DOING PHYSICS WITH PYTHON

QUANTUM MECHANICS

WAVE PARTICLE DUALITY

WAVE-LIKE PROPERTIES OF PARTICLES

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qm022.py **Double slit diffraction: visible EMR**
Electron diffraction pattern

Diffraction for visible light (400- 700 nm) from a double slit and corresponding electron diffraction pattern. Includes the function **colour(wL)** which returns the colour appropriate to a supplied wavelength. Is it assumed the supplied lambda is within the range 380-780 nm. Smaller or higher values are set notionally to the extreme values. All input measurements are in nanometres.

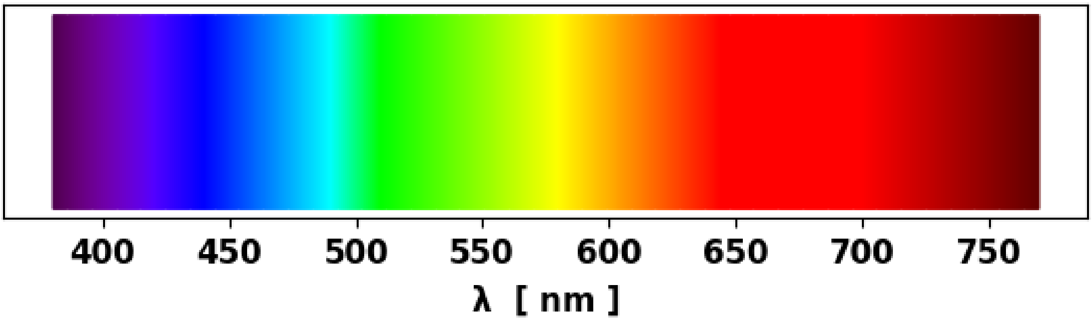


Fig. 1. Visible spectrum using the function colour(wL).

qmspectrum.py

Are electrons particles or waves ?

LIGHT BEHAVING AS WAVES

When light passes through very narrow apertures and falls on a screen, a diffraction / interference pattern consisting of a band of bright and dark regions is observed. The brightness (intensity) of light detected on the screen is proportional to the square of the amplitude of the wave. For a plane wave incident upon an aperture, we observe Fraunhofer diffraction when the screen distance is much larger than the width of the apertures.

The intensity of light reaching the screen for a **single slit** is given by the equation

(1) $$I_s = I_o \left(\frac{\sin \beta}{\beta} \right)^2$$

The intensity of light reaching the screen for a **double slit** is given by the equation

(2) $$I_d = I_o \left(\frac{\sin\beta}{\beta} \right)^2 \cos^2(\alpha) = I_s \cos^2(\alpha)$$

where

I intensity of the light [W.m^{-2}]

I_o maximum intensity [W.m^{-2}]

β $\beta = \frac{1}{2} k\, b \sin\theta$ [rad]

α $\alpha = \frac{1}{2} k\, a \sin\theta$ [rad]

k $k = \frac{2\pi}{\lambda}$ [rad.m^{-1}]

λ wavelength of light [m]

b slit width [m]

a slit separation

θ direction to point on screen from aperture [rad]

$$\theta = \frac{x}{D}$$

x position of screen [m]

D aperture screen distance [m]

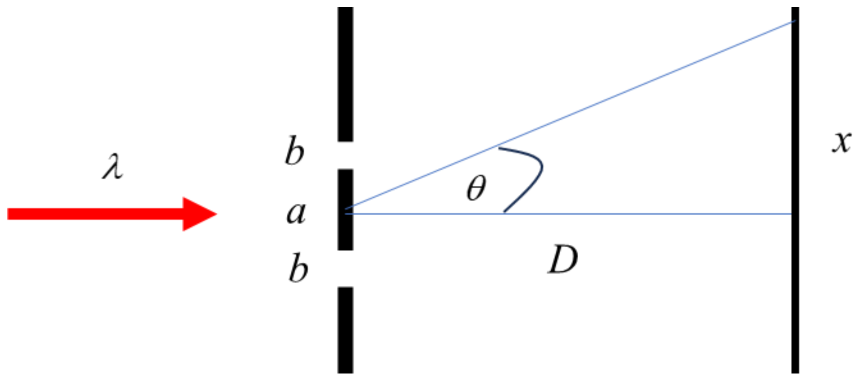


Fig. 2. Schematic diagram for a double-slit experiment.

