

DOING PHYSICS WITH MATLAB

ELECTRIC FIELD AND ELECTRIC POTENTIAL GAUSS'S LAW & ELECTRIC FLUX

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DOWNLOAD DIRECTORY FOR MATLAB SCRIPTS

Download and inspect the mscripts and make sure you can follow the structure of the programs.

cemVE015.m

Calculation of the total electric flux through the closed surface of a cube enclosing five charges randomly located inside the cube. The results of the simulation show clearly that Gauss's Law is satisfied.

simpson2d.m

[2D] computation of an integral using Simpson's rule. The function to be integrated must have an **ODD** number of the elements.

Function to give the integral of a function $f(x,y)$ using a two-dimensional form of Simpson's 1/3 rule. The format to call the function is

$$I_{xy} = \text{simpson2d}(f,ax,bx,ay,by)$$

http://www.physics.usyd.edu.au/teach_res/mp/doc/math_integration_2D.pdf

The electric flux ϕ_E of the electric field \vec{E} through a surface A is defined by the relationship

$$(1) \quad \phi_E = \int_A \vec{E} \cdot d\vec{A}$$

The dot product picks out the component of the electric field in the direction of the normal to the area. It is only the area in the plane perpendicular to \vec{E} that is important. Hence, we can expand the dot product given in equation 1 to give the flux as

$$(2) \quad \phi_E = \int_A \vec{E} \cdot d\vec{A} = \int_A (E_x dA_{yz} + E_y dA_{zx} + E_z dA_{xy})$$

where E_x, E_y, E_z are the Cartesian components of the electric field and dA_{yz} is an area element in the YZ plane and is perpendicular to the X axis. Similarly for dA_{zx} and dA_{xy} .

Gauss's Law states that for any closed surface the total flux Φ_E is proportional to the net charge $Q_{enclosed}$ enclosed by the surface.

$$(3) \quad \text{Gauss's Law} \quad \Phi_E = \oiint_A \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

Usually Gauss's Law is applied to situations in which symmetry plays an important role. However, using numerical integration techniques many more situations can be evaluated.

We will consider a set of 5 charges enclosed within a cubic volume element with side length $L = 10.0$ m. The flux through each of the six faces, aligned perpendicular to the Cartesian axes X, Y and Z is computed by evaluating the integral in equation 4 using a [2D] version of Simpson's rule

$$(4) \quad \phi = \pm \int_A E_n dA$$

E_n is the component of the electric field parallel to the normal to the surface. If the normal component of the electric field is in the direction of the outward normal then the + sign is used and if the normal component of the electric field is opposite to the direction of the outward normal, the – sign is used in equation 4.

The `meshgrid` function is used to setup a grid of NxN points for each square face of the cube. The normal component of the electric field from each charge is calculated at each grid point and then summed to give the net normal component of the electric field. The flux is then calculated using the function `simpson2d.m`. Once, the flux through each face has been determined, the sum of the fluxes gives the total flux Φ_E for the closed surface from which the charge enclosed is computed

$$(5) \quad Q_{enclosed} = \epsilon_0 \Phi_E$$

The charged enclosed $Q_{enclosed}$ is then compared to the sum of the charges Q_{total} inside the closed surface

$$(6) \quad Q_{total} = \sum_{n=1}^5 Q_n$$

On running the mscript `cemVE15.m`, with charges located at different location and using different values for the charges, the result always is in agreement with Gauss's Law

$$(7) \quad Q_{enclosed} = \epsilon_0 \Phi_E = Q_{total}$$

Section of the code to calculate the flux through one surface

```
% Side 5: nz0
p1 = G1; p2 = G2; p3 = 0;
E = zeros(N,N);
for n = 1 : NQ
    Rx = p1 - xQ(n);
    Ry = p2 - yQ(n);
    Rz = p3 - zQ(n);
    R = sqrt(Rx.^2 + Ry.^2 + Rz.^2);
    R3 = R.^3;
    E = E + (kC * Q(n) * Rz) ./R3;
end
flux(5) = -simpson2d(E,0,L,0,L);
```

