DOING PHYSICS WITH MATLAB

SOLIDS OF REVOLUTION

Ian Cooper
School of Physics, University of Sydney
ian.cooper@sydney.edu.au

DOWNLOAD DIRECTORY FOR MATLAB SCRIPTS

math_vol_02.m
math_vol_03.m
math_vol_04.m
math_vol_05.m
math_vol_06.m

mscripts used to produce plots for a function which defines a bounded region in the XY plane that when rotated through 360^o about a rotation axis parallel to a coordinate axis generates a **solid of revolution**. A sequence of plots of the region rotated through increasing angles can be used to create an animated gif. [3D] plots can be produced using the Matlab functions **plot3**, **cylinder** and **surf**.

The mscripts are "crudely" written, but they do illustrate the way in which [3D] plots can be generated for the solids of revolution. For different functions and limits, you need to change the mscript in a number of parts. Also, you need to enter the code for the function twice.

simpson1d.m

The function **simpson1d.m** can be called to integrate a function to compute the volume of the solid of revolution.

SOLIDS OF REVOLUTION

Solid figures can be produced by rotating bounded regions in the XY plane through 360°. A solid generated by the rotation is called a **solid of revolution**.

We will only consider solids of revolution that are generated by rotations about axes that are parallel to the X-axis or the Y-axis (coordinates axes). Using the **plot3** Matlab function we can create an animated gif image of the rotation of a function about lines parallel to a coordinate axis. An example is shown in figure (1).

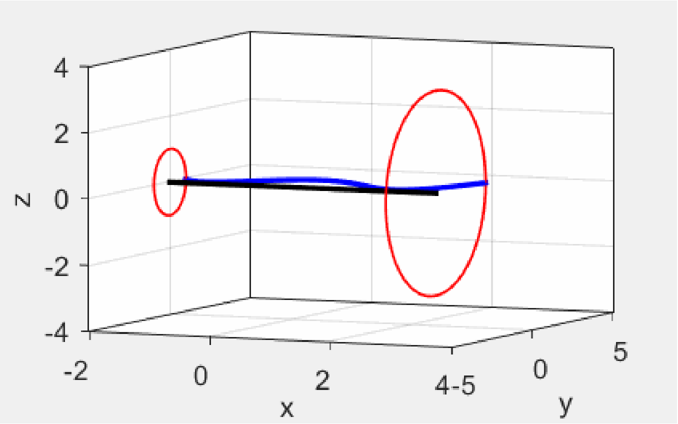

Fig. 1.
First image of the animation of a function about the X-axis to generate a solid of revolution.
math_vol_02.m

View an animation of the rotation of a function about the X-axis

ROTATIONS ABOUT THE X-AXIS ($y_R = 0$)

Let $y = f(x)$ be a single-valued continuous function where $f(x) \geq 0$ in the interval $x_a \leq x \leq x_b$. Consider the region ***R*** bounded by the function $y = f(x)$ and the X-axis ($y_R = 0$) for the interval $x_a \leq x \leq x_b$. When this region ***R*** is rotated about X-axis through the 360° rotation, a **solid of revolution** is generated. The volume V of the solid of revolution is given by

(1) $$V = \int_{x_a}^{x_b} A(x)\,dx$$ **rotation about X-axis**

The solid generated by the rotation must have a circular cross-section with radius $R(x)$. Therefore, the cross-sectional area $A(x)$ is given by

$$A(x) = \pi\, R(x)^2 \quad R(x) = y \quad A(x) = \pi\, y^2$$

The volume V of the solid of revolution is

(2) $$V = \pi \int_{x_a}^{x_b} R(x)^2\,dx = \pi \int_{x_a}^{x_b} y^2\,dx$$

disk method – rotation about X-axis

In the disk method, we sum up the volumes of an infinite number of infinitesimally thin circular disks to find the total volume of a solid. The solid has been decomposed into stacked circular disks, and by integrating the disk volumes we obtain the total volume.

EXAMPLES

To illustrate the graphical power of Matlab we can consider two and three dimensional plots of solids produced by the rotation of a function about lines parallel to a coordinate axis. As an example, we can find the volumes of the solids of revolution for the region bounded by the function $y = 2\sqrt{x}$, the X-axis and the vertical lines $x_a = 0$ and $x_b = 4$ for the following axes of rotation

(A) X-axis $y_R = 0$

(B) Y-axis $x_R = 0$

(C) $y_R = -2$

(D) $y_R = +2$ limits $x_a = 2$ and $x_b = 4$

(E) $y_R = -2$ limits $x_a = 2$ and $x_b = 4$

(A) ROTATION ABOUT THE X-AXIS $y_R = 0$

The mscript **math_vol_05.m** was used to create the following figures.

