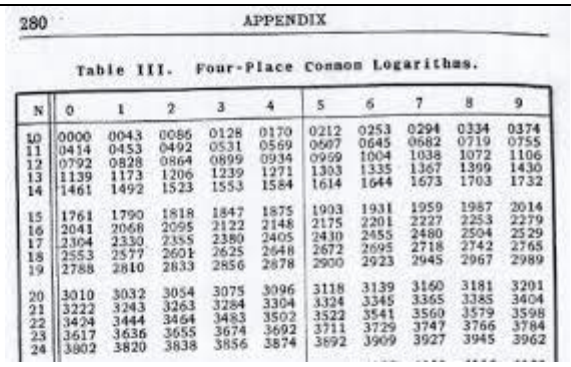
GitHub

Script

cemCh4Problems.m

We no longer use slides rule or log tables. With computer and software access available to everyone, we should be approaching the solving of traditional physics problems in an “up-to-date” fashion.





280 APPENDIX

Table III. Four-Place Common Logarithms.

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962

More realistic and more challenging problems should be encountered. Matlab is the “perfect” tool to solve many problems in electromagnetism. This Chapter will show how many traditional problems can be solved using Matlab. Many of the problems are taken from the excellent texts

Robert H. Good *Classical Electromagnetism*

David J. Griffiths *Introduction to Electrodynamics* (3rd Edition)

PROBLEM 1

Given the two vectors $\vec{A}(1,1)$ and $B(-1,1)$

1.1 Find the vector $\vec{C} = \vec{A} - \vec{B}$

1.2 Find the dot product $\vec{C} \bullet \vec{C}$

1.3 Find the angle θ between the vectors $\vec{A}$ and $\vec{B}$

1.4 Verify the law of cosines $C^2 = A^2 + B^2 - 2AB\cos\theta$

SOLUTION 1

```
%% PROBLEM 1
close all; clear all; clc


A = [1 1]
B = [-1 1]
Amag = norm(A)  →  1.4142
Bmag = norm(B)  →  1.4142
AdotB = dot(A,B)  →  0°
theta = acosd(AdotB/(Amag*Bmag))  →  90°
C = A-B  →  [2 0)
Cmag = norm(C)  →  2
CdotC = dot(C,C)  → 4
LHS = Cmag^2      → 4
RHS = Amag^2 + Bmag^2 - 2*Amag*Bmag*cosd(theta) →4
```

PROBLEM 2

Given the four vectors $\vec{A}(1,1,1)$ $\vec{B}(2,-1,1)$ $\vec{C}(-2,1,1)$ $\vec{D}(2,2\,0)$

Is the cross product associative?

$$\left(\vec{A}\times\vec{B}\right)\times\vec{C} \quad \vec{A}\times\left(\vec{B}\times\vec{C}\right)$$

$$\left(\vec{A}\times\vec{B}\right)\times\vec{D} \quad \vec{A}\times\left(\vec{B}\times\vec{D}\right)$$

SOLUTION 2

A = [1 1 1]

B = [2 -1 1]

C = [-2 1 1]

D = [2 2 0]

AB_C = cross(cross(A,B),C) → [4 4 4]

A_BC = cross(A,cross(B,C)) → [4 -2 -2]

AB_D = cross(cross(A,B),D) → [6 -6 2]

A_BD = cross(A,cross(B,D)) → [4 -8 4]

The cross product is **not** associative

PROBLEM 3

A box has base of 4x3 and sides 3x1 and 4x1. Calculate the angle of base diagonal with the diagonals of the two side faces.

SOLUTION 3

A = [4 3 0]   % base diagonal

B = [0 4 1]   % side 1 diagonal

C = [3 0 1]   % side 2 diagonal

Amag = norm(A) → 5

Bmag = norm(B) → 4.1231

Cmag = norm(C) → 3.163

AdotB = dot(A,B) → 12

AdotC = dot(A,C) → 12

theta1 = acosd(AdotB/(Amag*Bmag)) → 54.4^o

theta2 = acosd(AdotC/(Amag*Cmag)) → 40.6^o

PROBLEM 4

Find the displacement vector from a source point (3,9,8) to a field point (5,7,9). What is the unit vector in the direction of the displacement vector?