Examples

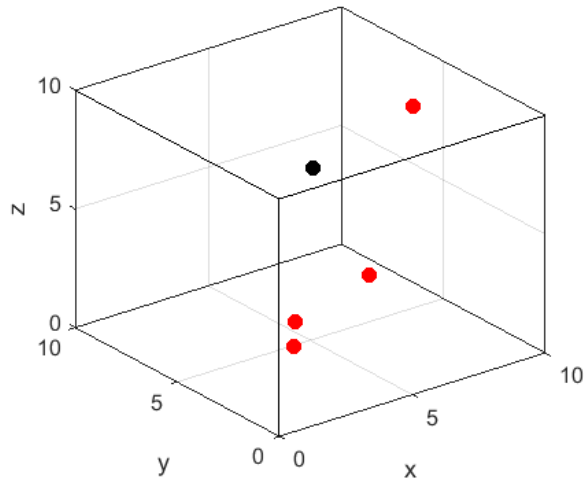
$Q_{\text{enclosed}} = 3.0000e-06$

$Q_{\text{total}} = 3.0000e-06$

flux =

1.0e+04 *

8.2153	6.9697	3.6703
5.6816	5.6700	3.6764



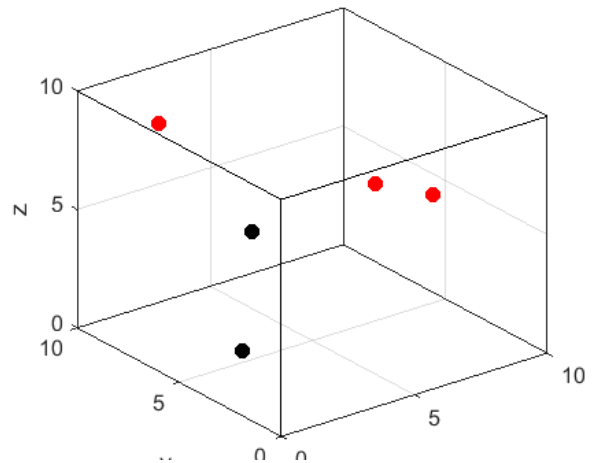
$Q_{\text{enclosed}} = -2.0000e-06$

$Q_{\text{total}} = -2.0000e-06$

flux =

1.0e+05 *

-2.2748	0.9457	-0.6398
0.0432	-0.6500	0.3168



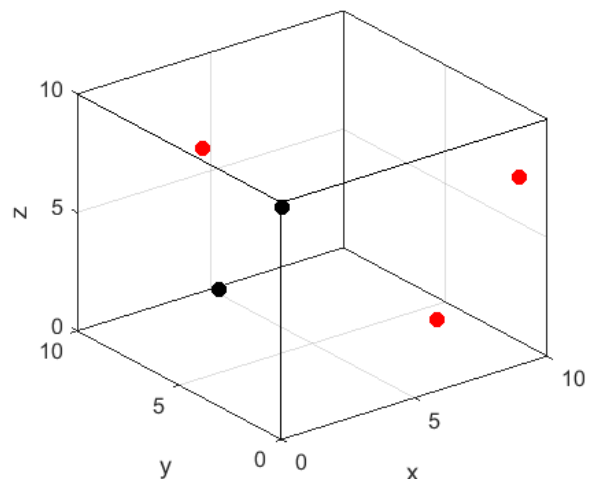
$Q_{\text{enclosed}} = -2.2242e-11$

$Q_{\text{total}} = 4.2352e-22$

flux =

1.0e+05 *

-0.3926	0.8684	1.6119
-1.0811	-1.2427	0.2360



If zero charge is enclosed then the total flux is also zero.

The flux can be thought of as the number of electric field lines passing through an element of area. Therefore, for a closed surface, the net number of field lines entering or exiting the closed area is proportional to the charge enclosed. The figure below shows the electrical potential and electric field lines for two positive charges: $Q_{left} = +20 \mu\text{C}$ and $Q_{right} = +60 \mu\text{C}$. The direction of the electric fields is away from the positive charges (red disks) and towards the boundaries of the [2D] space. The plot was done using the mscript [cemVE05.m](http://www.physics.usyd.edu.au/teach_res/mp/doc/cemVE05.m).

For more information on the visualization of the potential and electric field due to a variety of charge distribution go to:

http://www.physics.usyd.edu.au/teach_res/mp/doc/cemVEA.pdf

In the plot below, a red box represents a volume element.