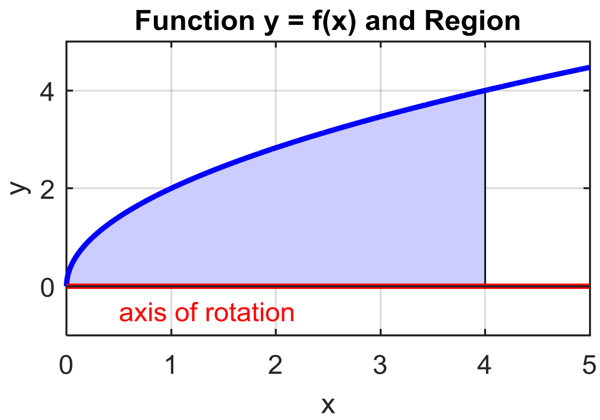

Fig. 2. A plot showing the function $y = f(x)$, the region ***R*** to be rotated and the axis of rotation.

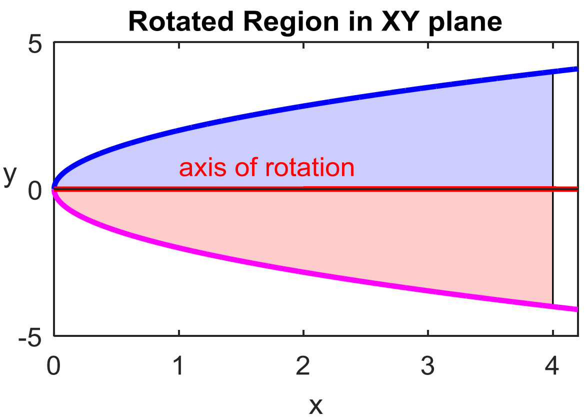

Fig. 3. A plot showing the function $y = f(x)$, the region ***R***, the region ***R*** rotated through 180^o and the axis of rotation.

Fig. 4. A [3D] plot showing the outer surface of the solid of revolution. The Matlab functions **cylinder** and **surf** were used to generate the [3D] plot.

The code for producing figure (4) in the mscript **math_vol_05.m** is

```
figure(6)    % [3D] plot ------------------------------------
set(gcf,'units','normalized','position',[0.35 0.4 0.22 0.22]);
[X,Y,Z] = cylinder(y-yR,100);
z = xA + Z.*(xB-xA);
surf(z,Y+yR,X);
axis equal
xlabel('X'); ylabel('Y'); zlabel('Z')
shading interp
box on
view(35,10);
axis off
set(gca,'Xlim',[0 4.2]);
set(gca,'Ylim',[-4.2 4.2]);
set(gca,'Zlim',[-4.2 4.2]);
```

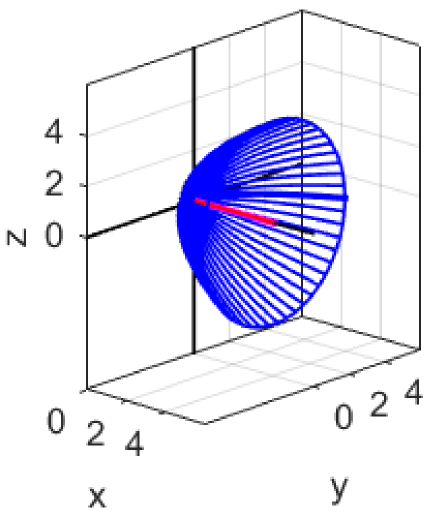

Fig. 5. A [3D] plot showing the outer surface of the solid of revolution. The Matlab function **plot3** was used to generate the [3D] plot.

The volume can be found by analytical means using equation (2). The volume of the solid of revolution about the X-axis is

(1) $$V = \pi \int_{x_a}^{x_b} y^2 \, dx$$ **Disk Method**

The limits of integration are $x_a = 0$ and $x_b = 4$

The function $y = f(x) \geq 0$ in the interval [0 4] is

$$y = 2\sqrt{x} \qquad y^2 = 4x$$

The volume of the cone is

$$V = 4\pi \int_0^4 x \, dx = 4\pi \left[\tfrac{1}{2} x^2 \right]_0^4 = 32\pi$$

An easy way to find the volume is to compute the integral numerically using [Simpson's rule](#).

```
% Volume calculation by disk method
fn = y.^2; a = xA; b = xB;
vol_pie = simpson1d(fn,a,b);

disp('volume/pie');
disp(vol_pie);
```

(B) ROTATIONS ABOUT THE Y-AXIS ($x_R = 0$)

The mscript **math_vol_05.m** was used to create the following figures.