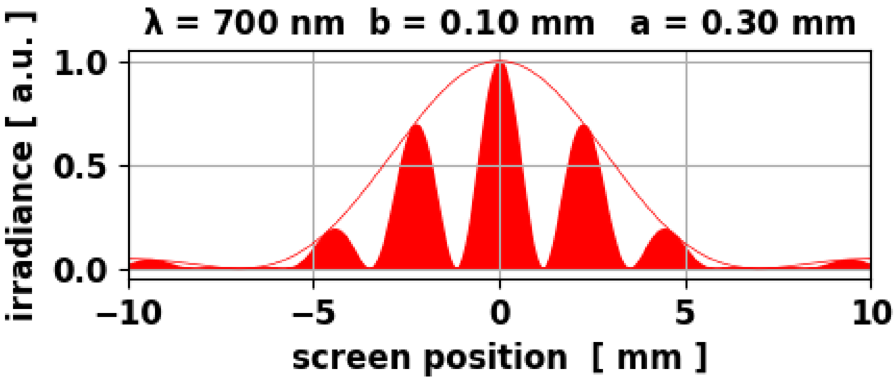
The following figures show the diffraction of visible light for the wavelengths 700, 600, 500 and 400 nm. The solid plot is the double-slit diffraction pattern and the envelope is for the single slit diffraction. Also, shown is the pattern you would observe by 50000 individual electrons sticking a detection screen. Each blue dot shows the location of an electron hitting the screen. Notice there are positions on the screen where electrons never hit.

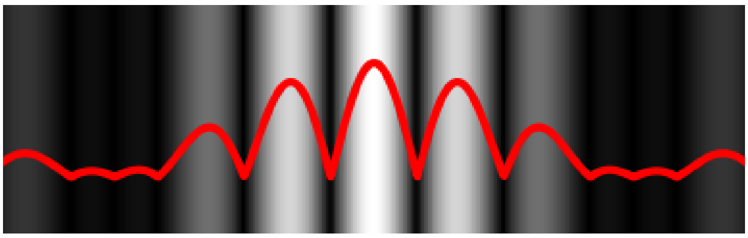
So, in experimental arrangements analogous to the two-slit interference for light, when a beam of electrons is incident upon a biprism (mimics two slits for light as the electrons can travel in two paths around a filament) and are detected upon a screen, an interference pattern is observed. We must conclude that the **electron** has **wave-like properties**. The **term matter** waves is often used to describe the wave-like properties of particles such as the electron .

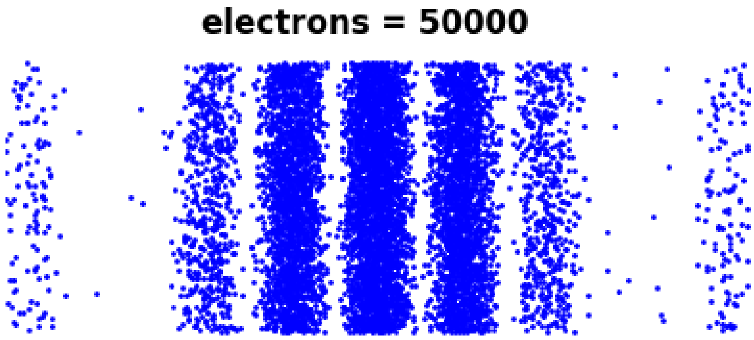
Simulation parameters

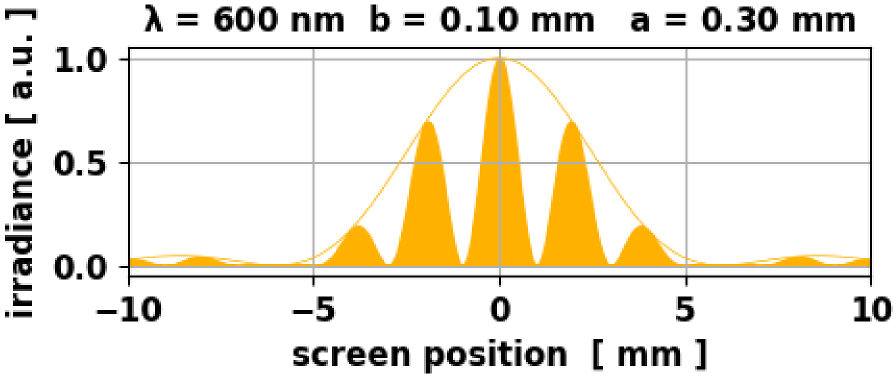
```
# Constants and Variables
wL = 700        # wavelength [ 400 - 700 nm]
b = 1e-4        # slit width  [ m ]
a = 3e-4        # slit separation [ m ]
L = 10e-3       # screen width  [ m ]
D = 1.0         # aperture to observation screen  [ m ]
N = 599         # grid points
```