SOLUTION 4

S = [3 9 8]   F = [5 7 9]

R = F – S → [2 -2 1]

Rmag = norm(R) → 3

Rhat = R./Rmag → [0.6667 -0.6667 0.3333]

PROBLEM 5

Find the gradient of the scalar displacement function

$$r = \left(x^2 + y^2 + z^2\right)^{1/2}$$

SOLUTION 5

syms r x y z drdx

r = sqrt(x^2 + y^2 + z^2) → (x^2 + y^2 + z^2)^(1/2)

drdx = diff(r,x) → x/(x^2 + y^2 + z^2)^(1/2)

drdy = diff(r,y) → y/(x^2 + y^2 + z^2)^(1/2)

drdz = diff(r,z) → z/(x^2 + y^2 + z^2)^(1/2)

$$\nabla r = \frac{x\hat{i} + y\,\hat{j} + z\,\hat{k}}{r} = \frac{\vec{r}}{r} = \hat{r}$$

This makes sense as the distance from the Origin increases rapidly in the radial direction and the maximum rate of increase is in that direction,

PROBLEM 6

Find the gradients of the functions $f(x,y,z)$

$$x^3 + y^4 + z^5$$
$$x\,y^2\,z^3$$
$$e^x \sin(y)\log(z)$$

SOLUTION 6

```
% comment the functions that are not used

syms f x y z drdx e

f = x^3 + y^4 + z^5   →  x^3 + y^4 + z^5

%f = x*y^2*z^3

%f = e^x*sin(y)*log(z)

drdx = diff(f,x)

drdy = diff(f,y)

drdz = diff(f,z)
```

$$\nabla\left(x^3 + y^4 + z^5\right) \;\to\; \left[3x^2 \;\; 4y^3 \; 5z^4\right]$$

$$\nabla\left(x\,y^2\,z^3\right) \;\to\; \left[y^2 z^3 \;\; 2xyz^3 \;\; 3x\,y^2 z^2\right]$$

$$\nabla\left(e^x \sin(y)\log(z)\right) \;\to$$

$$\left[e^x \log(e)\log(z)\sin(y) \quad e^x \cos(y)\log(z) \quad (e^x \sin(y))\,/\,z\right]$$

PROBLEM 7

Consider the displacement vector $\vec{r} = x\hat{i} + y\,\hat{j} + z\,\hat{k}$

Find ∇r^2 and $\nabla\left(\frac{1}{r}\right)$

SOLUTION 7

syms r1 r2 x y z

r2 = x^2 + y^2 + z^2   grad2_x = diff(r2,x)   grad2_y = diff(r2,y)

grad2_z = diff(r2,z)

r2 = x^2 + y^2 + z^2 → r^2

grad2_x = 2*x   grad2_y = 2*y   grad2_z = 2*z $\nabla\left(r^2\right) = 2\vec{r}$

r1 = 1/sqrt(r2)

grad1_x = diff(r1,x)   grad1_y = diff(r1,y)   grad1_z = diff(r1,z)

r1 = 1/(x^2 + y^2 + z^2)^(1/2) → $1/r$

grad1_x = -x/(x^2 + y^2 + z^2)^(3/2) → $-x/r^3$

grad1_y = -y/(x^2 + y^2 + z^2)^(3/2) → $-y/r^3$

grad1_z = -z/(x^2 + y^2 + z^2)^(3/2) → $-z/r^3$

$$\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3} = -\frac{1}{r^2}\hat{r}$$

Hence, we can conclude

$$\nabla\left(r^n\right) = n\,r^{n-1}\,\hat{r}$$

PROBLEM 8

A vector is given by

$$\vec{R} = x^2\,\hat{i} + 3xz^2\,\hat{j} - 2xz\,\hat{k}$$

Calculate its divergence, curl and Laplacian.

Calculate the Laplacian of the scalar function

$$A = x^2 + 2xy + 3z + 4$$

SOLUTION 8

syms x y z

R = [x^2*y*z^5   3*x*y^4*z^2   -2*x*z]

→ [x^2*y*z^5, 3*x*y^4*z^2, -2*x*z]

vars = [x y z];

divR = divergence(R,vars) → 12*x*y^3*z^2 + 2*x*y*z^5 - 2*x

crossR = cross(R,vars)

→ [3*x*y^4*z^3 + 2*x*y*z, - y*x^2*z^6 - 2*x^2*z,

x^2*y^2*z^5 - 3*x^2*y^4*z^2]

grad2_x = diff(R(1),x,2) → 2*y*z^5

grad2_y = diff(R(2),y,2) → 36*x*y^2*z^2

grad2_z = diff(R(3),z,2) → 0

$$\nabla^2\vec{R} = \left(2yz^5\right)\hat{i} + \left(36xy^2z^2\right)\hat{j} + 0\hat{k}$$