We can also visualize the solid of revolution about the Y-axis as shown in figures (6) and (7).

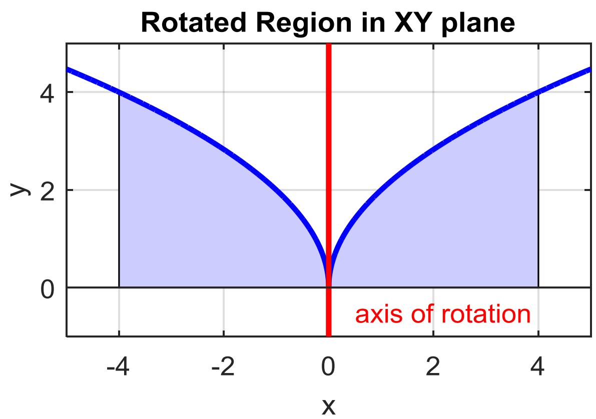

Fig. 6. A plot of the cross- section through the solid of revolution in the XY plane.

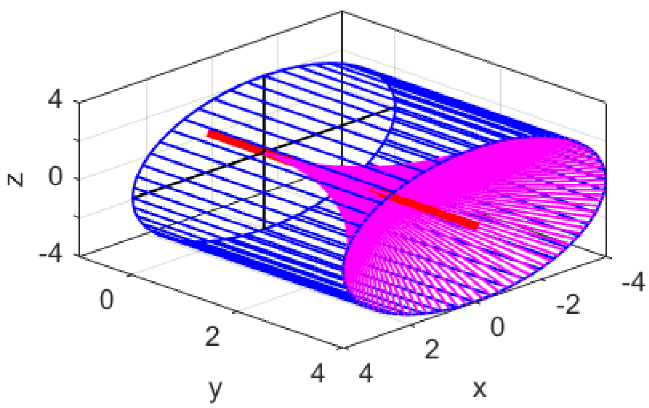

Fig. 7. A [3D] plot of the solid of revolution. The blue lines represent the outer surface of the solid and the magenta lines represent the inner surface.

(C) ROTATIONS ABOUT THE LINE y_R = -2

The mscript **math_vol_06.m** was used to create the following figures.

When the region is rotated about the line $y_R = -2$ which is parallel to the X-axis, a solid is generated with a hollow core as shown in the following figures.

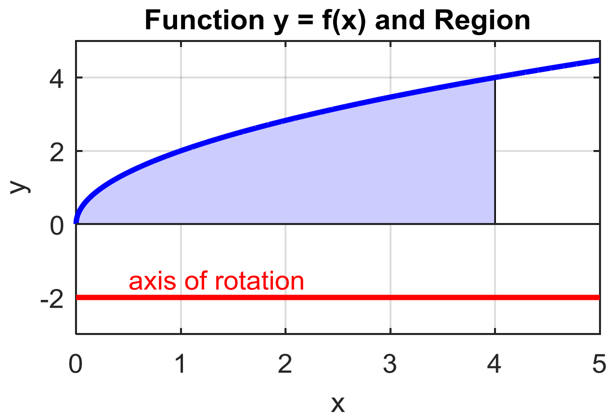

Fig. 8. A plot showing the function $y = f(x)$, the region ***R*** to be rotated and the axis of rotation.

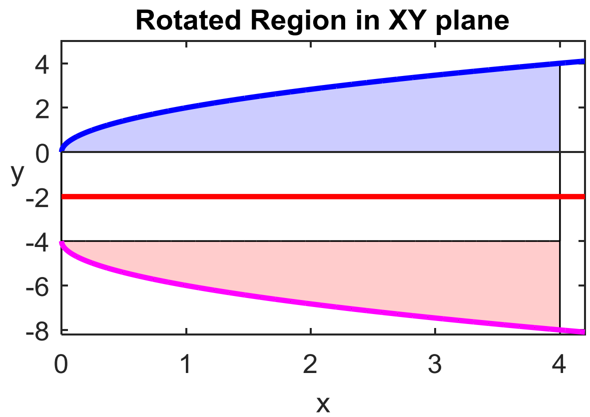

Fig. 9. A plot showing the function $y = f(x)$, the region ***R***, the region ***R*** rotated through 180° and the axis of rotation.