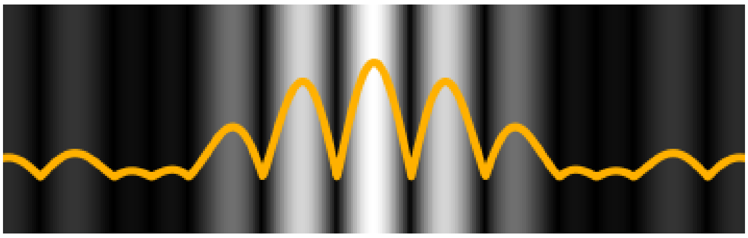
SIMULATIONS

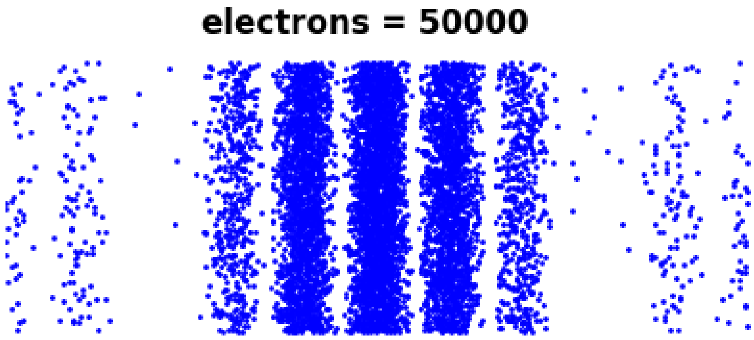


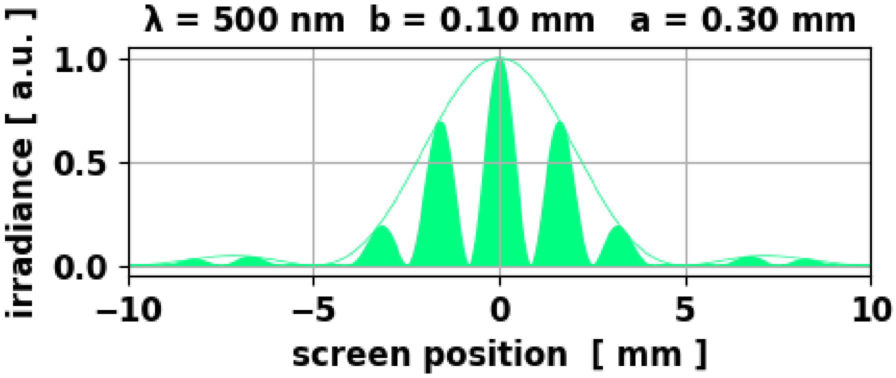


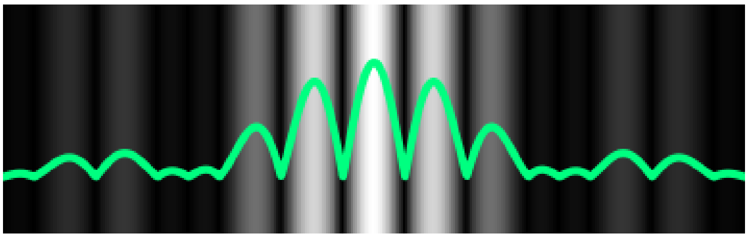


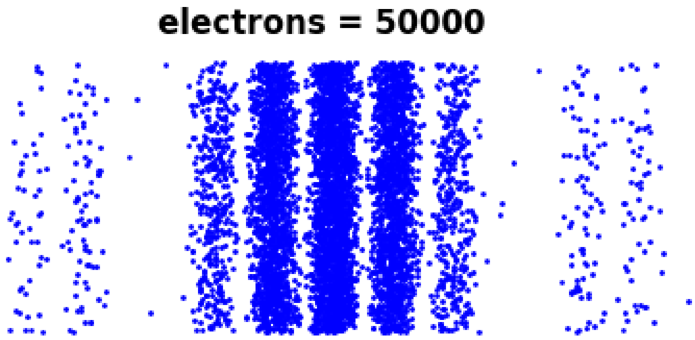

λ = 600 nm b = 0.10 mm   a = 0.30 mm
irradiance [a.u.]
1.0
0.5
0.0
−10
−5
0
5
10
screen position [mm]