A = -2*sin(x^2)*sin(4*y)*sin(3*z^3)

lapA = laplacian(A) →

32*sin(x^2)*sin(4*y)*sin(3*z^3) - 4*cos(x^2)*sin(4*y)*sin(3*z^3) -

36*z*sin(x^2)*sin(4*y)*cos(3*z^3) +

8*x^2*sin(x^2)*sin(4*y)*sin(3*z^3) +

162*z^4*sin(x^2)*sin(4*y)*sin(3*z^3)

PROBLEM 9

Graph the curl of the vector $\vec{V} = -y\hat{i} + x\,\hat{j} + z\hat{k}$

Also calculate its divergence and curl.

SOLUTION 9

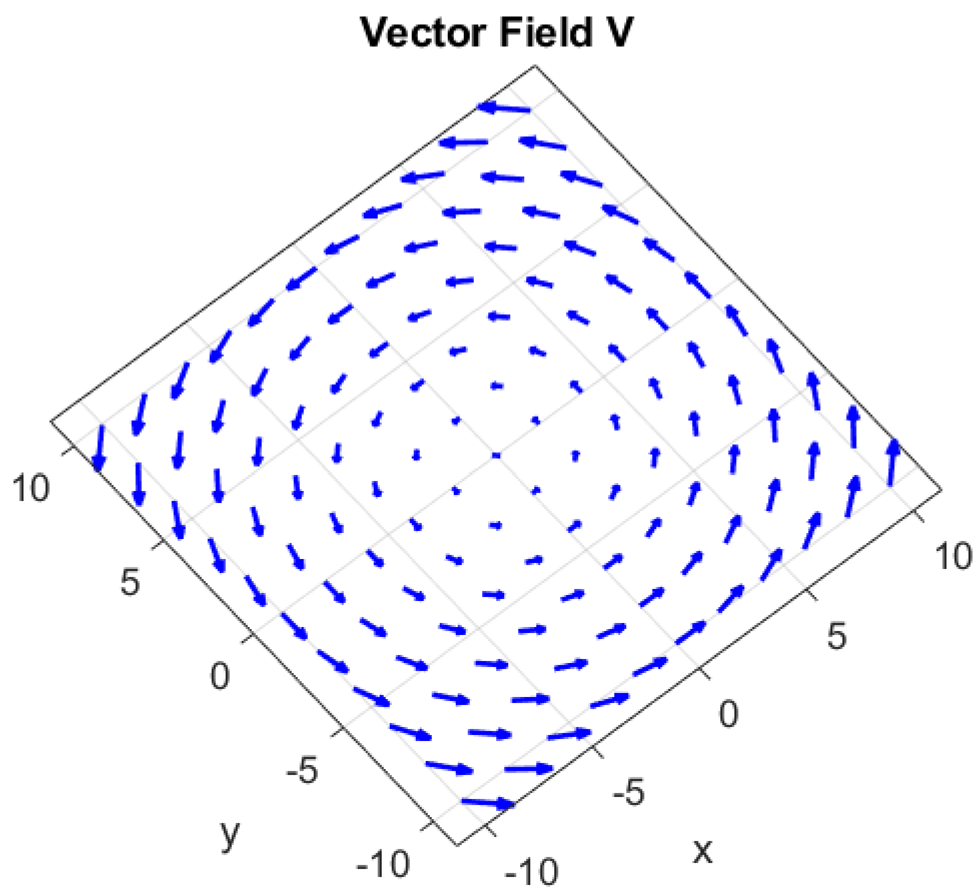
```
syms x y z
V = [-y x z]
vars = [x y z];
divV = divergence(V,vars)
crossV = cross(V,vars)

N = 201;
X = linspace(-10,10, N); Y = X; Z = X;
[xx, yy, zz] = meshgrid(X,Y,Z);
Vxx = -yy;   Vyy = xx;  Vzz = zz;

divV = divergence(xx, yy, zz, Vxx, Vyy, Vzz);
[curlVxx, curlVyy, curlVzz] = curl(xx, yy, zz, Vxx, Vyy, Vzz);
```

```matlab
% GRAPHICS
=============================================================
    minX = -10; minY = -10; maxX = 10; maxY = 10;
    dx = 1:20:N; dy = dx;   dz = 1;

    figure(1)
    set(gcf,'units','normalized','position',[0.05 0.2 0.3 0.4]);
    p1 = xx(dx,dy,dz); p2 = yy(dx,dy,dz); p3 = zz(dx,dy,dz);
    p4 = Vxx(dx,dy,dz); p5 = Vyy(dx,dy,dz); p6 = Vzz(dx,dy,dz);
    h = quiver3(p1, p2, p3, p4, p5, p6);
    set(h,'color',[0 0 1],'linewidth', 2);
    axis tight
    set(gca,'xLim',[minX, maxX]);
    set(gca,'yLim',[minY, maxY]);
    % set(gca,'zLim',[minZ, maxZ]);
    title('Vector Field V');
    xlabel('x'); ylabel('y'); zlabel('z');
    set(gca,'fontsize',14)
    view(-40,90)
    box on
    axis tight
```