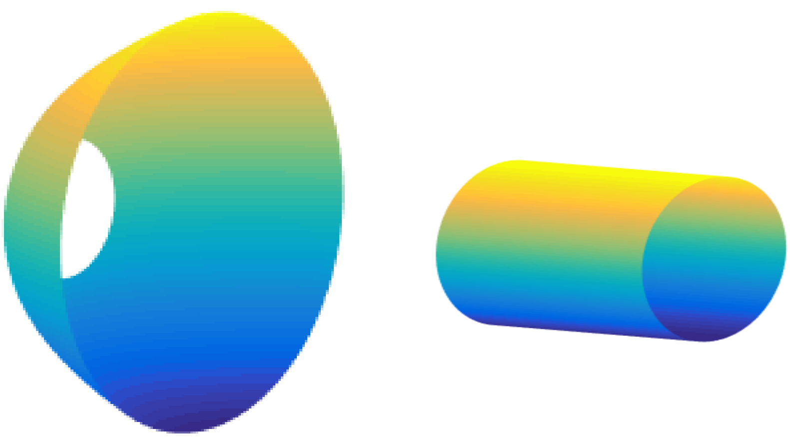

Fig. 10. A [3D] plot showing the outer surface and the inner surface of the solid of revolution.

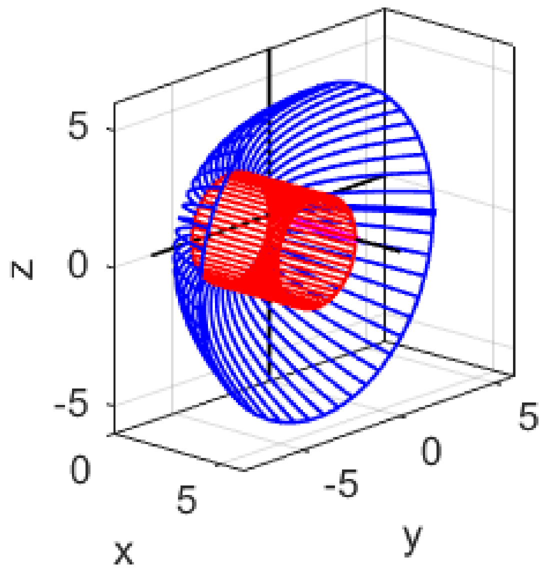

Fig. 11. A [3D] plot showing the outer surface (blue) and the inner surface (red) of the solid of revolution.

(D) ROTATIONS ABOUT THE LINE y_R = +2 x_a = 2 and x_b = 4

The mscript **math_vol_04.m** was used to create the following figures.

When the region is rotated about the line $y_R = -2$ which is parallel to the X-axis, a solid is generated with a hollow core as shown in the following figures.

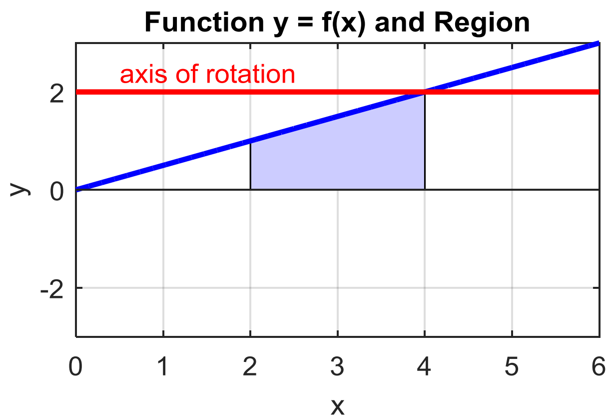

Fig. 12. A plot showing the function $y = f(x)$, the region ***R*** to be rotated and the axis of rotation.

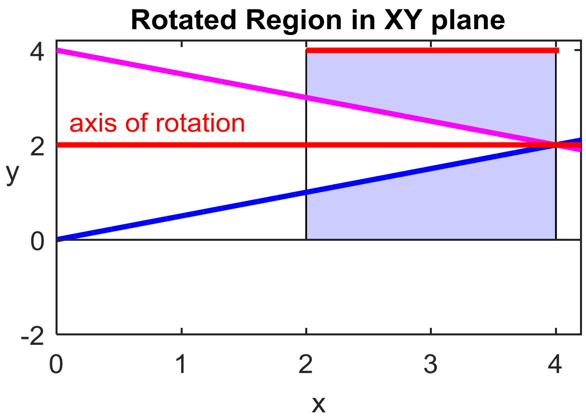

Fig. 13. A plot showing the function $y = f(x)$, the region ***R***, the region ***R*** rotated through 180^o and the axis of rotation.

