



DOING PHYSICS WITH PYTHON
**TIME-DEPENDENT QUANTUM-
MECHANICAL SCATTERING IN TWO-
DIMENSIONS**

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DOWNLOAD DIRECTORY FOR PYTHON SCRIPTS

- qm2DA.py** Propagation of a free [2D] Gaussian pulse
- qm2DB.py** Potential barrier: wall $U_0 > 0$ or cliff $U_0 < 0$
- qm2DC.py** Potential barrier: single slit
- qm2DD.py** Potential barrier: double slit

- qm2DE.py** Potential barrier: Coulomb repulsion

[GitHub](#)

[Google Drive](#)

[Finite Difference Time Development method](#)

Simulation `qm2DE.py`

Rutherford Scattering from a Coulomb Potential

The FDTD method is applied to the Rutherford scattering in two dimensions. Much of what is known about atoms, nuclei and subatomic particles has been gained from the results of scattering experiments. Usually, the experiments involve a localized electron beam of incident particles at the beginning, and a measured distribution of scattered particles at the end. The [2D] simulations provide a very intuitive picture of what happens in a scattering event. We can think of our simulations modelling the scattering of alpha particles from a stationary sample of gold foil. For alpha particle scattering, the dominant interaction between an alpha particle and a stationary gold nucleus is their **Coulomb repulsion**. The collision process is three-dimensional. However, the two-dimensional modelling conveys the essential features of the processes.

The [2D] Coulomb repulsive potential is given by

$$U(r) = -U_0 / r \quad r = \sqrt{(x - 0.5)^2 + (y - 0.5)^2} \quad U_0 > 0$$

The quantum-mechanical wavepackets spreads over time, so care much be taken in the starting coordinates of the initial wave packet.

The effect of different **impact parameters** is achieved by placing the initial wavepacket at different y coordinates (y_0). The results of calculations of the time evolution of the wavepacket are shown in the following figures for impact parameters

$$b = 0 \quad b = 0.02 \quad b = 0.05 \quad b = 0.09$$

The first figure shows the results of a “head-on” collision where $b = 0$, and $y_0 = 0.5$. Throughout the collision the wavepacket remains symmetrical about the x -axis since the impact parameter b is zero.

Viewing the time evolution of the wavepacket, we observe how the wavepacket spreads and develops a curved profile. This is due to the leading edge of the wavepacket being reflected by the scattering center and interfering with the rest of the wavepacket resulting in the severe distortion. The spreading is a result that the propagation of the wavepacket is determined by the range in values for the x and y components of the propagation constant k .

When $b \neq 0$ the wavepacket spreads asymmetrically and looks to be “squeezing” past the scattering center because of the strong distortion experienced by the repulsive Coulomb wall when it approaches the scattering center. The larger the impact parameter b , the overall distortion is less. For large impact parameters, the wavepacket is more deflected than scattered as there is less interference between the incoming and outgoing portions of the wavepacket.