V = [-y, x, z]

divV = 1

crossV = [x*z - y*z, x*z + y*z, - x^2 - y^2]


Vector Field V
10
5
0
-5
-10
y
10
5
0
-5
-10
x

PROBLEM 10

Calculate the divergence of the vector function

$$\vec{V} = -x\,y\,z\,\hat{i} + (x+y)\,z\,\hat{j} + x^3 y^5 z^6\,\hat{k}$$

SOLUTION 10

```
syms x y z
V = [-x*y*z  (x+y)*z  x^3*y^5*z^6]
vars = [x y z];
divV = divergence(V,vars)
```

```
V = [-x*y*z, z*(x + y), x^3*y^5*z^6]
divV = 6*x^3*y^5*z^5 - y*z + z
```

PROBLEM 11

A vector function is given by

$$\vec{V} = \left(\frac{1}{k}\right)\left(\cos(k\,x)\hat{i} + \sin(k\,y)\,\hat{j} + 0\,\hat{k}\right)$$

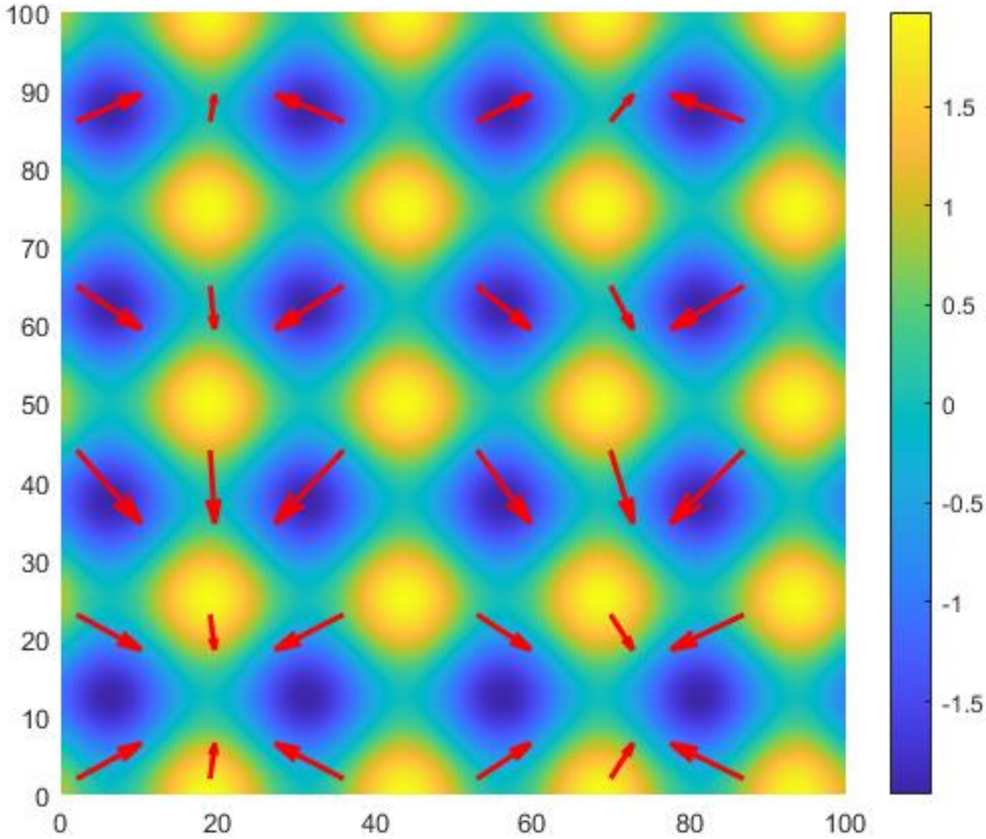
Calculate its divergence and plot the divergence function. Show the vector added to the plot using the quiver function.