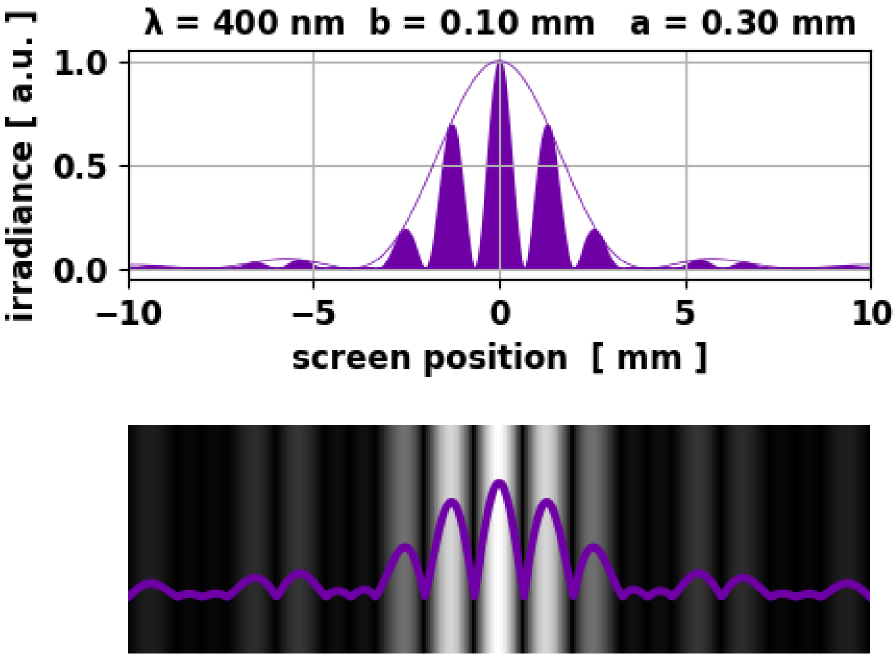


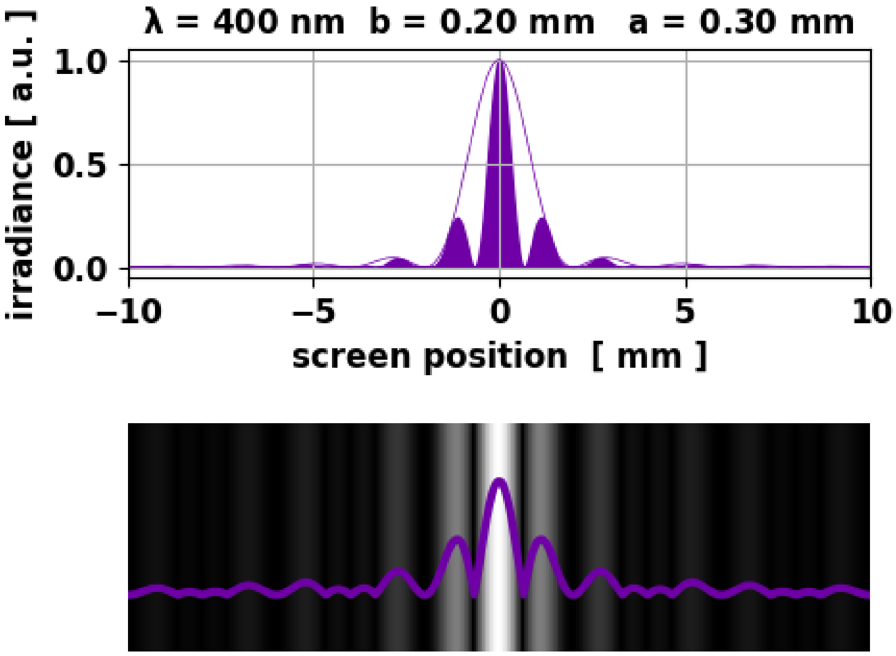

electrons = 50000


λ = 500 nm b = 0.10 mm a = 0.30 mm
irradiance [a.u.]
1.0
0.5
0.0
−10
−5
0
5
10
screen position [mm]




electrons = 50000


λ = 400 nm b = 0.10 mm   a = 0.30 mm
irradiance [a.u.]
1.0
0.5
0.0
−10
−5
0
5
10
screen position [mm]