Box 1: Net field lines entering or exiting box = 0 $\Rightarrow Q_{enclosed} = 0$

Box 2 surrounds Q_{left} :

Net field lines entering or exiting box = 10 $\Rightarrow Q_{enclosed} = +20 \mu\text{C}$

Box 3 surrounds Q_{right} :

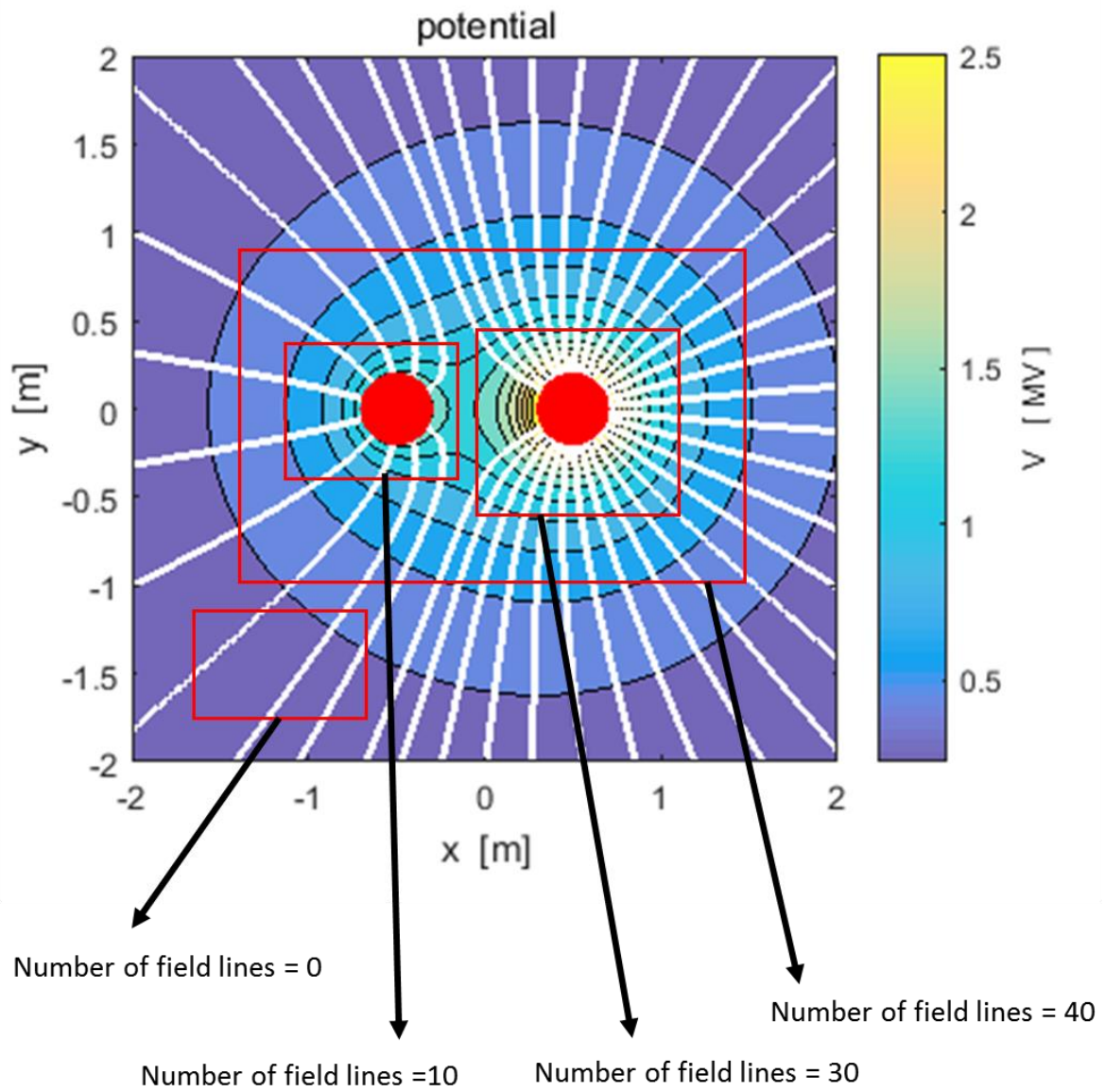
Net field lines entering or exiting box = 30 $\Rightarrow Q_{enclosed} = +60 \mu\text{C}$

(Three times the number of lines as Box 2)

Box 4 surrounds $Q_{left} + Q_{right}$:

Net field lines entering or exiting box = 40 $\Rightarrow Q_{enclosed} = +80 \mu\text{C}$

(Four times the number of lines as Box 2)



Plot created with the mscript **cemVE05.m**