SOLUTION 11

```
syms x y z k

V = [cos(k*x)/k (sin(k*y))/k 0]
vars = [x y z];
divV = divergence(V,vars)

lambda = 25;
k = 2*pi/lambda; num = 101;
X = linspace(0,100,num);
Y = X; Z = X;
[xx, yy, zz] = meshgrid(X,Y,Z);
Vxx = cos(k.*xx)/k;   Vyy = sin(k.*yy)/k;  Vzz =
zeros(num,num,num);
divV = divergence(xx, yy, zz, Vxx, Vyy, Vzz);
xP = xx(:,:,1); yP = yy(:,:,1); P = divV(:,:,1);

figure(1)
 set(gcf,'units','normalized','position',[0.05 0.2 0.3
0.4]);
 pcolor(xP,yP,P)
 shading("interp")
 colorbar; hold on

 xQ = 2:17:100; yQ = 2:21:100;

 [xxQ, yyQ] = meshgrid(xQ,yQ);
 Vx = cos(k*xxQ)./k; Vy = sin(k*yyQ)./k;

 h = quiver(xQ,yQ,Vx,Vy,'r','linewidth',2);
 set(h,'AutoScale','on', 'AutoScaleFactor',0.6)
```

V = [cos(k*x)/k, sin(k*y)/k, 0]

divV = cos(k*y) - sin(k*x)



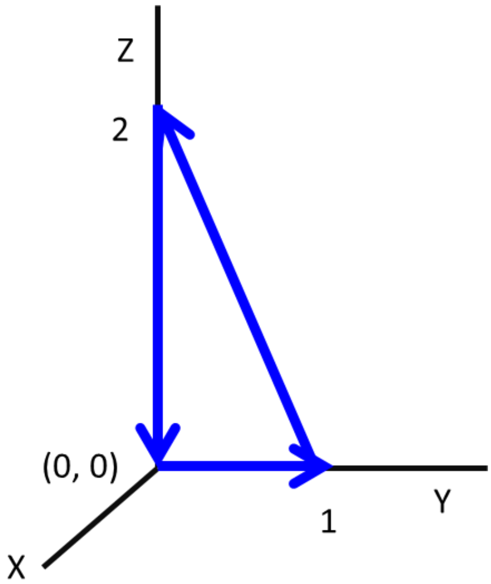
Note: the vector field spreads from areas of positive divergence (source) and is points in towards area of lower divergence values (sink).

PROBLEM 12

Compute the line integral of

$$\vec{V} = 6\hat{i} + y z^2 \hat{j} + (3y + z)\hat{k}$$

around the triangular path.



SOLUTION 12

Line integral

$$J = \int_{L_1}^{L_2} \vec{V} \bullet d\vec{L} = \int_{L_1}^{L_2} \left(V_x\, dx + V_y\, dy + V_z\, dz \right)$$

$$V_x = 6 \quad V_y = y z^2 \quad V_z = 3y + z$$

Path 1 $\quad x = 0 \quad z = 0 \Rightarrow J_1 = \int_0^1 V_y\, dy = \int_0^1 y z^2\, dy = 0$

Path 2 $\quad x = 0 \quad z = -2y + 2 \quad dz = -2dy$

$$J_2 = \int_1^0 \left(y(-2y+2)^2 + (y+2)(-2) \right) dy$$

Path 3 $\quad x = 0 \quad y = 0 \Rightarrow J_3 = \int_2^0 V_z\, dz = \int_2^0 z\, dz = -2$

We can use the Script **simpson1d.m** to evaluate the integral for path 2.

```
% Limits >>>>>
  xMin = 1;
  xMax = 0;

% X range
  num = 999;
  x = linspace(xMin,xMax,num);

% Function >>>>>
  F = x.*(-2.*x + 2).^2 -x -4;

% Value of integral
  S = simpson1d(F,xMin,xMax);
  fprintf('Integral S = %2.4f  \n',S)
```

Integral S = 4.1667

The line integral around the complete path is

$$J = 0 - 2 + 4.1667 = 2.1667$$