λ = 400 nm b = 0.20 mm   a = 0.30 mm
irradiance [a.u.]
1.0
0.5
0.0
−10
−5
0
5
10
screen position [mm]

PARTICLES BEHAVING AS WAVES

The electrons are individual particles when they strike a single point on the detection screen, but the distribution of the points on the screen gives an interference pattern which can only be attributed to a wave phenomenon. Hence, we can only conclude that electrons have this dual nature – they behave as particles or as waves. We can’t predict where a single electron will arrive on the screen. We only know the probability of where an electron will strike. This behaviour is typical of the quantum world and is a good example of the interplay between indeterminism and determinism.

The electron is represented by a mathematical function called the wavefunction $\Psi(\vec{r},t)$ which is a function of the position of the electron and time. The evolution of the wavefunction for a single electron is governed by the Schrodinger’s equation. However, this wavefunction is a complex quantity and can’t be measured directly. From it we can find the probability of locating the electron at some instant. The probability density is proportional to the real quantity $\left|\Psi(\vec{r},t)\right|^2$.

We can now interpret the irradiance given by equations 1 and 2 as a probability density for the electron striking the screen and the area under the curve being proportional to the probability of finding the electron. For a one-dimensional system, the probability of finding an electron between x_1 and x_2 at time t is given by

(3) $$probability \propto \int_{x_1}^{x_2} \left|\Psi(x,t)\right|^2 dx$$

and for the two-slit example, the probability of hitting a pixel at position (x, y) on the detection screen at time t is

(4) $$probability(pixel) \propto \left|\Psi(x,y,t)\right|^2 A$$

where A is the area of the pixel.

We can't predict where a particular electron will strike the screen but the pattern formed by many electrons is predicted by the Schrodinger equation which tells how Ψ spreads out from the slit to the screen. When a single electron leaves the slits and just before it strikes the screen, its wavefunction is spread out over a wide area which would cover many pixels, but only one pixel is triggered to respond, no other pixels respond. When a single pixel is triggered, we can interpret this in terms of news spreading out instantly from the responding pixel, telling all other pixels not to respond. This is an example of **quantum non-locality** – what happens at one place affects what happens at other places in a manner that can't be explained by communication at the speed of light (maximum speed at which any information can be transmitted). We say that when the electron is detected, its **wavefunction collapses**. In terms of quantum physics, a particle is interpreted as an entity which is found in only one place when its position is measured.

For a free particle (total energy E = kinetic energy K, potential energy U = 0) its wave nature is described by its **de Broglie wavelength** λ

$$\lambda = \frac{h}{p} \qquad (5)$$

where h is Planck's constant and p is the momentum of the particle. Diffraction experiments confirm that the wavelength given by equation 5 agrees with the wavelength as measured in these experiments.

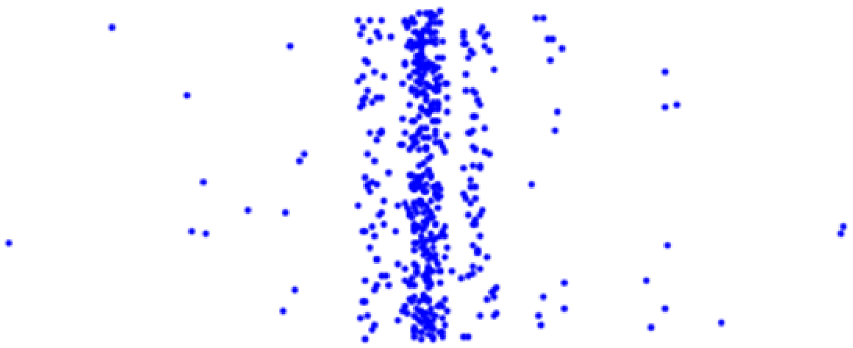
In is an easy task to modify the code **qm022.py** to animate the sequence of individual electrons hitting the detector screen.