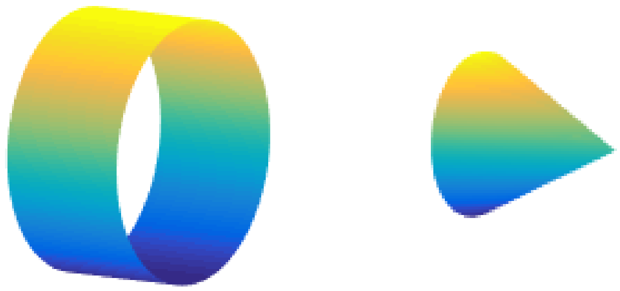

Fig. 14. A [3D] plot showing the outer surface and the inner surface of the solid of revolution.

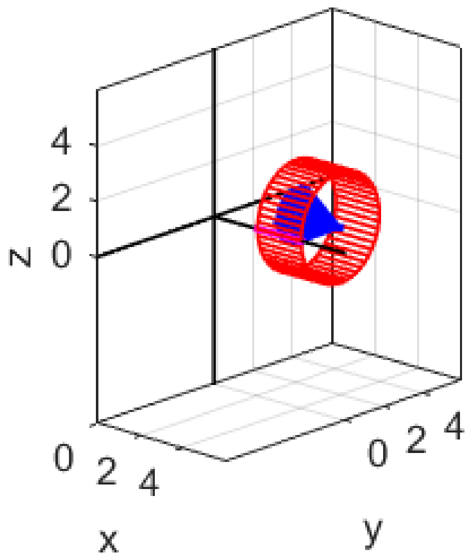

Fig. 15. A [3D] plot showing the outer surface (blue) and the inner surface (red) of the solid of revolution.

(E) ROTATIONS ABOUT THE LINE y_R = -2 x_a = 2 and x_b = 4

The mscript **math_vol_03.m** was used to create the following figures.

When the region is rotated about the line $y_R = -2$ which is parallel to the X-axis, a solid is generated with a hollow core as shown in the following figures.

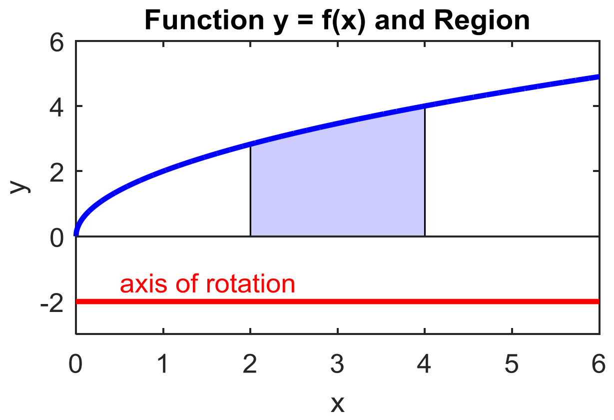

Fig. 16. A plot showing the function $y = f(x)$, the region ***R*** to be rotated and the axis of rotation.

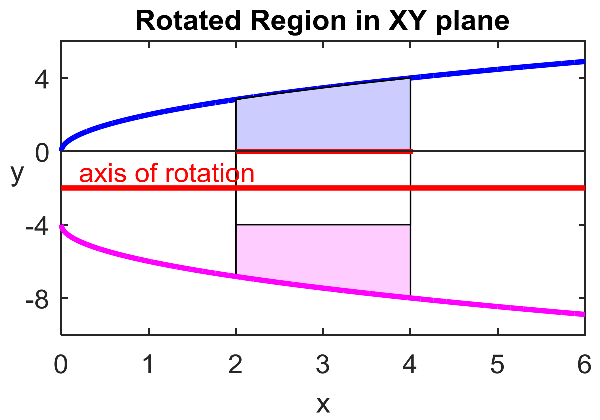

Fig. 17. A plot showing the function $y = f(x)$, the region ***R***, the region ***R*** rotated through 180^o and the axis of rotation.

Fig. 18. A [3D] plot showing the outer surface and the inner surface of the solid of revolution.

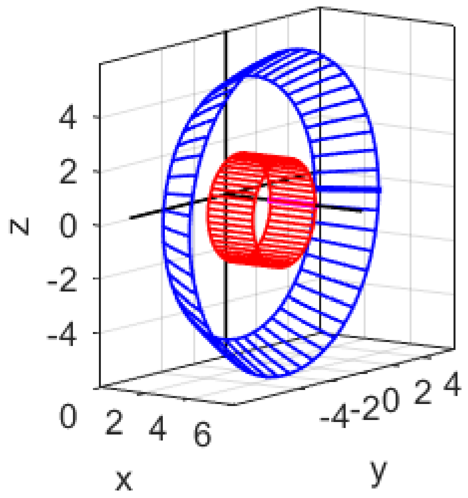

Fig. 19. A [3D] plot showing the outer surface (blue) and the inner surface (red) of the solid of revolution.