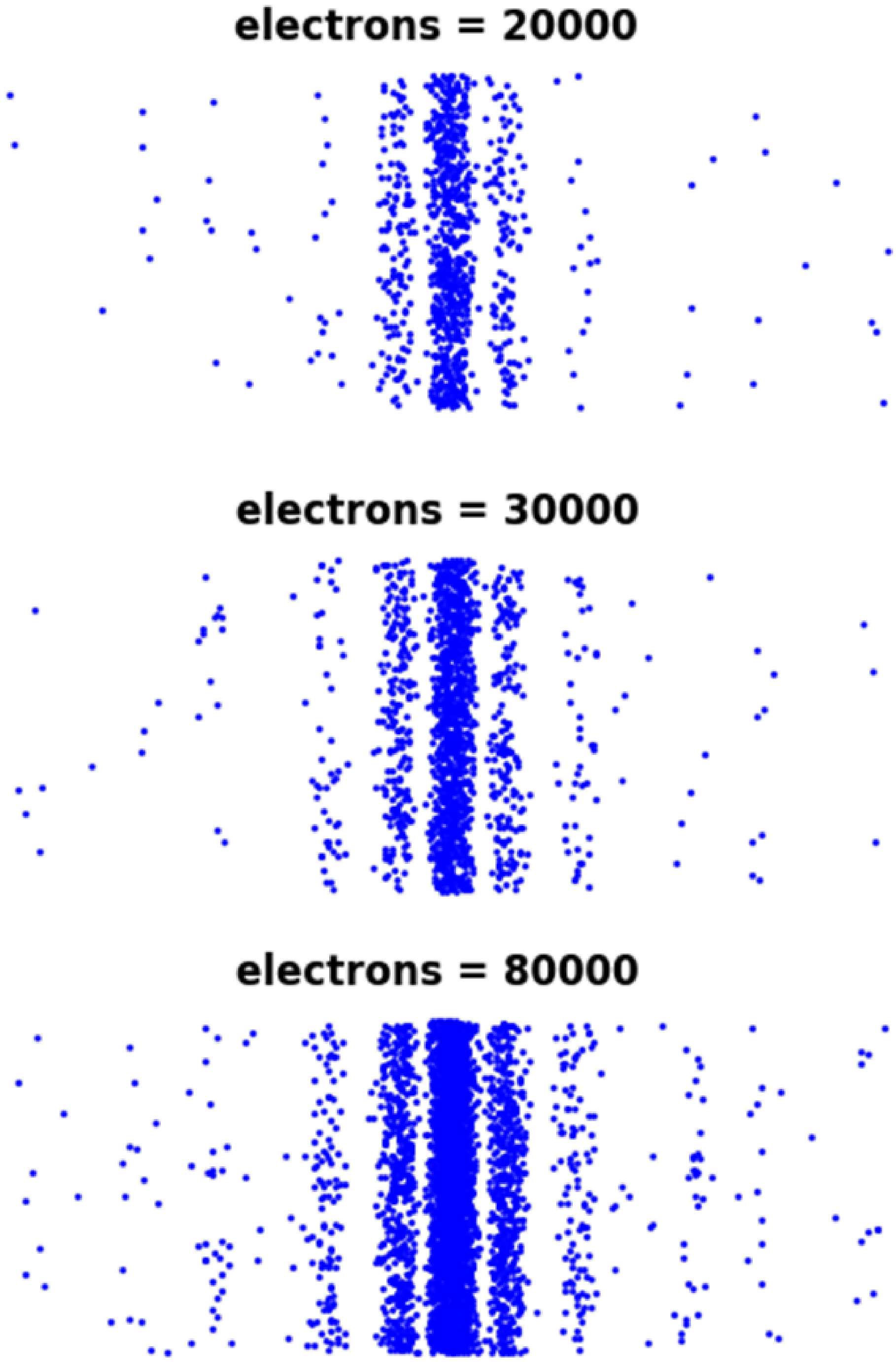
Below is a sequence of plots showing an increasing number of electrons being detected. Only after many thousands of impacts does the interference pattern immerge.

electrons = 500

electrons = 1000

electrons = 10000




electrons = 20000
electrons = 30000
electrons = 80000

The position of an electron is not known until it is measured. The electron does not spread out like the wave producing the interference pattern. The complex wavefunction gives a complete description of the electron. It is no longer sensible to think about the electron as a moving particle. $\left|\Psi(\vec{r},t)\right|^2$ tells us only the probability for finding the electron at a certain location.

The electrons propagate as waves (but not like classical waves) and are detected as particles – they display **wave-particle duality**.